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Tuesday, November 30, 2021 10:11 AM

Section 1

MANE 6313

Subsection 1

Week 15, Module A

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Apply the method of steepest descent.

Step 0

Read Example 11.1 on pages 493 - 497.

Step 1 - Find Coded Regression Model

Note that Example 11.1 uses both coded and natural (uncoded) models.

The engineer decides that the region of exploration for fitting the first-order model should be (30, 40) minutes of reaction time and (150, 160) Fahrenheit. To simplify the calculations, the independent variables will be coded to the usual $(-1, 1)$ interval. Thus, if ξ_1 denotes the **natural variable** time and ξ_2 denotes the **natural variable** temperature, then the **coded variables** are

$$x_1 = \frac{\xi_1 - 35}{5} \quad \text{and} \quad x_2 = \frac{\xi_2 - 155}{5}$$

→ half range of each variable

You must be very careful and ensure that for each step you are using the correct model. The coded model is

A first-order model may be fit to these data by least squares. Employing the methods for two-level designs, we obtain the following model in the coded variables:

$$\hat{y} = 40.44 + 0.775x_1 + 0.325x_2$$

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Step 2 - Take derivates

Calculate the derivate for each term in the model.

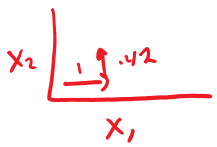
$$\frac{d\hat{y}}{dx_1} = 0.775$$

$$\frac{d\hat{y}}{dx_2} = 0.325$$

Step 3 - Determine slope

Consider the pair of variables x_1 and x_2 determine the slope.

To move away from the design center—the point ($x_1 = 0$, $x_2 = 0$)—along the path of steepest ascent, we would move 0.775 units in the x_1 direction for every 0.325 units in the x_2 direction. Thus, the path of steepest ascent passes through the point ($x_1 = 0, x_2 = 0$) and has a slope 0.325/0.775. The engineer decides to use 5 minutes of reaction time as the basic step size. Using the relationship between ξ_1 and x_1, we see that 5 minutes of reaction time is equivalent to a step in the *coded* variable x_1 of $\Delta x_1 = 1$. Therefore, the steps along the path of steepest ascent are $\Delta x_1 = 1.0000$ and $\Delta x_2 = (0.325/0.775) = 0.42$.

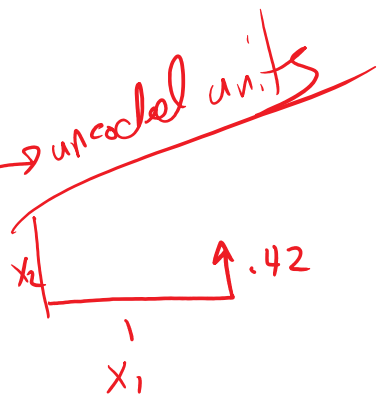


$$\text{slope} = \frac{\Delta x_2}{\Delta x_1} = \frac{d\hat{y}/dx_2}{d\hat{y}/dx_1} = \frac{0.325}{0.775} = 0.42$$

Step 4 - Determine step size

This step involves guess work. In Example 11. 1, a step of size of 1 coded unit in x_1 is selected. Notice that corresponds to a change of 5 in the x_1 direction of the uncoded model. Once the step size (in coded units) is select for x_1, use the slope to find the step size (in coded units) for x_2.

To move away from the design center—the point ($x_1 = 0$, $x_2 = 0$)—along the path of steepest ascent, we would move 0.775 units in the x_1 direction for every 0.325 units in the x_2 direction. Thus, the path of steepest ascent passes through the point ($x_1 = 0, x_2 = 0$) and has a slope 0.325/0.775. The engineer decides to use 5 minutes of reaction time as the basic step size. Using the relationship between ξ_1 and x_1, we see that 5 minutes of reaction time is equivalent to a step in the *coded* variable x_1 of $\Delta x_1 = 1$. Therefore, the steps along the path of steepest ascent are $\Delta x_1 = 1.0000$ and $\Delta x_2 = (0.325/0.775) = 0.42$.

