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Sunday, November 14, 2021 10:46 AM

Section 1

MANE 6313

Subsection 1

Week 13, Module B

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Describe designs for RSM.

Experimental Designs for Fitting Response Surfaces

A list of desirable features for an experimental design include:

1. Provides a reasonable distribution of data points (and hence information) throughout the region of interest
2. Allows model adequacy, **including lack of fit**, to be investigated
3. Allows experiments to be performed in blocks (if necessary)
4. Allows designs of higher order to be built up sequentially
5. Provides an internal estimate of error
6. Provides precise estimates of model coefficients (minimum variance)

7. Provides a good profile of the prediction variance throughout the experimental regions
8. Provides reasonable robustness against outliers or missing values
9. Does not require a large number of runs
10. Does not require too many levels of the independent variables
11. Ensures simplicity of calculation of the model parameters

$y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon$

First-order Models

- We want to use only models which have a minimum variance for $\left\{ \hat{\beta}_j \right\}$
- These designs are said to be *orthogonal first-order designs*. A first-order design is orthogonal if the off-diagonal elements of $(\mathbf{X}'\mathbf{X})$ are all zero.
- Orthogonal first-order designs include: 2^k factorial and fractional factorial designs and simplex designs.

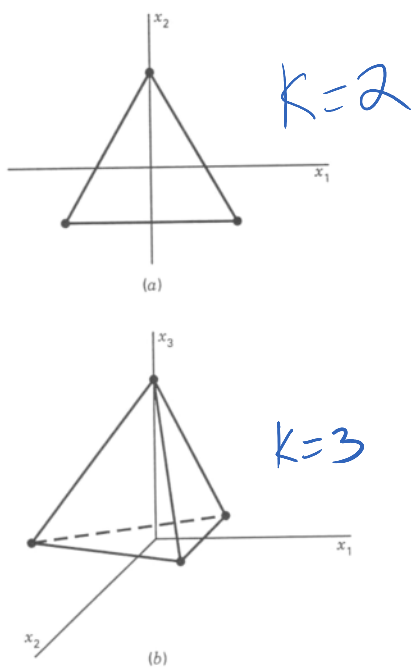

Figure 11-19 The simplex design for (*a*) $k = 2$ variables and (*b*) $k = 3$ variables.

Figure 1: image

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i x_i + \sum_{j=1}^{k} \beta_{jj} x_j^2 + \sum\sum \beta_{ij} x_i x_j + \varepsilon$$

Second-order Models

- The most popular second-order design is the central composite design (CCD)
- Generally, the CCD is obtained sequentially
  - An initial factorial or resolution V fractional factorial was performed with center points. A lack of fit test indicated that quadratic terms should be added.
  - The axial points are added
- The design is composed of three components: the factorial component with n_f observations, n_c center points and $2k$ axial points
- Selection of α, the distance from the center point to the axial points
- Set $\alpha = (n_f)^{1/4}$. This design is said to be rotatable.
- Set $\alpha = \sqrt{k}$. This design is said to be a *spherical CCD*. It has a better prediction variance than a rotatable CCD and is typically more common.
  - Generally three to five center points are recommended

