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Section 1

MANE 6313

Subsection 1

Week 12, Module D

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Employing test of hypothesis test on groups of variables.

Resources for the Week 12, Module D micro-lecture are:

- Week 12, Module D Micro-lecture
- Week 12, Module D Marked Notes

Applying Tests of Hypothesis for Groups of Variables

Tests for groups of regression coefficients

- We can examine the contribution of the regression sum of squares for a particular variable, say x_j, given that other variables $x_i (i \neq j)$ are included in the model
- We can also determine if the subset of regressor variables $x_1, x_2, \ldots, x_r (r < k)$ contribute to the model
- The full model is

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon$$

- We partition the regressors in two groups

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

non-statistically significant (x)
statistically significant terms (x_2 & x_3)

where $\beta_{\mathbf{1}}$ is $(r \times 1)$ and $\beta_{\mathbf{2}}$ is $[(p - r) + 1]$

- We can rewrite the **full model** as

$$\mathbf{y} = \mathbf{X_1}\beta_\mathbf{1} + \mathbf{X_2}\beta_\mathbf{2} + \varepsilon$$

- The regression sum of squares for the full model is denoted $SS_R(\beta)$ with p degrees of freedom
- We define the reduced model to be

$$\mathbf{y} = \mathbf{X_2}\beta_\mathbf{2}\varepsilon$$

- The regression sum of squares for the reduced model is $SS_r(\beta_\mathbf{2})$ with $p - r$ degrees of freedom.
- The regression sum of squares due to $\beta_\mathbf{1}$ given that β_2 is already in the model is

$$SS_R(\beta_\mathbf{1}|\beta_\mathbf{2}) = SS_R(\beta) - SS_R(\beta_2)$$

- The null hypothesis $\beta_1 = \mathbf{0}$ can be tested with the statistic

$$F_0 = \frac{SS_R(\beta_1|\beta_2)/r}{MS_E}$$

- Reject H_0 if $F_0 > F_{\alpha,r,n-p}$
- This is sometimes called the *partial F test* or extra sum of squares method

Step 1 - Fit Full Model

Regression Analysis: radius versus powder, rate, die (Full Model)

The regression equation is
radius = - 285 + 0.125 powder + 2.46 rate + 1.45 die

Predictor	Coef	SE Coef	T	P
Constant	-284.50	30.78	-9.24	0.001
powder	0.12500	0.08501	1.47	0.215
rate	2.4583	0.2834	8.68	0.001
die	1.4500	0.1133	12.79	0.000

S = 4.809 R-Sq = 98.4% R-Sq(adj) = 97.1%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	5575.0	1858.3	80.36	0.000
Residual Error	4	92.5	23.1		
Total	7	5667.5			

Source	DF	Seq SS
powder	1	50.0
rate	1	1740.5
die	1	3784.5

Step 2 - Fit Reduced Model

Regression Analysis: radius versus rate, die (Reduced Model)

The regression equation is
radius = - 263 + 2.46 rate + 1.45 die

Predictor	Coef	SE Coef	T	P
Constant	-263.25	30.17	-8.73	0.000
rate	2.4583	0.3146	7.81	0.001
die	1.4500	0.1258	11.52	0.000

S = 5.339   R-Sq = 97.5%   R-Sq(adj) = 96.5%

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	2	5525.0	2762.5	96.93	0.000
Residual Error	5	142.5	28.5		
Total	7	5667.5			

Source	DF	Seq SS
rate	1	1740.5
die	1	3784.5

Step 3 - Analysis

$SS_R(\beta) = 5575$ $\qquad SS_R(\beta_2) = 5525$

$MS_E = 23.1$

$$F_0 = \frac{(5575 - 5525)/(3-2)}{23.1} = 2.164$$

Reject if $F_0 > F_{.05,1,4} = 7.71$

Conclusion: fail to reject H_0 and conclude that $\beta_{powder} = 0$.

Figure 3: Analysis