

MANE 6313

Section 1

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1 / 13

Subsection 1

Week 2, Module B

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome

Review of (statistical) hypothesis testing.

Hypothesis Testing

H_0 : ~~accused~~ ^{accused} is innocent
 H_1 : accused is Not innocent

- A Statistical Hypothesis is a statement about the values of the parameters of a probability distribution. For example:

Decision
 1) ~~accept H_0~~ ^{fail to reject H_0}
 2) ~~Reject H_0~~ ^{Reject H_0}

$$H_0: \mu = 1.500$$

$$H_A: \mu \neq 1.500$$

(two-tailed)

$$H_0: p = .5$$

$$H_1: p \neq .5$$

- The Null Hypothesis is given by H_0 and is assumed to be true
- The Alternative Hypothesis is given by H_A . We are trying to gather evidence to support the claim of the alternative hypothesis
- Hypotheses may be either two-sided or one-sided

$$H_1: \mu < 1.5$$

$$H_1: \mu > 1.5$$

Hypothesis Testing

- You should be familiar with hypothesis testing.
- You are responsible for the following material:
 - Tests on means with variance known (Table 2-4)
 - Tests on means of normal distribution with variance unknown (Table 2-4)
 - Paired comparison test (section 2-5)
 - Tests on variances of normal distribution (Table 2-8)

Overview: Conducting a Test of Hypothesis

- ① Take a random sample from the population under study
- ② Compute the appropriate ~~test~~ statistic
- ③ Decide to either reject or fail to reject H_0

The set of values of the test statistic leading to the reject of H_0 is called the rejection region or critical region.

Errors in Hypothesis Testing

- A Type I error occurs when the null hypothesis is true but the decision is made to reject H_0

typically $\alpha = .05$

$$\alpha = P \{ \text{type I error} \} = P \{ \text{reject } H_0 | H_0 \text{ is true} \}$$

experimenter controls α

- A Type II error occurs when the null hypothesis is false but the decision is made not to reject H_0

but $\beta = P \{ \text{type II error} \} = P \{ \text{fail to reject } H_0 | H_0 \text{ is false} \}$

$$\beta = f(\alpha, n, \Delta)$$

H_0 : part is good
 H_1 : part is bad

Decision	Part is good	Part is bad
Good part	✓	type II error Consumer's risk
Bad part	type I error Producer's risk	✓

TABLE 4.1 Decision Errors in Hypothesis Testing

Decision	True H_0	False H_0
H_0 not rejected	No error with probability $1 - \alpha$	Type II error with probability β
H_0 rejected	Type I error with probability α	No error with probability $1 - \beta$

Source¹

H_0 : person is healthy
 H_1 : person is sick

Decision	healthy	Sick
healthy	✓	type II error sick but had healthy
Sick	type I error healthy but had sick	✓

¹DeVor, Chang, Sutherland (2007). *Statistical Quality Design and Control: Contemporary Concepts and Methods*, 2nd edition. Pearson: Prentice-Hall.

Questions

- ① Which type of error is the producer's risk?
- ② Which type of error is the consumer's risk?

Classical Approach

Steps taken from ²

- ① Find parameter of interest
- ② State null hypothesis, H_0
- ③ State alternative hypothesis, H_1
- ④ Calculate test statistic
- ⑤ Construct rejection region
- ⑥ State conclusion(s)

²Montgomery and Runger (2014). *Applied Statistics and Probability for Engineers*, 6th edition. John Wiley & Sons.

P-value Approach

- Similar in structure to classical approach
- ① Find parameter of interest
- ② State null hypothesis, H_0
- ③ State alternative hypothesis, H_1
- ④ Calculate p -value
- ⑤ State conclusion(s)
- Decision rule
 - If $p\text{-value} < \alpha$ reject H_0
 - else ($p\text{-value} > \alpha$) fail to reject H_0

