25.4.20

$$\int_{x_0}^{x_{10}} f(x)dx=\frac{5h}{299376}\{16067(f_0+f_{10})$$
$$+106300(f_1+f_9)-48525(f_2+f_8)+272400(f_3+f_7)$$
$$-260550(f_4+f_6)+427368f_5\}$$
$$-\frac{1346350}{326918592}f^{(12)}(\xi)h^{13}$$

Newton-Cotes Formulas (Open Type)

25.4.21

$$\int_{x_0}^{x_3} f(x)dx=\frac{3h}{2}(f_1+f_2)+\frac{f^{(2)}(\xi)h^3}{4}$$

25.4.22

$$\int_{x_0}^{x_4} f(x)dx=\frac{4h}{3}(2f_1-f_2+2f_3)+\frac{28f^{(4)}(\xi)h^5}{90}$$

25.4.23

$$\int_{x_0}^{x_5} f(x)dx=\frac{5h}{24}(11f_1+f_2+f_3+11f_4)+\frac{95f^{(4)}(\xi)h^5}{144}$$

25.4.24

$$\int_{x_0}^{x_6} f(x)dx=\frac{6h}{20}(11f_1-14f_2+26f_3-14f_4+11f_5)$$
$$+\frac{41f^{(6)}(\xi)h^7}{140}$$

25.4.25

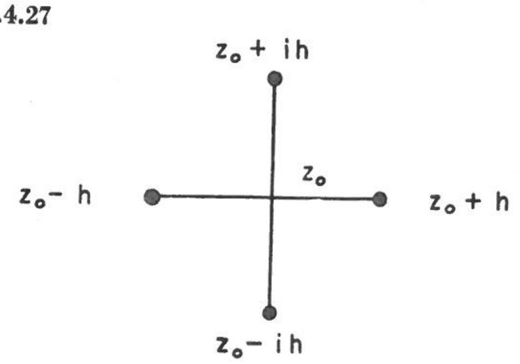
$$\int_{x_0}^{x_7} f(x)dx=\frac{7h}{1440}(611f_1-453f_2+562f_3+562f_4$$
$$-453f_5+611f_6)+\frac{5257}{8640}f^{(6)}(\xi)h^7$$

25.4.26

$$\int_{x_0}^{x_8} f(x)dx=\frac{8h}{945}(460f_1-954f_2+2196f_3-2459f_4$$
$$+2196f_5-954f_6+460f_7)+\frac{3956}{14175}f^{(8)}(\xi)h^9$$

Five Point Rule for Analytic Functions

25.4.27



$$\int_{z_0-h}^{z_0+h} f(z)dz=\frac{h}{15}\{24f(z_0)+4[f(z_0+h)+f(z_0-h)]$$
$$-[f(z_0+ih)+f(z_0-ih)]\}+R$$

$|R|\leq\frac{|h|^7}{1890}\operatorname*{Max}_{z\in S}|f^{(6)}(z)|$, S designates the square with vertices $z_0+i^kh(k=0,1,2,3)$; h can be complex.

Chebyshev's Equal Weight Integration Formula

25.4.28 $$\int_{-1}^{1} f(x)dx=\frac{2}{n}\sum_{i=1}^{n} f(x_i)+R_n$$

Abscissas: x_i is the i^{th} zero of the polynomial part of

$$x^n\exp\left[\frac{-n}{2\cdot 3x^2}-\frac{n}{4\cdot 5x^3}-\frac{n}{6\cdot 7x^4}-\ldots\right]$$

(See **Table 25.5** for x_i.)

