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Section 1

MANE 6313

Subsection 1

Week 12, Module C

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Using inference for linear regression models.

Resources for the Week 12, Module C micro-lecture are:

- Week 12, Module C Micro-lecture
- Week 12, Module C Marked Notes

Inference for Regression Models

Tests on Individual Regression Coefficients

Example $K = 3$

$\alpha_{ind} = .05$

$\alpha_{overall} = 1 - (1-\alpha)^K$

$= 1 - (1 - .05)^3$

$= 1 - .953$

$= \underline{.1426}$

- We can test $H_0 : \beta_j = 0$ vs. $H_a : \beta_j \neq 0$

$$\frac{\hat{\beta}_j}{\sqrt{\hat{\sigma}^2 C_{jj}}} \sim t_{n-k-1}$$

where C_{jj} is the (jj)th element of $(\mathbf{X}'\mathbf{X})^{-1}$

- or equivalently

$$\frac{\hat{\beta}_j}{se(\hat{\beta}_j)} \sim t_{n-k-1}$$

- If $H_0 : \beta_j = 0$ is not rejected, we can remove the variable x_j from the model

C.I. on the Individual Regression Coefficients

- Straightforward to construct

$$\frac{\hat{\beta}_j - \beta}{\sqrt{\hat{\sigma}^2 C_{jj}}} \sim t_{n-p} \quad j = 0, 1, 2, \ldots, k$$

- A $100(1-\alpha)\%$ confidence interval for β_j is

$$\hat{\beta}_j - t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2 C_{jj}} \leq \beta_j \leq \hat{\beta}_j + t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2 C_{jj}}$$

- or equivalently

$$\hat{\beta}_j - t_{\alpha/2,n-p} se(\hat{\beta}_j) \leq \beta_j \leq \hat{\beta}_j + t_{\alpha/2,n-p} se(\hat{\beta}_j)$$

C.I. on the Mean response

- We can find a confidence interval on the mean response at a point $x_0' = [1, x_{01}, x_{02}, \ldots, x_{x0k}]$.
- Note that

$$\begin{aligned} \mu_{y|\mathbf{x_0}} = & \quad \beta_0 + \beta_1 x_{01} + \beta_2 x_{02} + \cdots + \beta_k x_{0k} \\ \hat{y}(\mathbf{x_0}) = & \quad \mathbf{X}_0' \widehat{\beta} \end{aligned}$$

- The variance of $\hat{y}(\mathbf{x_0})$ is

$$V\left[\hat{y}(\mathbf{x_0})\right] = \sigma^2 \mathbf{x}_0' (\mathbf{X}'\mathbf{X})^{-1} \mathbf{x_0}$$

- A $100(1-\alpha)\%$ confidence interval for the mean response is

$$\begin{aligned} \hat{y}(\mathbf{x_0}) - & \quad t_{\alpha/2, n-p} \sqrt{\hat{\sigma}^2 \mathbf{x}_0' (\mathbf{X}'\mathbf{X})^{-1} \mathbf{x_0}} \\ \leq & \quad \mu_{y|x_0} \leq \hat{y}(\mathbf{x_0}) + t_{\alpha/2, n-p} \sqrt{\hat{\sigma}^2 \mathbf{x}_0' (\mathbf{X}'\mathbf{X})^{-1} \mathbf{x_0}} \end{aligned}$$

Prediction Intervals

- We can use the regression equation to predict values at points other than those in the design matrix. In general, we only want to interpolate; not extrapolate
- Consider the point $x_0' = [1, x_{01}, x_{02}, \ldots, x_{x0k}]$. A point estimate for y is

$$\hat{y}(\mathbf{x_0}) = \mathbf{x_0}'\widehat{\beta}$$

- A $100(1-\alpha)\%$ prediction interval for this observation is

$$\begin{aligned}\hat{y}(\mathbf{x_0}) - & \quad t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2(1+\mathbf{x_0}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x_0})} \\ \leq & \quad y_0 \leq \hat{y}(\mathbf{x_0}) + t_{\alpha/2,n-p}\sqrt{\hat{\sigma}^2(1+\mathbf{x_0}'(\mathbf{X}'\mathbf{X})^{-1}\mathbf{x_0})}\end{aligned}$$

- Most packages will perform this function for you.

Minitab Demonstration

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-284.5	30.8	-9.24	0.001	
x1	0.1250	0.0850	1.47	0.215	1.00
x2	2.458	0.283	8.68	0.001	1.00
x3	1.450	0.113	12.79	0.000	1.00

not statistically significant

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Regression Equation

radius = -284.5 + 0.1250 x1 + 2.458 x2 + 1.450 x3

Settings

Variable	Setting
x1	175
x2	20
x3	230

$\hat{Y}$

Prediction

Fit	SE Fit	95% CI	95% PI
120.042	1.92708	(114.691, 125.392)	(105.658, 134.425)

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$Se(\hat{Y})$