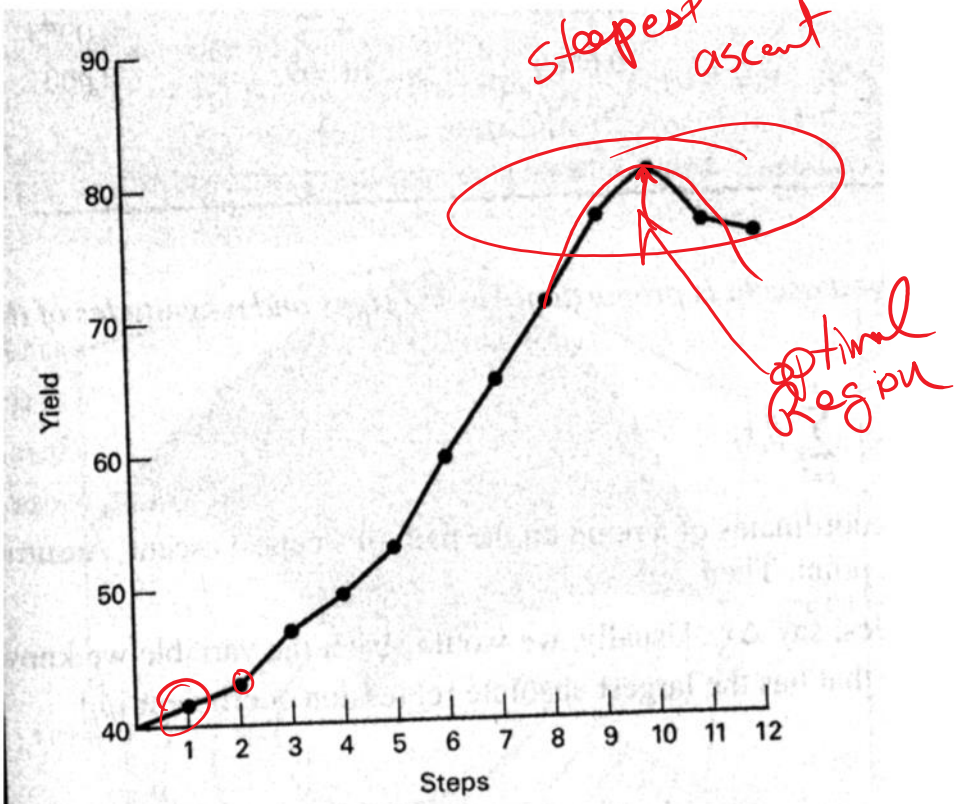


Step 5 - Construct Table 11.3 and conduct experiments

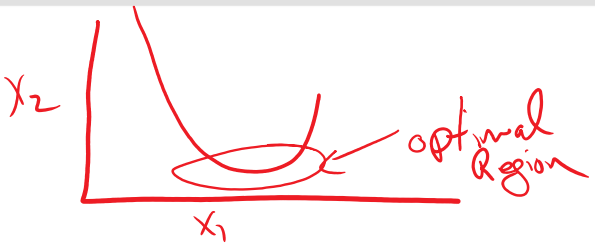
■ **TABLE 11.3**
Steepest Ascent Experiment for Example 11.1

Steps	Coded Variables x_1	Coded Variables x_2	Natural Variables ξ_1	Natural Variables ξ_2	Response y
Origin	0	0	35	155	
Δ	1.00	0.42	5	2	
Origin + Δ	1.00	0.42	40	157	41.0
Origin + 2Δ	2.00	0.84	45	159	42.9
Origin + 3Δ	3.00	1.26	50	161	47.1
Origin + 4Δ	4.00	1.68	55	163	49.7
Origin + 5Δ	5.00	2.10	60	165	53.8
Origin + 6Δ	6.00	2.52	65	167	59.9
Origin + 7Δ	7.00	2.94	70	169	65.0
Origin + 8Δ	8.00	3.36	75	171	70.4
Origin + 9Δ	9.00	3.78	80	173	77.6
Origin + 10Δ	10.00	4.20	85	175	80.3
Origin + 11Δ	11.00	4.62	90	179	76.2
Origin + 12Δ	12.00	5.04	95	181	75.1

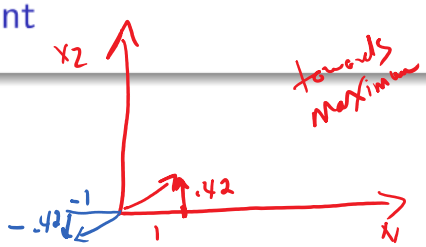
Step 6 - Search for optimal region



■ FIGURE 11.5 Yield versus steps along the path



Method of steepest descent



towards minimum

find direction: multiplied derivatives by -1