For $n=8$ and $n\geq 10$ some of the zeros are complex.
Remainder:

$$R_n=\int_{-1}^{+1}\frac{x^{n+1}}{(n+1)!}f^{(n+1)}(\xi)dx$$
$$-\frac{2}{n(n+1)!}\sum_{i=1}^{n} x_i^{n+1}f^{(n+1)}(\xi_i)$$

where $\xi=\xi(x)$ satisfies $0\leq\xi\leq x$ and $0\leq\xi_i\leq x_i$

$$(i=1,\ldots,n)$$

Integration Formulas of Gaussian Type

(For Orthogonal Polynomials see chapter **22**)

Gauss' Formula

25.4.29 $$\int_{-1}^{1} f(x)dx=\sum_{i=1}^{n} w_i f(x_i)+R_n$$

Related orthogonal polynomials: Legendre polynomials $P_n(x)$, $P_n(1)=1$

Abscissas: x_i is the i^{th} zero of $P_n(x)$

Weights: $w_i=2/(1-x_i^2)[P_n'(x_i)]^2$ *
(See **Table 25.4** for x_i and w_i.)

$$R_n=\frac{2^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3}f^{(2n)}(\xi)\qquad(-1<\xi<1)$$

Gauss' Formula, Arbitrary Interval

25.4.30 $$\int_a^b f(y)dy=\frac{b-a}{2}\sum_{i=1}^{n} w_i f(y_i)+R_n$$

$$y_i=\left(\frac{b-a}{2}\right)x_i+\left(\frac{b+a}{2}\right)$$

*See page II.

Related orthogonal polynomials: $P_n(x)$, $P_n(1)=1$
Abscissas: x_i is the i^{th} zero of $P_n(x)$
* Weights: $w_i=2/(1-x_i^2)\,[P_n'(x_i)]^2$

* $$R_n=\frac{(b-a)^{2n+1}(n!)^4}{(2n+1)[(2n)!]^3}f^{(2n)}(\xi)$$

Radau's Integration Formula

25.4.31

$$\int_{-1}^{1} f(x)dx=\frac{2}{n^2}f_{-1}+\sum_{i=1}^{n-1} w_i f(x_i)+R_n$$

Related polynomials:

$$\frac{P_{n-1}(x)+P_n(x)}{x+1}$$

Abscissas: x_i is the i^{th} zero of

$$\frac{P_{n-1}(x)+P_n(x)}{x+1}$$

Weights:

$$w_i=\frac{1}{n^2}\frac{1-x_i}{[P_{n-1}(x_i)]^2}=\frac{1}{1-x_i}\frac{1}{[P'_{n-1}(x_i)]^2}$$

Remainder:

$$R_n=\frac{2^{2n-1}\cdot n}{[(2n-1)!]^3}[(n-1)!]^4 f^{(2n-1)}(\xi) \qquad (-1<\xi<1)$$

Lobatto's Integration Formula

25.4.32

$$\int_{-1}^{1} f(x)dx=\frac{2}{n(n-1)}[f(1)+f(-1)]+\sum_{i=2}^{n-1} w_i f(x_i)+R_n$$

Related polynomials: $P'_{n-1}(x)$

Abscissas: x_i is the $(i-1)^{\text{st}}$ zero of $P'_{n-1}(x)$

Weights:

$$w_i=\frac{2}{n(n-1)[P_{n-1}(x_i)]^2} \qquad (x_i\neq\pm 1)$$

(See **Table 25.6** for x_i and w_i.)

Remainder:

$$R_n=\frac{-n(n-1)^3 2^{2n-1}[(n-2)!]^4}{(2n-1)[(2n-2)!]^3}f^{(2n-2)}(\xi) \qquad (-1<\xi<1)$$

*See page II.

25.4.33 $$\int_0^1 x^k f(x)dx=\sum_{i=1}^{n} w_i f(x_i)+R_n$$

Related orthogonal polynomials:

$$q_n(x)=\sqrt{k+2n+1}\,P_n^{(k,0)}(1-2x)$$

(For the Jacobi polynomials $P_n^{(k,0)}$ see chapter **22.**)

Abscissas:

x_i is the i^{th} zero of $q_n(x)$

Weights:

$$w_i=\left\{\sum_{j=0}^{n-1}[q_j(x_i)]^2\right\}^{-1}$$ *

(See **Table 25.8** for x_i and w_i.)

