In practice, 2^{k-p} fractional factorial designs with $N = 4, 8, 16$, and 32 runs are highly useful. Table 8.28 summarizes these designs, identifying how many factors can be used with each design to obtain various types of screening experiments. For example, the 16-run design is a full factorial for 4 factors, a one-half fraction for 5 factors, a resolution IV fraction for 6 to 8 factors, and a resolution III fraction for 9 to 15 factors. All of these designs may be constructed using the methods discussed in this chapter, and many of their alias structures are shown in Appendix Table VIII.

8.10 Problems

8.1 Suppose that in the chemical process development experiment described in Problem 6.11, it was only possible to run a one-half fraction of the 2^4 design. Construct the design and perform the statistical analysis, using the data from replicate I.

8.2 Suppose that in Problem 6.19, only a one-half fraction of the 2^4 design could be run. Construct the design and perform the analysis, using the data from replicate I.

8.3 Consider the plasma etch experiment described in Example 6.1. Suppose that only a one-half fraction of the design could be run. Set up the design and analyze the data.

8.4 Problem 6.30 describes a process improvement study in the manufacturing process of an integrated circuit. Suppose that only eight runs could be made in this process. Set up an appropriate 2^{5-2} design and find the alias structure. Use the appropriate observations from Problem 6.28 as the observations in this design and estimate the factor effects. What conclusions can you draw?

8.5 ***Continuation of Problem 8.4.*** Suppose you have made the eight runs in the 2^{5-2} design in Problem 8.4. What additional runs would be required to identify the factor effects that are of interest? What are the alias relationships in the combined design?

8.6 In Example 6.10, a 2^4 factorial design was used to improve the response rate to a credit card mail marketing offer. Suppose that the researchers had used the 2^{4-1} fractional factorial design with $I = ABCD$ instead. Set up the design and select the responses for the runs from the full factorial data in Example 6.6. Analyze the data and draw conclusions. Compare your findings with those from the full factorial in Example 6.6.

8.7 ***Continuation of Problem 8.6.*** In Problem 6.6, we found that all four main effects and the two-factor AB interaction were significant. Show that if the alternate fraction ($I = -ABCD$) is added to the 2^{4-1} design in Problem 8.6 that the analysis of the results from the combined design produce results identical to those found in Problem 6.6.

8.8 ***Continuation of Problem 8.6.*** Reconsider the 2^{4-1} fractional factorial design with $I = ABCD$ from Problem 8.6. Set a partial fold over of this fraction to isolate the AB interaction. Select the appropriate set of responses from the full factorial data in Example 6.6 and analyze the resulting data.

H **8.9** R. D. Snee ("Experimenting with a Large Number of Variables," in *Experiments in Industry: Design, Analysis and Interpretation of Results*, by R. D. Snee, L. B. Hare, and J. B. Trout, Editors, ASQC, 1985) describes an experiment in which a 2^{5-1} design with $I = ABCDE$ was used to investigate the effects of five factors on the color of a chemical product. The factors are A = solvent/reactant, B = catalyst/reactant, C = temperature, D = reactant purity, and E = reactant pH. The responses obtained are as follows:

$e = -0.63$	$d = 6.79$
$a = 2.51$	$ade = 5.47$
$b = -2.68$	$bde = 3.45$
$abe = 1.66$	$abd = 5.68$
$c = 2.06$	$cde = 5.22$
$ace = 1.22$	$acd = 4.38$
$bce = -2.09$	$bcd = 4.30$
$abc = 1.93$	$abcde = 4.05$

(a) Prepare a normal probability plot of the effects. Which effects seem active?

(b) Calculate the residuals. Construct a normal probability plot of the residuals and plot the residuals versus the fitted values. Comment on the plots.

(c) If any factors are negligible, collapse the 2^{5-1} design into a full factorial in the active factors. Comment on the resulting design, and interpret the results.

