

$$\begin{aligned}Ax &= b \\ A'Ax &= A'b \\ I_x &= A'b \\ x &= A^{-1}b\end{aligned}$$

$$\begin{aligned}y &= X\beta \\ \bar{x}'y &= (\bar{x}'x)\beta \\ \bar{x}'y &= I\beta \\ \bar{x}'y &= \beta\end{aligned}$$

$$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 8 & 7 \end{pmatrix} =$$

$$35-48 = -13 \quad 28-54 = -26 \quad 32-45 = -13$$

$$14-24 = -10 \quad 12-9(8) = -20 \quad 18-9(2) = -10$$

$$12-12 = 0 \quad 6-12 = -6 \quad 5-8 = -3$$

Matrix of minors

$$= \begin{pmatrix} -13 & -26 & -13 \\ -10 & -20 & -10 \\ 0 & -6 & -3 \end{pmatrix}$$

Matrix of Cofactors

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

$$\begin{pmatrix} -13 & 26 & -13 \\ 10 & -20 & 10 \\ -3 & 6 & 3 \end{pmatrix}$$

matrix of minors after applying matrix of Cofactors

$$\begin{pmatrix} -13 & 10 & -3 \\ 26 & -20 & 6 \\ -13 & 10 & -3 \end{pmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 9 & 8 & 7 & 9 & 8 \end{vmatrix}$$

$$= 1(5)7 + 2(6)9 + 3(4)8 - [9(5)3 + 8(6)1 + 7(4)2]$$

$$= 239 - [135 + 48 + 56]$$

$$= 239 - 239 = 0!$$

Final Step

$$\frac{1}{|A|} \begin{pmatrix} -13 & 10 & -3 \\ 24 & -20 & 6 \\ -13 & 10 & -3 \end{pmatrix}$$

for a Matrix:

$$\begin{array}{c} [A \mid I] \\ \text{"Elementary Row Operations"} \\ [I \mid A^{-1}] \end{array}$$

$$\begin{array}{c} \begin{bmatrix} 1 & 2 & | & 1 & 0 \\ 15 & -6 & | & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 1 & 0 & | & A^{-1} \\ 0 & 1 & | & \end{bmatrix} \end{array}$$

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But we can only do these **"Elementary Row Operations"**:

- **swap** rows
- **multiply** or divide each element in a row by a constant
- replace a row by **adding** or subtracting a multiple of another row to it

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$$\left[\begin{array}{ccc|ccc} 5 & 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array} \right] \xrightarrow{\text{Add}}$$

Add row 2 to row 1,

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$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.2 & 0.2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0.2 & -0.3 & 0 \end{array} \right] \xrightarrow{R_2 - R_3} \text{Swap}$$

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$$\begin{array}{cccccc} 1 & 0 & 0 & .2 & .2 & 0 \\ 0 & 1 & 0 & -.2 & .3 & 1 \\ 0 & 0 & 1 & 0.2 & -.3 & 0 \end{array}$$