Remainder:

$$R_n=\frac{f^{(2n)}(\xi)}{(k+2n+1)(2n)!}\left[\frac{n!(k+n)!}{(k+2n)!}\right]^2 \qquad (0<\xi<1)$$

25.4.34

$$\int_0^1 f(x)\sqrt{1-x}\,dx=\sum_{i=1}^{n} w_i f(x_i)+R_n$$

Related orthogonal polynomials:

$$\frac{1}{\sqrt{1-x}}P_{2n+1}(\sqrt{1-x}),\ P_{2n+1}(1)=1$$

Abscissas: $x_i=1-\xi_i^2$ where ξ_i is the i^{th} positive zero of $P_{2n+1}(x)$.
Weights: $w_i=2\xi_i^2 w_i^{(2n+1)}$ where $w_i^{(2n+1)}$ are the Gaussian weights of order $2n+1$.
Remainder:

$$R_n=\frac{2^{4n+3}[(2n+1)!]^4}{(2n)!(4n+3)[(4n+2)!]^2}f^{(2n)}(\xi) \qquad (0<\xi<1)$$

25.4.35

$$\int_a^b f(y)\sqrt{b-y}\,dy=(b-a)^{3/2}\sum_{i=1}^{n} w_i f(y_i)$$

$$y_i=a+(b-a)x_i$$

Related orthogonal polynomials:

$$\frac{1}{\sqrt{1-x}}P_{2n+1}(\sqrt{1-x}),\ P_{2n+1}(1)=1$$

Abscissas: $x_i=1-\xi_i^2$ where ξ_i is the i^{th} positive zero of $P_{2n+1}(x)$.
Weights: $w_i=2\xi_i^2 w_i^{(2n+1)}$ where $w_i^{(2n+1)}$ are the Gaussian weights of order $2n+1$.

Table 25.4 ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

Abscissas$=\pm x_i$ (Zeros of Legendre Polynomials) Weight Factors$=w_i$

$\pm x_i$	w_i
$n=2$	
0.57735 02691 89626	1.00000 00000 00000
$n=3$	
0.00000 00000 00000	0.88888 88888 88889
0.77459 66692 41483	0.55555 55555 55556
$n=4$	
0.33998 10435 84856	0.65214 51548 62546
0.86113 63115 94053	0.34785 48451 37454
$n=5$	
0.00000 00000 00000	0.56888 88888 88889
0.53846 93101 05683	0.47862 86704 99366
0.90617 98459 38664	0.23692 68850 56189
$n=6$	
0.23861 91860 83197	0.46791 39345 72691
0.66120 93864 66265	0.36076 15730 48139
0.93246 95142 03152	0.17132 44923 79170
$n=7$	
0.00000 00000 00000	0.41795 91836 73469
0.40584 51513 77397	0.38183 00505 05119
0.74153 11855 99394	0.27970 53914 89277
0.94910 79123 42759	0.12948 49661 68870

$\pm x_i$	w_i
$n=8$	
0.18343 46424 95650	0.36268 37833 78362
0.52553 24099 16329	0.31370 66458 77887
0.79666 64774 13627	0.22238 10344 53374
0.96028 98564 97536	0.10122 85362 90376
$n=9$	
0.00000 00000 00000	0.33023 93550 01260
0.32425 34234 03809	0.31234 70770 40003
0.61337 14327 00590	0.26061 06964 02935
0.83603 11073 26636	0.18064 81606 94857
0.96816 02395 07626	0.08127 43883 61574
$n=10$	
0.14887 43389 81631	0.29552 42247 14753
0.43339 53941 29247	0.26926 67193 09996
0.67940 95682 99024	0.21908 63625 15982
0.86506 33666 88985	0.14945 13491 50581
0.97390 65285 17172	0.06667 13443 08688
$n=12$	
0.12523 34085 11469	0.24914 70458 13403
0.36783 14989 98180	0.23349 25365 38355
0.58731 79542 86617	0.20316 74267 23066
0.76990 26741 94305	0.16007 83285 43346
0.90411 72563 70475	0.10693 93259 95318
0.98156 06342 46719	0.04717 53363 86512

