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Sunday, November 14, 2021 10:23 AM

Section 1

MANE 6313

Subsection 1

Week 13, Module A

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Describe response surface methodology.

Introduction to RSM

- *Response Surface Methodology* is a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response
- Consider a two-variable function where

$$y = f(x_1, x_2) + \epsilon$$

- The expected response function is

$$E(y) = f(x_1, x_2) = \eta$$

eta

- Thus the function η is often called the response surface.
- The response surface is often shown graphically

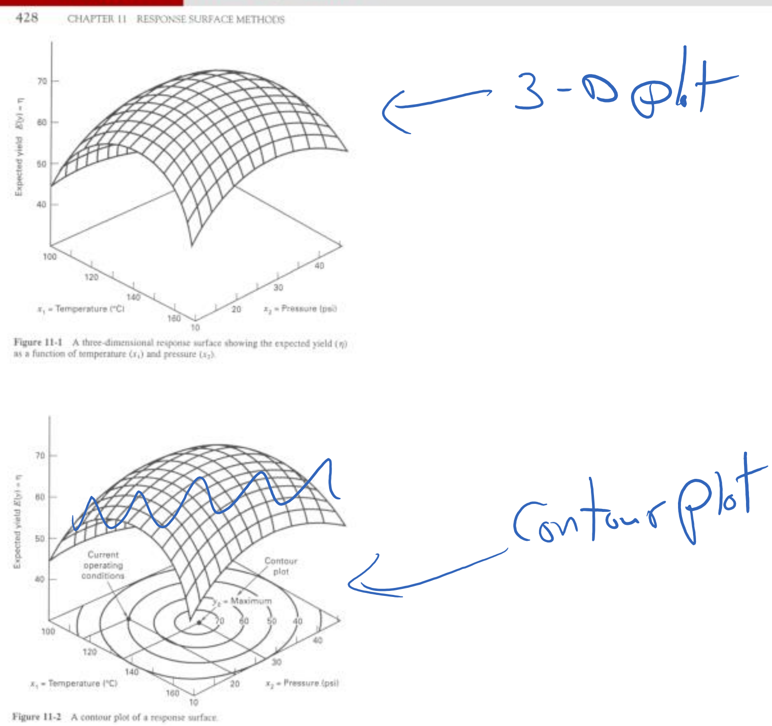


Figure 1: image

- In general the function η is unknown.
- We will approximate η with low-order polynomial functions.
- A first-order model is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon$$

- A second-order model is

Quadratic

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \sum_{i<j} \sum \beta_{ij} x_i x_j + \epsilon$$

interactions

- The method of least squares will be used to estimate the parameters, β

Sequential Approach

- The use of RSM often requires sequential analysis
- Most of the time, you will not be operating at (or possibly near) an optimal region
- Perform an initial experiment, often first-order design (screening)
- Determine direction towards optimum point → search for region with optimum point
- Conduct another experiment nearer to the optimum point
- Repeat until in the neighborhood of the optimum
- Conduct an experiment using a second-order design

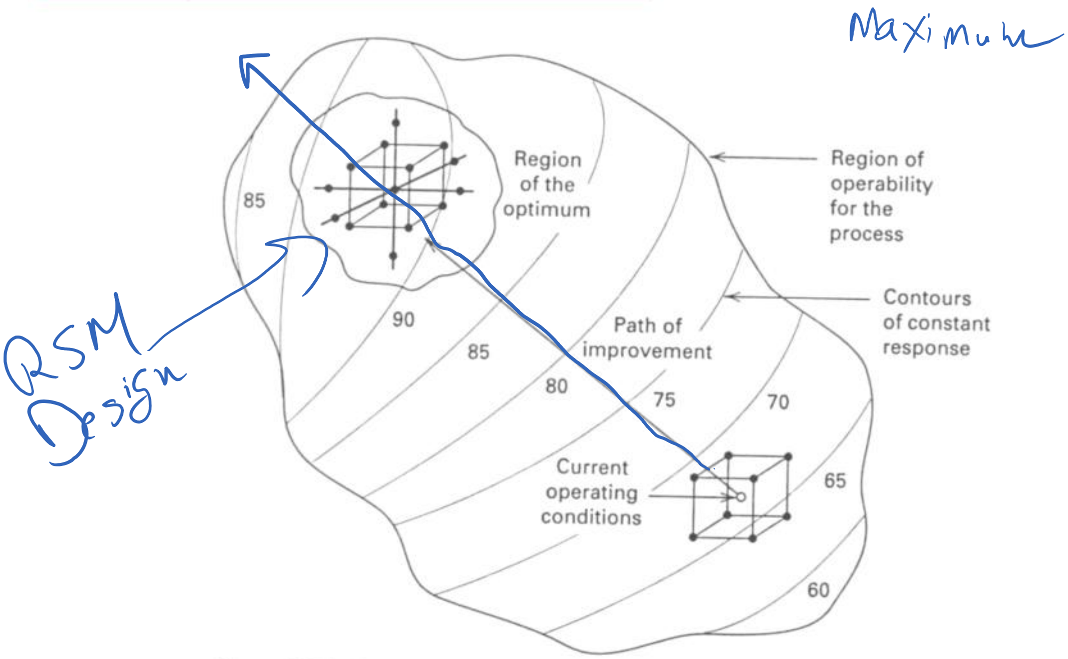


Figure 11-3 The sequential nature of RSM.