8.10 An article by J. J. Pignatiello Jr. and J. S. Ramberg in the *Journal of Quality Technology* (Vol. 17, 1985, pp. 198–206) describes the use of a replicated fractional factorial to investigate the effect of five factors on the free height of leaf springs used in an automotive application. The factors are A = furnace temperature, B = heating time, C = transfer time, D = hold down time, and E = quench oil temperature. The data are shown in Table P8.1.

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(c) If this design were run in two blocks with the AB interaction confounded with blocks, the run d would be in the block where the sign on AB is ______. Answer either − or +.

8.48 Consider the following design:

Std	A	B	C	D	E	y
1	−1	−1	−1	1	1	40
2	1	−1	−1	−1	1	10
3	−1	1	−1	−1	−1	30
4	1	1	−1	1	−1	20
5	−1	−1	1	−1	−1	40
6	1	−1	1	1	−1	30
7	−1	1	1	1	1	20
8	1	1	1	−1	1	30

(a) What is the generator for column D?

(b) What is the generator for column E?

(c) If this design were folded over, what is the resolution of the combined design?

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8.49 In an article in *Quality Engineering* ("An Application of Fractional Factorial Experimental Designs," 1988, Vol. 1, pp. 19–23), M. B. Kilgo describes an experiment to determine the effect of CO_2 pressure (A), CO_2 temperature (B), peanut moisture (C), CO_2 flow rate (D), and peanut particle size (E) on the total yield of oil per batch of peanuts (y). The levels that she used for these factors are shown in Table P8.11. She conducted the 16-run fractional factorial experiment shown in Table P8.12.

■ **TABLE P8.11**
Factor Levels for the Experiment in Problem 8.49

Coded Level	A, Pressure (bar)	B, Temp, (°C)	C, Moisture (% by weight)	D, Flow (liters/min)	E, Part. Size (mm)
−1	415	25	5	40	1.28
1	550	95	15	60	4.05

(a) What type of design has been used? Identify the defining relation and the alias relationships.

(b) Estimate the factor effects and use a normal probability plot to tentatively identify the important factors.

(c) Perform an appropriate statistical analysis to test the hypotheses that the factors identified in part (b) have a significant effect on the yield of peanut oil.

■ **TABLE P8.12**
The Peanut Oil Experiment

A	B	C	D	E	y
415	25	5	40	1.28	63
550	25	5	40	4.05	21
415	95	5	40	4.05	36
550	95	5	40	1.28	99
415	25	15	40	4.05	24
550	25	15	40	1.28	66
415	95	15	40	1.28	71
550	95	15	40	4.05	54
415	25	5	60	4.05	23
550	25	5	60	1.28	74
415	95	5	60	1.28	80
550	95	5	60	4.05	33
415	25	15	60	1.28	63
550	25	15	60	4.05	21
415	95	15	60	4.05	44
550	95	15	60	1.28	96

(d) Fit a model that could be used to predict peanut oil yield in terms of the factors that you have identified as important.

(e) Analyze the residuals from this experiment and comment on model adequacy.

8.50 A 16-run fractional factorial experiment in 10 factors on sand-casting of engine manifolds was conducted by engineers at the Essex Aluminum Plant of the Ford Motor Company and described in the article "Evaporative Cast Process 3.0 Liter Intake Manifold Poor Sandfill Study," by D. Becknell (*Fourth Symposium on Taguchi Methods*, American Supplier Institute, Dearborn, MI, 1986, pp. 120–130). The purpose was to determine which of 10 factors has an effect on the proportion of defective castings. The design and the resulting proportion of nondefective castings $\hat{p}$ observed on each run are shown in Table P8.13. This is a resolution III fraction with generators $E = CD$, $F = BD$, $G = BC$, $H = AC$, $J = AB$, and $K = ABC$. Assume that the number of castings made at each run in the design is 1000.

(a) Find the defining relation and the alias relationships in this design.

(b) Estimate the factor effects and use a normal probability plot to tentatively identify the important factors.

(c) Fit an appropriate model using the factors identified in part (b).