$\pm x_i$	w_i
$n=16$	
0.09501 25098 37637 440185	0.18945 06104 55068 496285
0.28160 35507 79258 913230	0.18260 34150 44923 588867
0.45801 67776 57227 386342	0.16915 65193 95002 538189
0.61787 62444 02643 748447	0.14959 59888 16576 732081
0.75540 44083 55003 033895	0.12462 89712 55533 872052
0.86563 12023 87831 743880	0.09515 85116 82492 784810
0.94457 50230 73232 576078	0.06225 35239 38647 892863
0.98940 09349 91649 932596	0.02715 24594 11754 094852
$n=20$	
0.07652 65211 33497 333755	0.15275 33871 30725 850698
0.22778 58511 41645 078080	0.14917 29864 72603 746788
0.37370 60887 15419 560673	0.14209 61093 18382 051329
0.51086 70019 50827 098004	0.13168 86384 49176 626898
0.63605 36807 26515 025453	0.11819 45319 61518 417312
0.74633 19064 60150 792614	0.10193 01198 17240 435037
0.83911 69718 22218 823395	0.08327 67415 76704 748725
0.91223 44282 51325 905868	0.06267 20483 34109 063570
0.96397 19272 77913 791268	0.04060 14298 00386 941331
0.99312 85991 85094 924786	0.01761 40071 39152 118312
$n=24$	
0.06405 68928 62605 626085	0.12793 81953 46752 156974
0.19111 88674 73616 309159	0.12583 74563 46828 296121
0.31504 26796 96163 374387	0.12167 04729 27803 391204
0.43379 35076 26045 138487	0.11550 56680 53725 601353
0.54542 14713 88839 535658	0.10744 42701 15965 634783
0.64809 36519 36975 569252	0.09761 86521 04113 888270
0.74012 41915 78554 364244	0.08619 01615 31953 275917
0.82000 19859 73902 921954	0.07334 64814 11080 305734
0.88641 55270 04401 034213	0.05929 85849 15436 780746
0.93827 45520 02732 758524	0.04427 74388 17419 806169
0.97472 85559 71309 498198	0.02853 13886 28933 663181
0.99518 72199 97021 360180	0.01234 12297 99987 199547

Compiled from P. Davis and P. Rabinowitz, Abscissas and weights for Gaussian quadratures of high order, J. Research NBS **56,** 35–37, 1956, RP2645; P. Davis and P. Rabinowitz, Additional abscissas and weights for Gaussian quadratures of high order. Values for $n=64$, 80, and 96, J. Research NBS **60,** 613–614, 1958, RP2875; and A. N. Lowan, N. Davids, and A. Levenson, Table of the zeros of the Legendre polynomials of order 1–16 and the weight coefficients for Gauss' mechanical quadrature formula, Bull. Amer. Math. Soc. **48,** 739–743, 1942 (with permission).

