

Test 1 – MANE 3351
Manufacturing Engineering Analysis
November 4, 2019

1. This examination is closed book and contains 6 pages,
2. You are provided a formula sheet needed to work this exam and this exam
3. You will need a calculator,
4. You have 75 minutes to complete this exam,
5. The points are clearly labelled,
6. Good Luck!

Name: _____

SID: _____

1. (20 points) **Gaussian Quadrature**

Use 3-point Gaussian Quadrature to evaluate

$$\int_1^3 e^{-\sin^2 x} dx$$

. Use the truncated Gaussian points and weights shown below.

Table 25.4 **ABSCISSAS AND WEIGHT FACTORS FOR GAUSSIAN INTEGRATION**

$$\int_{-1}^{+1} f(x) dx \approx \sum_{i=1}^n w_i f(x_i)$$

Abcissas= $\pm x_i$ (Zeros of Legendre Polynomials)			Weight Factors= w_i		
$\pm x_i$		w_i	$\pm x_i$		w_i
$n=2$			$n=8$		
0.57735	02691	89626	1.00000	00000	00000
$n=3$			0.18343	46424	95650
0.00000			0.52553	24099	16329
0.77459			0.79666	64774	13627
$n=4$			0.96028	98564	97536
			$n=9$		
			0.00000	00000	00000
			0.33023	93550	01260
			0.81234	70770	40003

Note that $\sin^2 x = \frac{1}{2}[1 - \cos(2x)]$ and make sure to work in radians (not in degrees).

2. (20 points) **Romberg Integration**

Answer the following questions regarding Romberg integration.

- (a) (16 points) Compute two rows of Romberg's algorithm to evaluate

$$\int_1^2 \left(x + \frac{1}{x} \right)^2 dx.$$

Clearly indicate your answer.

- (b) (2 points) If the exact answer is 2.19314718, calculate the absolute and percentage error for your best estimate from part a.

- (c) (2 points) Name the two numerical algorithms that Romberg Integration is based upon.

3. (20 points) **Newton-Raphson**

- (a) (16 points) Calculate three iterations of the Newton-Raphson algorithm to find the root of $f(x) = 2x^3 - 11.7x^2 + 17.7X - 5$ starting at $x_0 = 3$.

- (b) (2 points) Name the main advantage and main disadvantage of the Newton-Raphson algorithm.

- (c) (2 points) Suggest another root finding algorithm with performance similar to the Newton-Raphson method that does not require knowledge of the derivative.

4. (20 points) **False Position**

Calculate four iterations of the false position method to find the root of

$$f(x) = x^{10} - 1$$

between $x = 0$ and 1.3 .

5. (20 points) **Bisection Method**

- (a) (16 points) Calculate four iterations of the bisection method find the root of

$$f(x) = x^4 - 8x^3 - 35x^2 + 50x - 1001$$

with starting values of $a = 4.5$ and $b = 6.0$.

- (b) (4 points) Name one advantage and one disadvantage of the bisection method.