Section 1

MANE 3351

Subsection 1

Lecture 12, September 29

Classroom Management

Agenda

- Lab today in ENGR 2.268 at 3:30 pm
- GitHub Accounts
  - Sign up for free student GitHub account at https://education.github.com/students
  - Register your student GitHub account with MANE 3351 GitHub classroom email
- Lecture

Subsection 2

Resources

Handouts

- Lecture 12 Slides
- Lecture 12 Marked Slides

Calendar

Lecture	Date	Content
1	August 23	Welcome to Class, Syllabus
2	August 25	Error Analysis
3	August 30	Introduction to Jupyter Notebook
4	September 1	Markdown
5	September 6	Labor Day Holiday
6	September 8	Taylor Polynomials, Homework 1
7	September 13	Roots of Equations, Bisection Method
8	September 15	Bisection Error Analysis
9	September 20	False Position Algorithm
10	September 22	Newton Raphson Algorithm
11	September 27	Secant Method
12	September 29	Numerical Integration

Assignments

- Homework 3 (assigned 9/22/2021, due 9/30/2021)

Looking Ahead

Lecture	Date	Content
13	October 4	Simpson's Rule
14	October 6	Romberg Integration
15	October 11	Gaussian Quadrature (end of material for Test 1)
16	October 13	Vectors
17	October 18	Vector Operations
18	October 20	Test 1

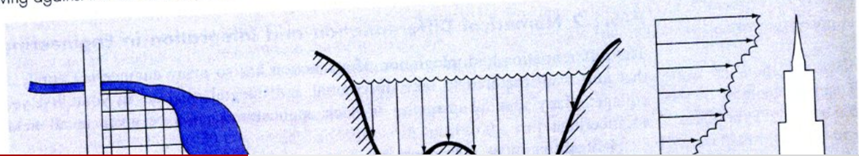
Lecture 12, September 29

Today's topic is numerical integration. This is a major new topic after root finding.

Introduction

- In layman's terms, an integral calculates the area under a curve
- Frequently used in engineering analysis

FIGURE PT6.8
Examples of how integration is used to evaluate areas in engineering applications. (*a*) A surveyor might need to know the area of a field bounded by a meandering stream and two roads. (*b*) A water-resource engineer might need to know the cross-sectional area of a river. (*c*) A structural engineer might need to determine the net force due to a nonuniform wind blowing against the side of a skyscraper.



In electrical field theory, it is proved that the magnetic field induced by a current flowing in a circular loop of wire has intensity

$$H(x) = \frac{4Ir}{r^2 - x^2} \int_0^{\pi/2} \left[1 - \left(\frac{x}{r} \right)^2 \sin^2 \theta \right]^{1/2} d\theta$$

where I is the current, r the radius of the loop, and x the distance from the center to the point where the magnetic intensity is being computed ($0 \leqq x \leqq r$). If I, r, and x are given, we have a nasty integral to evaluate. It is an **elliptic integral** and not expressible in terms of familiar functions. But H *can* be computed precisely by the methods of this chapter. For example, if $I = 15.3$, $r = 120$, and $x = 84$, we find $H = 1.35566\,1135$ accurate to nine decimals.

Figure 2: Another Integration Example

Definitions

Cheney and Kincaid (2004)[a] provide the following definitions

- **Indefinite integral :** $\int x^2 \, dx = \frac{1}{3}x^3 + C$
- **Definite integral:** $\int x^2 \, dx = \frac{8}{3}$

---

[a]Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*

Numerical Integration

Kiusalaas (2013)[a] suggest three major approaches to numerical integration that we will investigate:

1. Newton-Cotes
   a. Trapezoid rule (n=1)
   b. Simpson's rule (n=2)
   c. 3/8 Simpson's rule (n=3)
2. Romberg Integration
3. Gaussian Quadrature

Note: there are many different techniques for numerical integration than the ones listed above

[a] Kiusalaas, J. (2013), *Numerical Methods in Engineering with Python 3*

Newton-Cotes Formulas

Kiusalass (2013)[a] provide the following illustration to explain Newton-Cotes techniques