Table 25.4

ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

Abscissas $= \pm x_i$ (Zeros of Legendre Polynomials) Weight Factors $= w_i$

$\pm x_i$	w_i
$n=32$	
0.04830 76656 87738 316235	0.09654 00885 14727 800567
0.14447 19615 82796 493485	0.09563 87200 79274 859419
0.23928 73622 52137 074545	0.09384 43990 80804 565639
0.33186 86022 82127 649780	0.09117 38786 95763 884713
0.42135 12761 30635 345364	0.08765 20930 04403 811143
0.50689 99089 32229 390024	0.08331 19242 26946 755222
0.58771 57572 40762 329041	0.07819 38957 87070 306472
0.66304 42669 30215 200975	0.07234 57941 08848 506225
0.73218 21187 40289 680387	0.06582 22227 76361 846838
0.79448 37959 67942 406963	0.05868 40934 78535 547145
0.84936 76137 32569 970134	0.05099 80592 62376 176196
0.89632 11557 66052 123965	0.04283 58980 22226 680657
0.93490 60759 37739 689171	0.03427 38629 13021 433103
0.96476 22555 87506 430774	0.02539 20653 09262 059456
0.98561 15115 45268 335400	0.01627 43947 30905 670605
0.99726 38618 49481 563545	0.00701 86100 09470 096600
$n=40$	
0.03877 24175 06050 821933	0.07750 59479 78424 811264
0.11608 40706 75255 208483	0.07703 98181 64247 965588
0.19269 75807 01371 099716	0.07611 03619 00626 242372
0.26815 21850 07253 681141	0.07472 31690 57968 264200
0.34199 40908 25758 473007	0.07288 65823 95804 059061
0.41377 92043 71605 001525	0.07061 16473 91286 779695
0.48307 58016 86178 712909	0.06791 20458 15233 903826
0.54946 71250 95128 202076	0.06480 40134 56601 038075
0.61255 38896 67980 237953	0.06130 62424 92928 939167
0.67195 66846 14179 548379	0.05743 97690 99391 551367
0.72731 82551 89927 103281	0.05322 78469 83936 824355
0.77830 56514 26519 387695	0.04869 58076 35072 232061
0.82461 22308 33311 663196	0.04387 09081 85673 271992
0.86595 95032 12259 503821	0.03878 21679 74472 017640
0.90209 88069 68874 296728	0.03346 01952 82547 847393
0.93281 28082 78676 533361	0.02793 70069 80023 401098
0.95791 68192 13791 655805	0.02224 58491 94166 957262
0.97725 99499 83774 262663	0.01642 10583 81907 888713
0.99072 62386 99457 006453	0.01049 82845 31152 813615
0.99823 77097 10559 200350	0.00452 12770 98533 191258
$n=48$	
0.03238 01709 62869 362033	0.06473 76968 12683 922503
0.09700 46992 09462 698930	0.06446 61644 35950 082207
0.16122 23560 68891 718056	0.06392 42385 84648 186624
0.22476 37903 94689 061225	0.06311 41922 86254 025657
0.28736 24873 55455 576736	0.06203 94231 59892 663904
0.34875 58862 92160 738160	0.06070 44391 65893 880053
0.40868 64819 90716 729916	0.05911 48396 98395 635746
0.46690 29047 50958 404545	0.05727 72921 00403 215705
0.52316 09747 22233 033678	0.05519 95036 99984 162868
0.57722 47260 83972 703818	0.05289 01894 85193 667096
0.62886 73967 76513 623995	0.05035 90355 53854 474958
0.67787 23796 32663 905212	0.04761 66584 92490 474826
0.72403 41309 23814 654674	0.04467 45608 56694 280419
0.76715 90325 15740 339254	0.04154 50829 43464 749214
0.80706 62040 29442 627083	0.03824 13510 65830 706317
0.84358 82616 24393 530711	0.03477 72225 64770 438893
0.87657 20202 74247 885906	0.03116 72278 32798 088902
0.90587 91367 15569 672822	0.02742 65097 08356 948200
0.93138 66907 06554 333114	0.02357 07608 39324 379141
0.95298 77031 60430 860723	0.01961 61604 57355 527814
0.97059 15925 46247 250461	0.01557 93157 22943 848728
0.98412 45837 22826 857745	0.01147 72345 79234 539490
0.99353 01722 66350 757548	0.00732 75539 01276 262102
0.99877 10072 52426 118601	0.00315 33460 52305 838633

