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Section 1

MANE 6313

Subsection 1

Week 2, Module F

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome

Select correct two sample test of hypothesis.

Inference for Two Population

mean or Variance

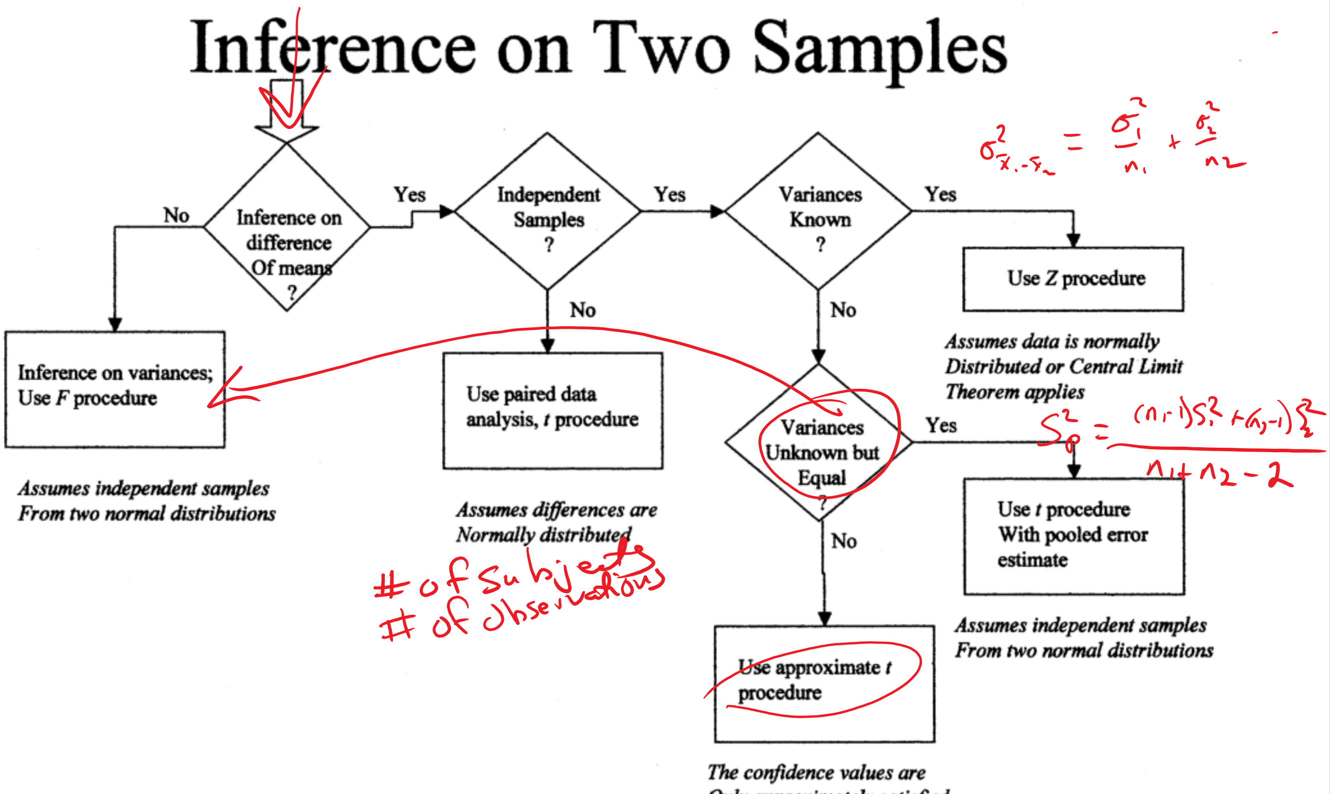

Table 2.4[1]

■ **TABLE 2.4**
Tests on Means with Variance Known

Hypothesis	Test Statistic	Fixed Significance Level Criteria for Rejection	*P*-Value
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$		$\lvert Z_0 \rvert > Z_{\alpha/2}$	$P = 2[1 - \Phi(\lvert Z_0 \rvert)]$
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$	$Z_0 = \dfrac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}$	$Z_0 < -Z_\alpha$	$P = \Phi(Z_0)$
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$		$Z_0 > Z_\alpha$	$P = 1 - \Phi(Z_0)$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$	2-sample z-test	$\lvert Z_0 \rvert > Z_{\alpha/2}$	$P = 2[1 - \Phi(\lvert Z_0 \rvert)]$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	$Z_0 = \dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$Z_0 < -Z_\alpha$	$P = \Phi(Z_0)$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$		$Z_0 > Z_\alpha$	$P = 1 - \Phi(Z_0)$

[1] Montgomery (2017). *Design and Analysis of Experiments, 9th edition.* John Wiley & Sons.

