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Section 1

MANE 3351

Subsection 1

Lecture 11, September 27

Classroom Management

Agenda

- Update on Raspberry Pi VM
- GitHub Accounts
  - Sign up for free student GitHub account at https://education.github.com/students
  - Register your student GitHub account with MANE 3351 GitHub classroom email
- Lecture
- No lab today!

Subsection 2

Resources

Handouts

- Lecture 11 Slides
- Lecture 11 Marked Slides

Calendar

Lecture	Date	Content
1	August 23	Welcome to Class, Syllabus
2	August 25	Error Analysis
3	August 30	Introduction to Jupyter Notebook
4	September 1	Markdown
5	September 6	Labor Day Holiday
6	September 8	Taylor Polynomials, Homework 1
7	September 13	Roots of Equations, Bisection Method
8	September 15	Bisection Error Analysis
9	September 20	False Position Algorithm
10	September 22	Newton Raphson Algorithm
11	September 27	Secant Method

Assignments

- Homework 3 (assigned 9/22/2021, due 9/30/2021)

Looking Ahead

Lecture	Date	Content
12	September 29	Numerical Integration, Newton-Cotes
13	October 4	Simpson's Rule
14	October 6	Romberg Integration
15	October 11	Gaussian Quadrature (end of material for Test 1)
16	October 13	Vectors
17	October 18	Vector Operations
18	October 20	Test 1

Lecture 11, September 27

Today's topic is the secant method. The secant method does not utilize derivative information as Newton's method does. The secant method is also similar to the false position method.

Geometric Inspiration

Cheney and Kincaid (2004)[a] demonstrate the geometric inspiration for secant method

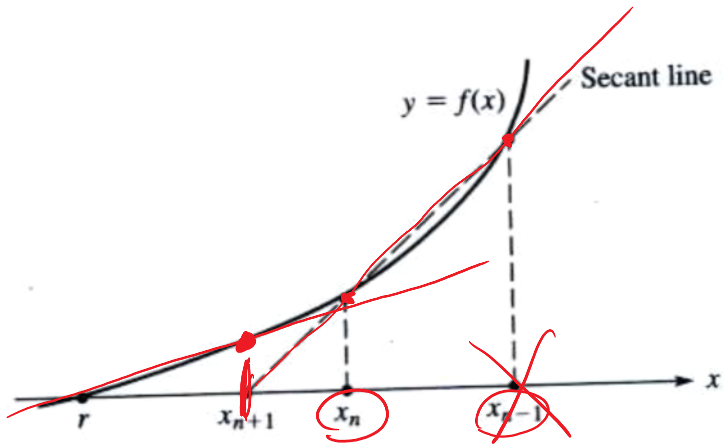


FIGURE 3.6
Secant method

Figure 1: Secant method

[a]Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*

Secant Method

- The formula is simply

$$x_{n+1} = x_n - g(x_n)\frac{x_n - x_{n-1}}{g(x_n) - g(x_{n-1})}$$

$\frac{\Delta x}{\Delta y} \to$ slope

Example Problem Used for Newton's Method

- Consider a new function, $f(x) = e^x + 2^{-x} + 2\cos(x) - 6 = 0$

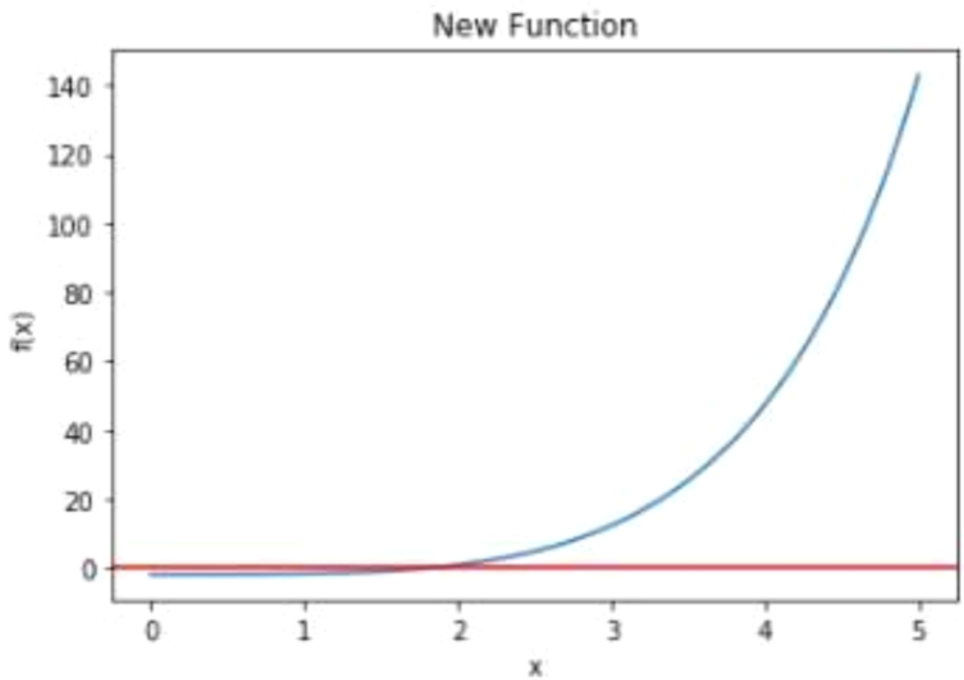


Figure 2: Example Function for Newton's method

Pseudo-code

Brin (2020)[a] provides the following pseudo-code

Secant Method (pseudo-code)

A straightforward implementation of the secant method can easily be inefficient due to the number of times g appears in formula on page 67. The pseudo-code below takes great care not to compute each value of g more than once. If it seems more complicated than necessary, this is likely the source of the complication.