Table 25.4

ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

Abscissas $= \pm x_i$ (Zeros of Legendre Polynomials) Weight Factors $= w_i$

$n=64$

$\pm x_i$	w_i
0.02435 02926 63424 432509	0.04869 09570 09139 720383
0.07299 31217 87799 039450	0.04857 54674 41503 426935
0.12146 28192 96120 554470	0.04834 47622 34802 957170
0.16964 44204 23992 818037	0.04799 93885 96458 307728
0.21742 36437 40007 084150	0.04754 01657 14830 308662
0.26468 71622 08767 416374	0.04696 81828 16210 017325
0.31132 28719 90210 956158	0.04628 47965 81314 417296
0.35722 01583 37668 115950	0.04549 16279 27418 144480
0.40227 01579 63991 603696	0.04459 05581 63756 563060
0.44636 60172 53464 087985	0.04358 37245 29323 453377
0.48940 31457 07052 957479	0.04247 35151 23653 589007
0.53127 94640 19894 545658	0.04126 25632 42623 528610
0.57189 56462 02634 034284	0.03995 37411 32720 341387
0.61115 53551 72393 250249	0.03855 01531 78615 629129
0.64896 54712 54657 339858	0.03705 51285 40240 046040
0.68523 63130 54233 242564	0.03547 22132 56882 383811
0.71988 18501 71610 826849	0.03380 51618 37141 609392
0.75281 99072 60531 896612	0.03205 79283 54851 553585
0.78397 23589 43341 407610	0.03023 46570 72402 478868
0.81326 53151 22797 559742	0.02833 96726 14259 483228
0.84062 92962 52580 362752	0.02637 74697 15054 658672
0.86599 93981 54092 819761	0.02435 27025 68710 873338
0.88931 54459 95114 105853	0.02227 01738 08383 254159
0.91052 21370 78502 805756	0.02013 48231 53530 209372
0.92956 91721 31939 575821	0.01795 17157 75697 343085
0.94641 13748 58402 816062	0.01572 60304 76024 719322
0.96100 87996 52053 718919	0.01346 30478 96718 642598
0.97332 68277 89910 963742	0.01116 81394 60131 128819
0.98333 62538 84625 956931	0.00884 67598 26363 947723
0.99101 33714 76744 320739	0.00650 44579 68978 362856
0.99634 01167 71955 279347	0.00414 70332 60562 467635
0.99930 50417 35772 139457	0.00178 32807 21696 432947

$n=80$

$\pm x_i$	w_i
0.01951 13832 56793 997654	0.03901 78136 56306 654811
0.05850 44371 52420 668629	0.03895 83959 62769 531199
0.09740 83984 41584 599063	0.03883 96510 59051 968932
0.13616 40228 09143 886559	0.03866 17597 74076 463327
0.17471 22918 32646 812559	0.03842 49930 06959 423185
0.21299 45028 57666 132572	0.03812 97113 14477 638344
0.25095 23583 92272 120493	0.03777 63643 62001 397490
0.28852 80548 84511 853109	0.03736 54902 38730 490027
0.32566 43707 47701 914619	0.03689 77146 38276 008839
0.36230 47534 99487 315619	0.03637 37499 05835 978044
0.39839 34058 81969 227024	0.03579 43939 53416 054603
0.43387 53708 31756 093062	0.03516 05290 44747 593496
0.46869 66151 70544 477036	0.03447 31204 51753 928794
0.50280 41118 88784 987594	0.03373 32149 84611 522817
0.53614 59208 97131 932020	0.03294 19393 97645 401383
0.56867 12681 22709 784725	0.03210 04986 73487 773148
0.60033 06228 29751 743155	0.03121 01741 88114 701642
0.63107 57730 46871 966248	0.03027 23217 59557 980661
0.66085 98989 86119 801736	0.02928 83695 83267 847693
0.68963 76443 42027 600771	0.02825 98160 57276 862397
0.71736 51853 62099 880254	0.02718 82275 00486 380674
0.74400 02975 83597 272317	0.02607 52357 67565 117903
0.76950 24201 35041 373866	0.02492 25357 64115 491105
0.79383 27175 04605 449949	0.02373 18828 65930 101293
0.81695 41386 81463 470371	0.02250 50902 46332 461926
0.83883 14735 80255 275617	0.02124 40261 15782 006389
0.85943 14066 63111 096977	0.01995 06108 78141 998929
0.87872 25676 78213 828704	0.01862 68142 08299 031429
0.89667 55794 38770 683194	0.01727 46520 56269 306359
0.91326 31025 71757 654165	0.01589 61835 83725 688045
0.92845 98771 72445 795953	0.01449 35080 40509 076117
0.94224 27613 09872 674752	0.01306 87615 92401 339294
0.95459 07663 43634 905493	0.01162 41141 20797 826916
0.96548 50890 43799 251452	0.01016 17660 41103 064521
0.97490 91405 85727 793386	0.00868 39452 69260 858426
0.98284 85727 38629 070418	0.00719 29047 68117 312753
0.98929 13024 99755 531027	0.00569 09224 51403 198649
0.99422 75409 65688 277892	0.00418 03131 24694 895237
0.99764 98643 98237 688900	0.00266 35335 89512 681669
0.99955 38226 51630 629880	0.00114 49500 03186 941534