Table 2.5[2]

TABLE 2.5

Tests on Means of Normal Distributions, Variance Unknown

Hypothesis	Test Statistic	Fixed Significance Level Criteria for Rejection	*P*-Value
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$		$\lvert t_0 \rvert > t_{\alpha/2,n-1}$	sum of the probability above t_0 and below $-t_0$
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$	$t_0 = \dfrac{\bar{y} - \mu_0}{S/\sqrt{n}}$	$t_0 < -t_{\alpha,n-1}$	probability below t_0
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$		$t_0 > t_{\alpha,n-1}$	probability above t_0
	if $\sigma_1^2 = \sigma_2^2$, unknown		
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$	$t_0 = \dfrac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ $v = n_1 + n_2 - 2$	$\lvert t_0 \rvert > t_{\alpha/2,v}$	sum of the probability above t_0 and below $-t_0$
	if $\sigma_1^2 \neq \sigma_2^2$		
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	$t_0 = \dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$	$t_0 < -t_{\alpha,v}$	probability below t_0
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$	$v = \dfrac{\left(\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right)^2}{(S_1^2/n_1)^2 \quad (S_2^2/n_2)^2}$	$t_0 > t_{\alpha,v}$	probability above t_0

approximate t-test

adjusted d.f. round

Table 2.8[3]

■ TABLE 2.8
Tests on Variances of Normal Distributions

Hypothesis	Test Statistic	Fixed Significance Level Criteria for Rejection
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 \neq \sigma_0^2$		$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 = \dfrac{(n-1)S^2}{\sigma_0^2}$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$
$H_0: \sigma^2 = \sigma_0^2$ $H_1: \sigma^2 > \sigma_0^2$		$\chi_0^2 > \chi^2_{\alpha,n-1}$
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 \neq \sigma_2^2$	$F_0 = \dfrac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha/2,n_1-1,n_2-1}$ or $F_0 < F_{1-\alpha/2,n_1-1,n_2-1}$
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 < \sigma_2^2$	$F_0 = \dfrac{S_2^2}{S_1^2}$	$F_0 > F_{\alpha,n_2-1,n_1-1}$
$H_0: \sigma_1^2 = \sigma_2^2$ $H_1: \sigma_1^2 > \sigma_2^2$	$F_0 = \dfrac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha,n_1-1,n_2-1}$

F-test

[3] Montgomery (2017). *Design and Analysis of Experiments, 9th edition.* John Wiley & Sons.

Montgomery & Runger[4]