Assumptions: g has a root at $\hat{x}$. g is differentiable in a neighborhood of $\hat{x}$. x_0 and x_1 are sufficiently close to $\hat{x}$.

Input: Initial values x_0 and x_1; function g; desired accuracy tol; maximum number of iterations N.

Step 1: Set $y_0 = g(x_0)$; $y_1 = g(x_1)$

Step 2: For $j = 1 \ldots N$ do Steps 3-5:

- **Step 3:** Set $x = x_1 - y_1 \frac{x_1 - x_0}{y_1 - y_0}$;
- **Step 4:** If $|x - x_1| \leq tol$ then return x;
- **Step 5:** Set $x_0 = x_1$; $y_0 = y_1$; $x_1 = x$; $y_1 = g(x_1)$

Step 6: Print "Method failed. Maximum iterations exceeded."

Output: Approximation x near exact fixed point, or message of failure.

Figure 3: Secant Method pseud-ocode

[a] Brin, L, (2020), *Tea Time Numerical Analysis: Experiences in Mathematics, 3rd edition*

Convergence

- Brin (2020)[a] study the performance of Secant Method
- The secant method converges with order $\frac{1+\sqrt{5}}{2} = 1.62$
- Not quite as fast as Newton's Method (quadratic)

[a] Brin, L, (2020), *Tea Time Numerical Analysis: Experiences in Mathematics, 3rd edition*

Similarity to False Position

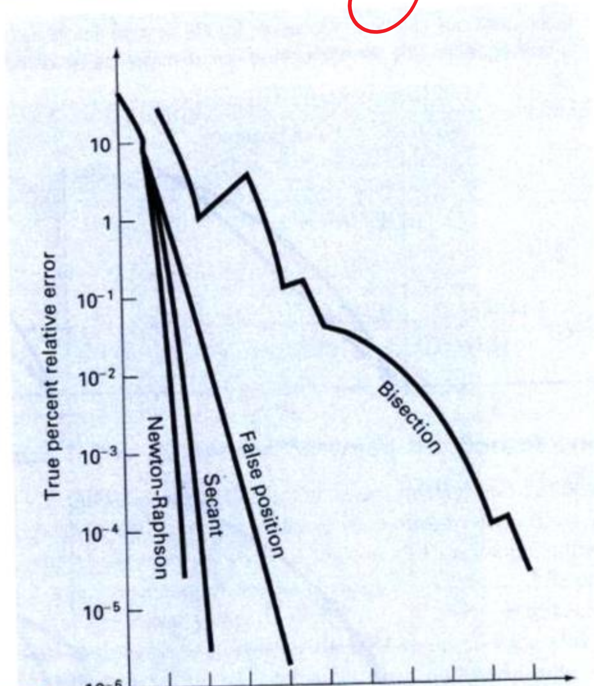
- Secant method: $x_{n+1} = x_n - g(x_n)\frac{x_n - x_{n-1}}{g(x_n) - g(x_{n-1})}$
- False Position method: $x_r = x_u - \frac{f(x_u)(x_l - x_u)}{f(x_l) - f(x_u)}$
- Secant method is not guaranteed to bracket result when starting

Speed of Convergence

Chapra and Canale (2015)[a] compared the performance of various root finding methods

Not ~~ex~~ a general result

FIGURE 6.9
Comparison of the true percent relative errors ε_t for the methods to determine the roots of $f(x) = e^{-x} - x$.



Jupyter Notebook Example

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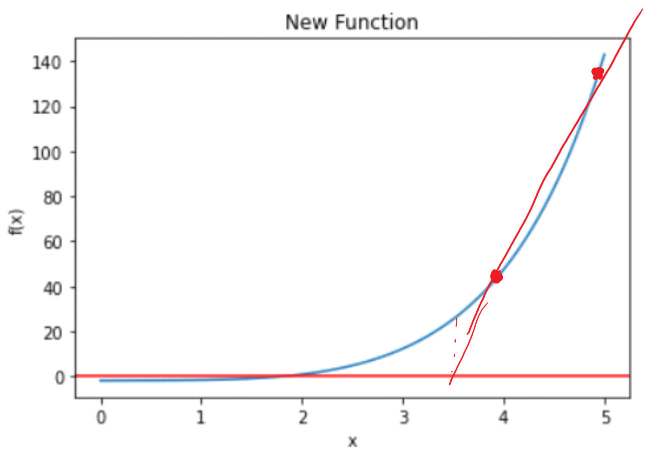

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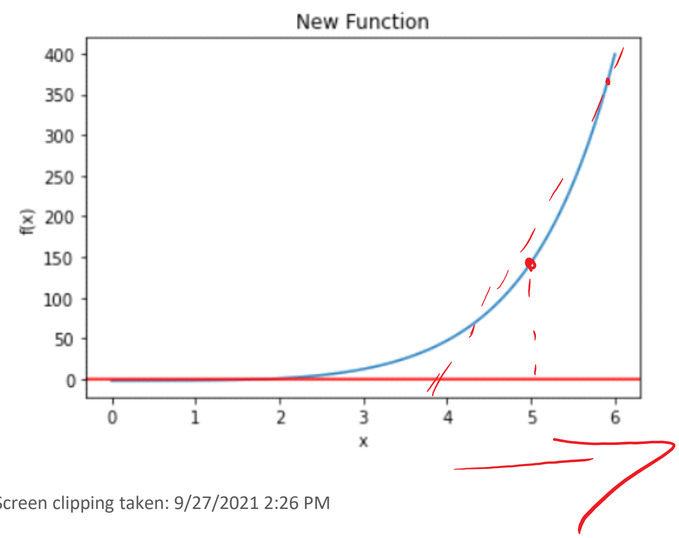

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