Figure 2: image

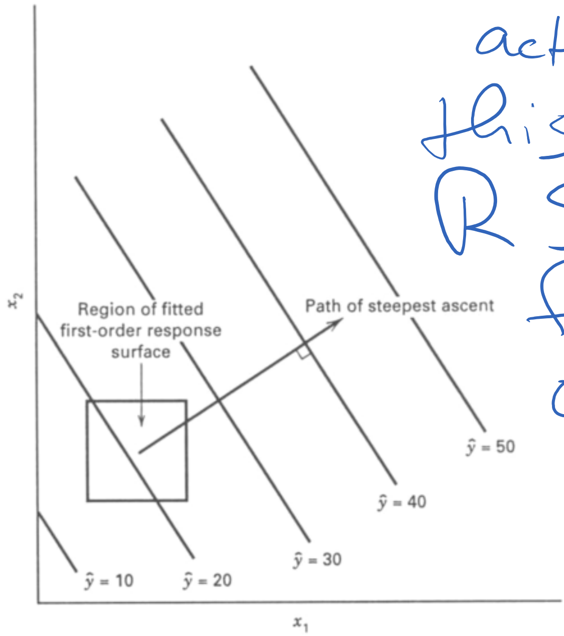


Figure 11-4 First-order response surface and path of steepest ascent.

actually see this predicted RS from 1st order model

Figure 3: image

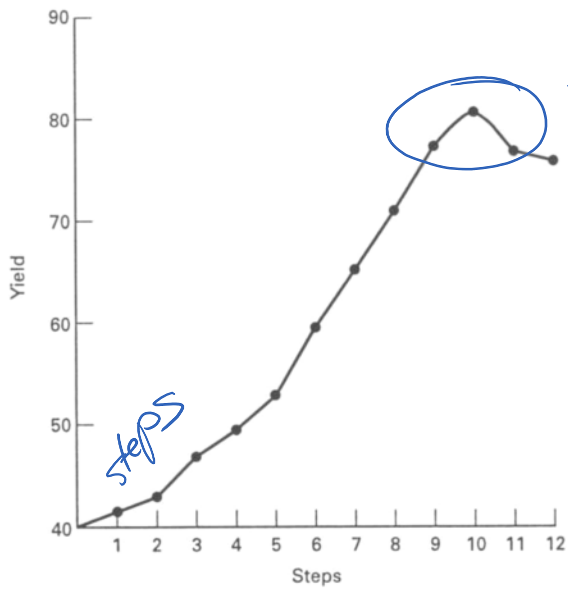
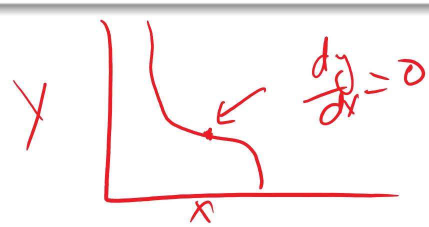


Figure 11-5 Yield versus steps along the path of steepest ascent for Example 11-1.

Figure 4: image

Analysis of the 2nd-order Response Surface

- Suppose that we wish to find the levels of $x_1, x_2, \ldots, x_k$ that optimize the predicted response
- The point, if it exists, will be the set of $x_1, x_2, \ldots, x_k$ for which $\partial\hat{y}/\partial x_1 = \partial\hat{y}/\partial x_2 = \partial\hat{y}/\partial x_k = 0$
- This point is called the *stationary point*.
- Based upon our knowledge of calculus what are the possible types of stationary points? max, min, inflection
- How do we determine if a stationary point is an optimal point?