Summary of Two-Sample Hypothesis-Testing Procedures

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis	Fixed Significance Level Criteria for Rejection	P-value	O.C. Curve Parameter
1.	$H_0: \mu_1 - \mu_2 = \Delta_0$ σ_1^2 and σ_2^2 known	$z_0 = \dfrac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$H_1: \mu_1 - \mu_2 \neq \Delta_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$P = 2[1 - \Phi(\lvert z_0 \rvert)]$	$d = \dfrac{\lvert \mu_1 - \mu_2 - \Delta_0 \rvert}{\sqrt{\sigma_1^2 + \sigma_2^2}}$
			$H_1: \mu_1 - \mu_2 > \Delta_0$	$z_0 > z_\alpha$	Probability above z_0 $P = 1 - \Phi(z_0)$	$d = \dfrac{\mu_1 - \mu_2 - \Delta_0}{\sqrt{\sigma_1^2 + \sigma_2^2}}$
			$H_1: \mu_1 - \mu_2 < \Delta_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	$d = \dfrac{\mu_2 - \mu_1 - \Delta_0}{\sqrt{\sigma_1^2 + \sigma_2^2}}$
2.	$H_0: \mu_1 - \mu_2 = \Delta_0$ $\sigma_1^2 = \sigma_2^2$ unknown	$t_0 = \dfrac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$	$H_1: \mu_1 - \mu_2 \neq \Delta_0$	$\lvert t_0 \rvert > t_{\alpha/2, n_1+n_2-2}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	$d = \lvert \Delta - \Delta_0 \rvert / 2\sigma$
			$H_1: \mu_1 - \mu_2 > \Delta_0$	$t_0 > t_{\alpha, n_1+n_2-2}$	Probability above t_0	$d = (\Delta - \Delta_0)/2\sigma$
			$H_1: \mu_1 - \mu_2 < \Delta_0$	$t_0 < -t_{\alpha, n_1+n_2-2}$	Probability below t_0	$d = (\Delta_0 - \Delta)/2\sigma$ where $\Delta = \mu_1 - \mu_2$
3.	$H_0: \mu_1 - \mu_2 = \Delta_0$ $\sigma_1^2 \neq \sigma_2^2$ unknown	$t_0 = \dfrac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$ $v = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}}$	$H_1: \mu_1 - \mu_2 \neq \Delta_0$	$\lvert t_0 \rvert > t_{\alpha/2, v}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	3–4
			$H_1: \mu_1 - \mu_2 > \Delta_0$	$t_0 > t_{\alpha, v}$	Probability above t_0	3–4
			$H_1: \mu_1 - \mu_2 < \Delta_0$	$t_0 < -t_{\alpha, v}$	Probability below t_0	3–4
4.	Paired data $H_0: \mu_D = 0$	$t_0 = \dfrac{\bar{d}}{s_d/\sqrt{n}}$	$H_1: \mu_d \neq 0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	3–4
			$H_1: \mu_d > 0$	$t_0 > t_{\alpha, n-1}$	Probability above t_0	3–4
			$H_1: \mu_d < 0$	$t_0 < -t_{\alpha, n-1}$	Probability below t_0	3–4
5.	$H_0: \sigma_1^2 = \sigma_2^2$	$f_0 = s_1^2/s_2^2$	$H_1: \sigma_1^2 \neq \sigma_2^2$	$f_0 > f_{\alpha/2, n_1-1, n_2-1}$ or $f_0 < f_{1-\alpha/2, n_1-1, n_2-1}$	See text Section 10-5.2.	$\lambda = \sigma_1/\sigma_2$
			$H_1: \sigma_1^2 > \sigma_2^2$	$f_0 > f_{\alpha, n_1-1, n_2-1}$		$\lambda = \sigma_1/\sigma_2$
6.	$H_0: p_1 = p_2$	$z_0 = \dfrac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}(1-\hat{p})\left[\dfrac{1}{n_1} + \dfrac{1}{n_2}\right]}}$	$H_1: p_1 \neq p_2$	$\lvert z_0 \rvert > z_{\alpha/2}$	$P = 2[1 - \Phi(z_0)]$	3–4
			$H_1: p_1 > p_2$	$z_0 > z_\alpha$	Probability above z_0 $P = 1 - \Phi(z_0)$	3–4
				$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	3–4

Montgomery & Runger[5]

Summary of Two-Sample Confidence Interval Procedures

Case	Problem Type	Point Estimate	
1.	Difference in two means μ_1 and μ_2, variances σ_1^2 and σ_2^2 known	$\bar{x}_1 - \bar{x}_2$	$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$
2.	Difference in means of two normal distributions $\mu_1 - \mu_2$, variances $\sigma_1^2 = \sigma_2^2$ and unknown	$\bar{x}_1 - \bar{x}_2$	$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,n_1+n_2-2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu$ $\le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,n_1+n_2-2}s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ where $s_p = \sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}}$
3.	Difference in means of two normal distributions $\mu_1 - \mu_2$, variances $\sigma_1^2 \ne \sigma_2^2$ and unknown	$\bar{x}_1 - \bar{x}_2$	$\bar{x}_1 - \bar{x}_2 - t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + t_{\alpha/2,v}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ where $v = \frac{(s_1^2/n_1 + s_2^2/n_2)^2}{\frac{(s_1^2/n_1)^2}{n_1-1} + \frac{(s_2^2/n_2)^2}{n_2-1}}$
4.	Difference in means of two normal distributions for paired samples $\mu_D = \mu_1 - \mu_2$	$\bar{d}$	$\bar{d} - t_{\alpha/2,n-1}s_d/\sqrt{n} \le \mu_D \le \bar{d} + t_{\alpha/2,n-}$
5.	Ratio of the variances σ_1^2/σ_2^2 of two normal distributions	$\frac{s_1^2}{s_2^2}$	$\frac{s_1^2}{s_2^2} f_{1-\alpha/2,n_2-1,n_1-1} \le \frac{\sigma_1^2}{\sigma_2^2} \le \frac{s_1^2}{s_2^2} f_{\alpha/2,n_2-}$ where $f_{1-\alpha/2,n_2-1,n_1-1} = \frac{1}{f_{\alpha/2,n_1-1,n_2-1}}$
6.	Difference in two proportions of two binominal parameters $p_1 - p_2$	$\hat{p}_1 - \hat{p}_2$	$\hat{p}_1 - \hat{p}_2 - z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-}{n_2}}$ $\le p_1 - p_2 \le \hat{p}_1 - \hat{p}_2 + z_{\alpha/2}\sqrt{\frac{\hat{p}_1(1-}{n_1}}$