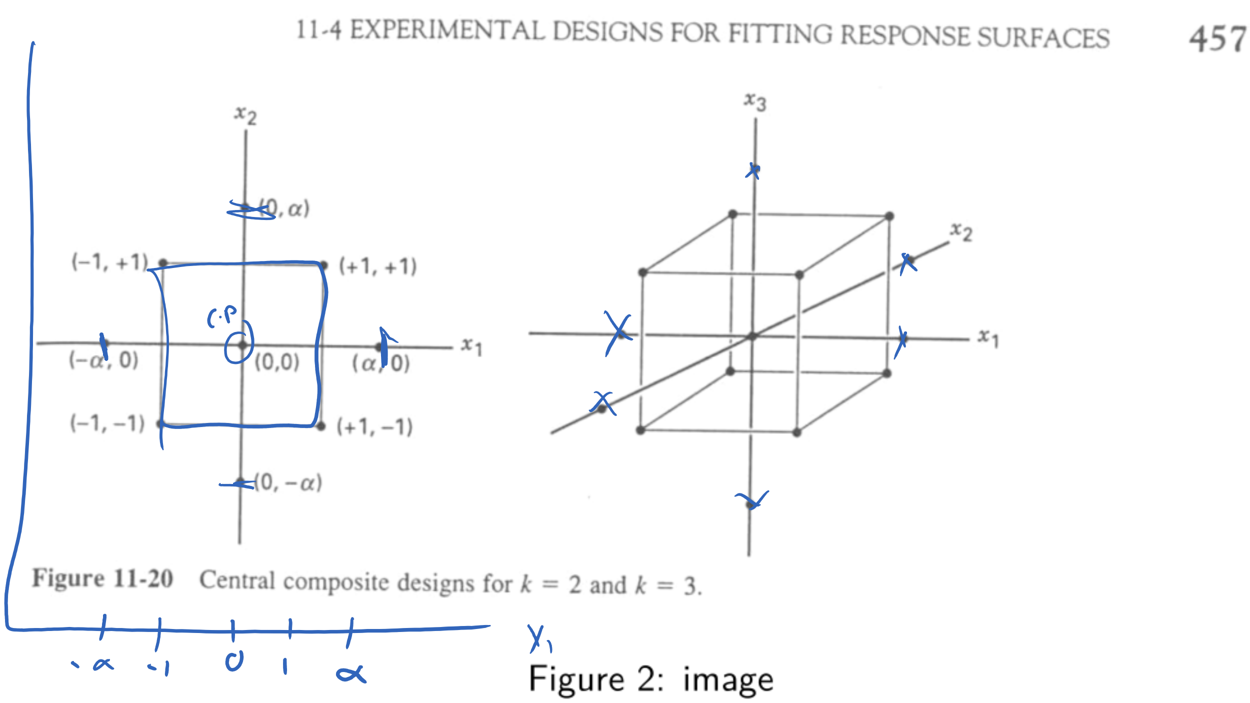

Figure 11-20 Central composite designs for $k = 2$ and $k = 3$.

Figure 2: image

Box-Behnken Design

- Box and Behnken proposed some three-level designs for fitting response models
- These designs are typically very efficient in terms of number of runs required
- Some of the designs are spherical
- Not requiring the extreme points of the design space may be an advantage

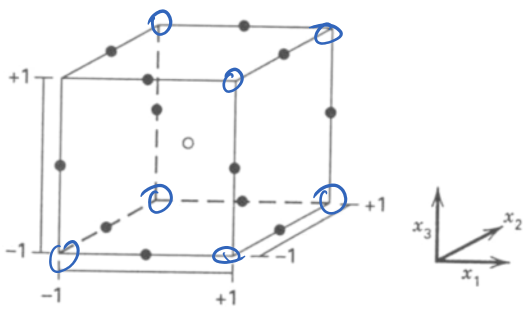

Figure 11-22 A Box–Behnken design for three factors.

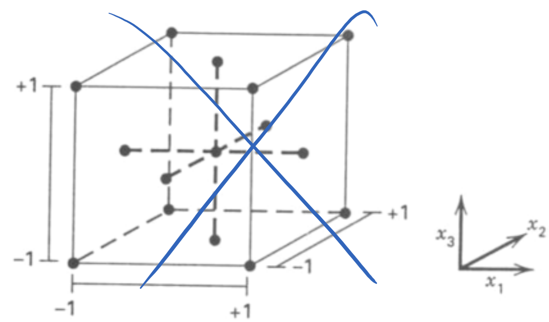

Figure 11-23 A face-centered central composite design for $k = 3$.

Figure 3: image

Face-centered CCD

- If the area of interest is a cube instead of a sphere, we set the axial points at the surface of the cube.
- That is, $\alpha = 1$
- This design is not rotatable
- The number of center points required is less and typically is set to 2

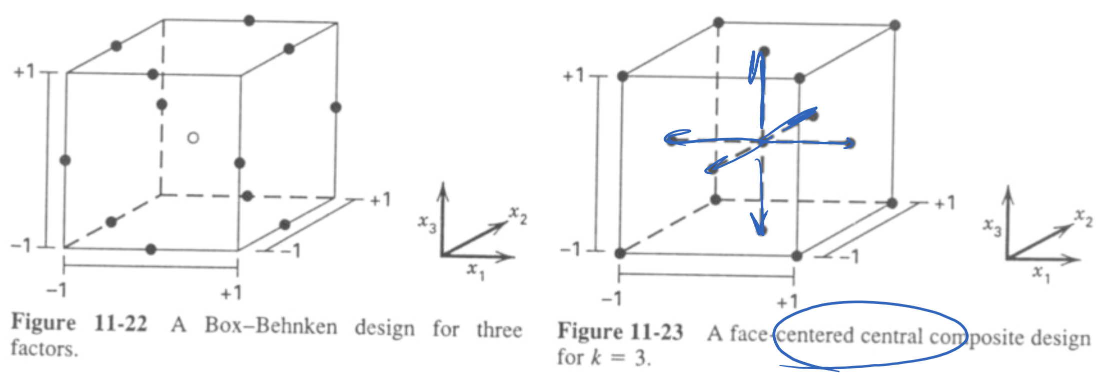

Figure 4: image