1) 2nd derivative (Hessian)

2) Response optimizer (minitab)

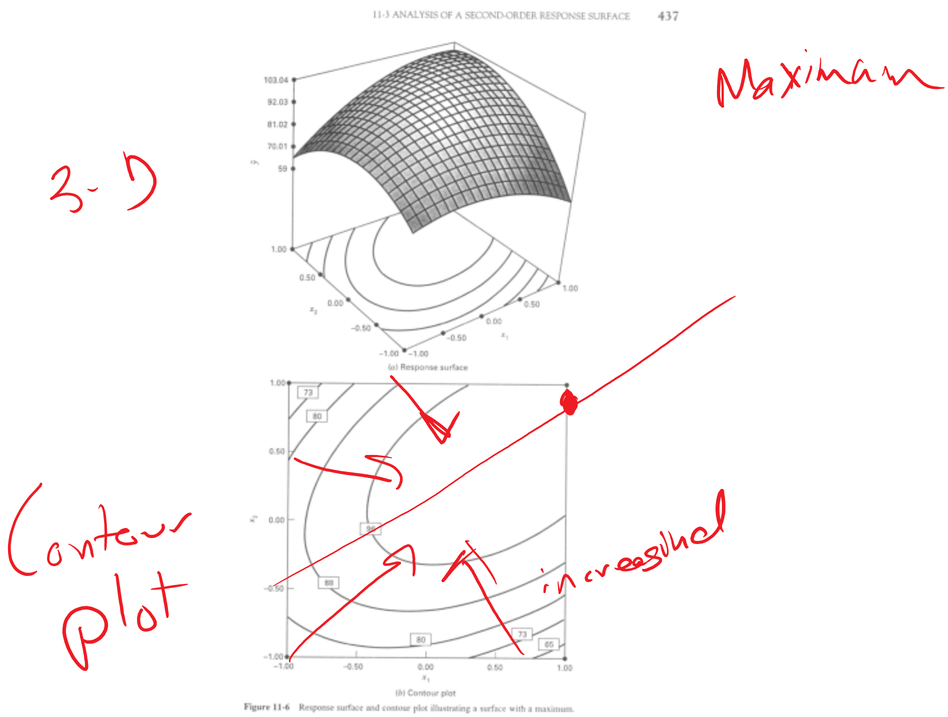


Figure 5: image

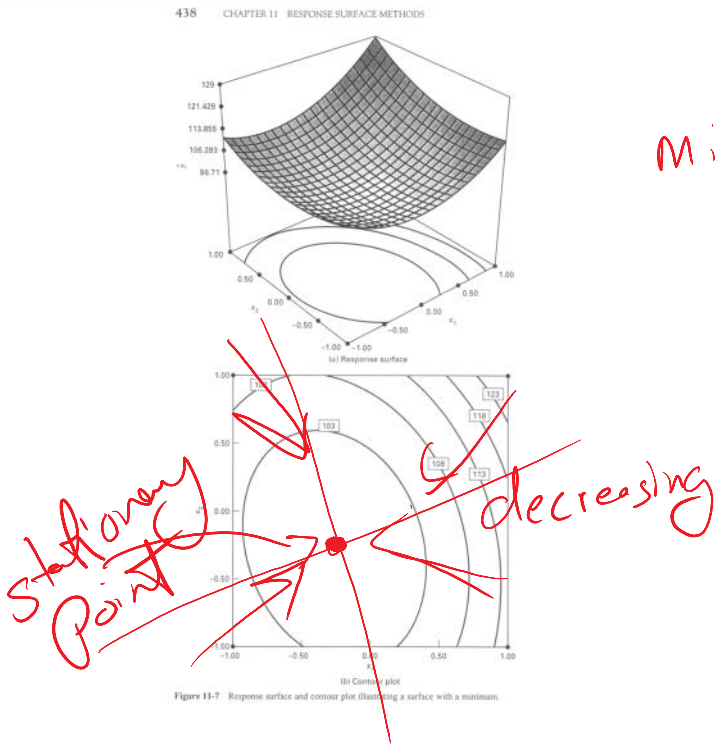
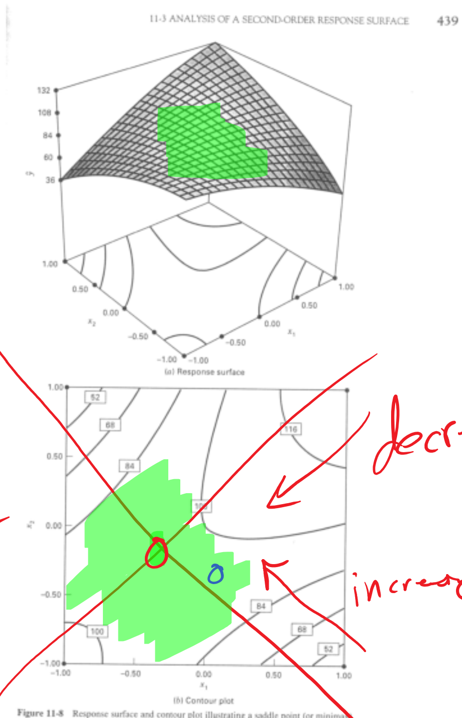


Figure 6: image

Saddle point



inflection / Saddle point

decreasing

increasing

Figure 7: image