Table 25.4

ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION

$$\int_{-1}^{+1} f(x)dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

Abscissas $=\pm x_i$ (Zeros of Legendre Polynomials) Weight Factors $=w_i$

$n=96$

$\pm x_i$	w_i
0.01627 67448 49602 969579	0.03255 06144 92363 166242
0.04881 29851 36049 731112	0.03251 61187 13868 835987
0.08129 74954 64425 558994	0.03244 71637 14064 269364
0.11369 58501 10665 920911	0.03234 38225 68575 928429
0.14597 37146 54896 941989	0.03220 62047 94030 250669
0.17809 68823 67618 602759	0.03203 44562 31992 663218
0.21003 13104 60567 203603	0.03182 87588 94411 006535
0.24174 31561 63840 012328	0.03158 93307 70727 168558
0.27319 88125 91049 141487	0.03131 64255 96861 355813
0.30436 49443 54496 353024	0.03101 03325 86313 837423
0.33520 85228 92625 422616	0.03067 13761 23669 149014
0.36569 68614 72313 635031	0.03029 99154 20827 593794
0.39579 76498 28908 603285	0.02989 63441 36328 385984
0.42547 89884 07300 545365	0.02946 10899 58167 905970
0.45470 94221 67743 008636	0.02899 46141 50555 236543
0.48345 79739 20596 359768	0.02849 74110 65085 385646
0.51169 41771 54667 673586	0.02797 00076 16848 334440
0.53938 81083 24357 436227	0.02741 29627 26029 242823
0.56651 04185 61397 168404	0.02682 68667 25591 762198
0.59303 23647 77572 080684	0.02621 23407 35672 413913
0.61892 58401 25468 570386	0.02557 00360 05349 361499
0.64416 34037 84967 106798	0.02490 06332 22483 610288
0.66871 83100 43916 153953	0.02420 48417 92364 691282
0.69256 45366 42171 561344	0.02348 33990 85926 219842
0.71567 68123 48967 626225	0.02273 70696 58329 374001
0.73803 06437 44400 132851	0.02196 66444 38744 349195
0.75960 23411 76647 498703	0.02117 29398 92191 298988
0.78036 90438 67433 217604	0.02035 67971 54333 324595
0.80030 87441 39140 817229	0.01951 90811 40145 022410
0.81940 03107 37931 675539	0.01866 06796 27411 467385
0.83762 35112 28187 121494	0.01778 25023 16045 260838
0.85495 90334 34601 455463	0.01688 54798 64245 172450
0.87138 85059 09296 502874	0.01597 05629 02562 291381
0.88689 45174 02420 416057	0.01503 87210 26994 938006
0.90146 06353 15852 341319	0.01409 09417 72314 860916
0.91507 14231 20898 074206	0.01312 82295 66961 572637
0.92771 24567 22308 690965	0.01215 16046 71088 319635
0.93937 03397 52755 216932	0.01116 21020 99838 498591
0.95003 27177 84437 635756	0.01016 07705 35008 415758
0.95968 82914 48742 539300	0.00914 86712 30783 386633
0.96832 68284 63264 212174	0.00812 68769 25698 759217
0.97593 91745 85136 466453	0.00709 64707 91153 865269
0.98251 72635 63014 677447	0.00605 85455 04235 961683
0.98805 41263 29623 799481	0.00501 42027 42927 517693
0.99254 39003 23762 624572	0.00396 45543 38444 686674
0.99598 18429 87209 290650	0.00291 07318 17934 946408
0.99836 43758 63181 677724	0.00185 39607 88946 921732
0.99968 95038 83230 766828	0.00079 67920 65552 012429