

Printout

Sunday, August 29, 2021 10:13 AM

Section 1

MANE 6313

Subsection 1

Week 2, Module E

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome

Analyze a one-sample chi-squared test.

Example: One-Sample Chi-squared Test¹

9-85. + Reconsider the percentage of titanium in an alloy used in aerospace castings from Exercise 8-52. Recall that $s = 0.37$ and $n = 51$.

- (a) Test the hypothesis $H_0: \sigma = 0.25$ versus $H_1: \sigma \neq 0.25$ using $\alpha = 0.05$. State any necessary assumptions about the underlying distribution of the data. Find the P -value.
- (b) Explain how you could answer the question in part (a) by constructing a 95% two-sided confidence interval for σ .

¹Montgomery and Runger (2014). *Applied Statistics and Probability for Engineers*, 6th edition. John Wiley & Sons.

Classical Approach

$$H_0: \sigma = .25$$

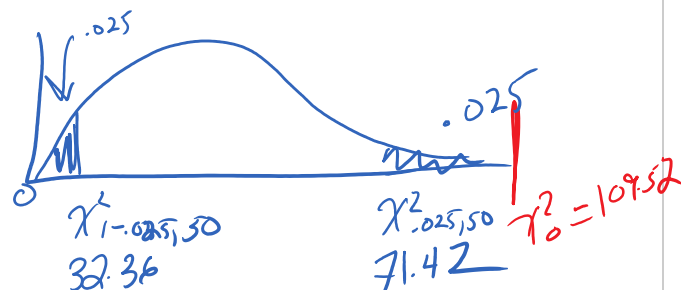
$$H_1: \sigma \neq .25 \quad (\alpha = .05)$$

$$n = 51$$

$$s = .37$$

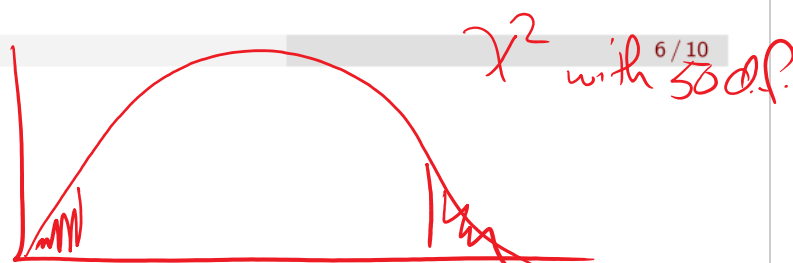
Test Statistics $\chi^2_0 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(51-1)(.37)^2}{(.25)^2} = 109.52$

Rejection Region



Conclusion:
Reject H_0

P-values



$$\begin{aligned} p\text{-value} &= 2(1 - F_{\chi^2_{50}}(109.52)) \\ &= 4.99 \times 10^{-6} \approx 0.0 \end{aligned}$$

Rejection Region & Conclusion^a

^aMontgomery and Runger (2014). *Applied Statistics and Probability for Engineers*, 6th edition. John Wiley & Sons.

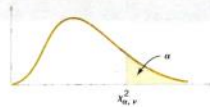


TABLE • IV Percentage Points $\chi^2_{\alpha, r}$ of the Chi-Squared Distribution

α r	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67

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Chi-squared Test using Minitab

WORKSHEET 1

Test and CI for One Variance

Method

σ : standard deviation of Sample

The Bonett method cannot be calculated for summarized data.

The chi-square method is valid only for the normal distribution.

Descriptive Statistics

N	StDev	Variance	95% CI for σ using Chi-Square
51	0.370	0.137	(0.310, 0.460)

Test

Null hypothesis $H_0: \sigma = 0.25$

Alternative hypothesis $H_1: \sigma \neq 0.25$

Test

Recall $\sigma_0 = .25$
 If σ_0 is in the confidence interval, we fail to reject H_0
 If σ_0 is not in the interval, Reject H_0

P-value Approach

WORKSHEET 1

Test and CI for One Variance

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40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49

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$$\chi^2_{.975, 50} = 32.36$$

$$\chi^2_{.025, 50} = 71.42$$