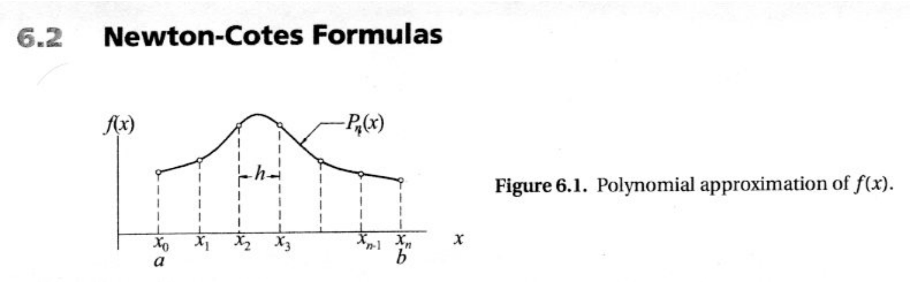


Figure 3: Newton Cotes Approach

[a]Kiusalaas, J. (2013), *Numerical Methods in Engineering with Python 3*

Trapezoid Rule

Chapra and Canale (2015)[a] provide the figure shown below illustrating the trapezoid rule

FIGURE 21.4
Graphical depiction of the trapezoidal rule.

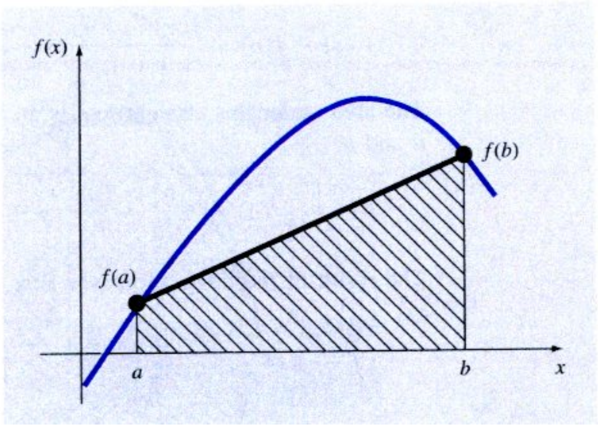


Figure 4: Trapezoid Rule

Trapezoid Rule, continued

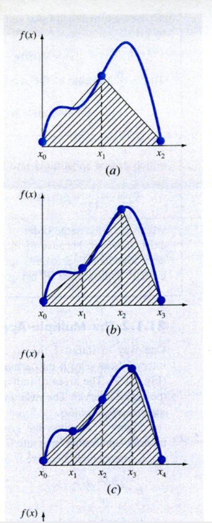
Chapra and Canale (2015)[a] provided the following formulae

- $I = (b-a)\frac{f(a)+f(b)}{2}$
- $E = -\frac{1}{12}f''(\xi)(b-a)^3$

[a] Chapra, S., and Canale, R., (2015), *Numerical Methods for Engineers, 7th edition*

Multiple Applications of the Trapezoid Rule

Typically, the region form a to b is sub-divided into multiple regions and then the Trapezoid Rule for each region is applied. Chapra and Canale (2015)[a] illustrate this concept.



Uniform Spacing

Cheney and Kincaid (2004)[a] the following formula for composite (multiple) applications of the Trapezoid Rule

$$\int_a^b = f(x)dx \approx T(f;P) = h\left\{\sum_{i=1}^{n-1} f\left(x_i\right) + \frac{1}{2}\left[f\left(x_0\right) + f\left(x_n\right)\right]\right\}$$

[a]Cheny, W., and Kincaid, D., (2004), *Numerical Mathematics and Computer, 5th edition*

Pseudo-code

Cheney and Kincaid (2004)[a] provided the following pseudo-code for the composite trapezoid rule

program *Trapezoid*
integer parameter $n \leftarrow 60$
real parameter $a \leftarrow 0,\ b \leftarrow 1$
integer i
real h, sum, x
$h \leftarrow (b - a)/n$
$sum \leftarrow \frac{1}{2}[f(a) + f(b)]$
for $i = 1$ **to** $n - 1$ **do**
  $x \leftarrow a + ih$
  $sum \leftarrow sum + f(x)$
end for
$sum \leftarrow (sum)h$
output *sum*
end *Trapezoid*

Figure 6: Trapezoid Rule Pseudo-code

Jupyter Notebook Demonstration