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Section 1

MANE 6313

Subsection 1

Week 2, Module G

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome

Analyze an F-test.

Example: F-test[1]

10-33. The overall distance traveled by a golf ball is tested by hitting the ball with Iron Byron, a mechanical golfer with a swing that is said to emulate the distance hit by the legendary champion, Byron Nelson. Ten randomly selected balls of two different brands are tested and the overall distance measured. The data follow:

Brand 1: 275, 286, 287, 271, 283, 271, 279, 275, 263, 267
Brand 2: 258, 244, 260, 265, 273, 281, 271, 270, 263, 268

F-test

(a) Is there evidence that overall distance is approximately normally distributed? Is an assumption of equal variances justified?
(b) Test the hypothesis that both brands of ball have equal mean overall distance. Use $\alpha = 0.05$. What is the P-value?
(c) Construct a 95% two-sided CI on the mean difference in overall distance for the two brands of golf balls.
(d) What is the power of the statistical test in part (b) to detect a true difference in mean overall distance of 5 yards?
(e) What sample size would be required to detect a true difference in mean overall distance of 3 yards with power of approximately 0.75?

Classical Approach

WORKSHEET 2

Descriptive Statistics: Brand 1, Brand 2

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Brand 1	10	0	275.70	2.54	8.03	263.00	270.00	275.00	283.75	287.00
Brand 2	10	0	265.30	3.18	10.04	244.00	259.50	266.50	271.50	281.00

$H_0: \frac{\sigma_1^2}{\sigma_2^2} = 1$

$H_1: \frac{\sigma_1^2}{\sigma_2^2} \neq 1 \quad (\alpha = .05)$

Test Statistic

$F_0 = \frac{S_1^2}{S_2^2} = \frac{8.03^2}{10.04^2} = 0.6397$

Rejection Region & Conclusion[a]

[a]Montgomery and Runger (2014). *Applied Statistics and Probability for Engineers, 6th edition.* John Wiley & Sons.

$$F_0 = \frac{S_1^2}{S_2^2} = 0.6397$$

Rejection Region

$F_0 > F_{\alpha/2, n_1-1, n_2-1}$

or

$F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$

$F_{.025, 7, 9} = 4.03$

$$F_{.975, n_1-1, n_2-1} = \frac{1}{F_{.025, n_2-1, n_1-1}}$$

$$= \frac{1}{4.03} = .2481$$

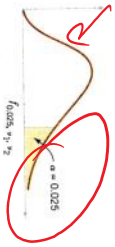
fail to reject H_0, accept assumption of equal variances

TABLE • V1 Percentage Points f_{α,n_1,n_2} of the *F* Distribution (*Continued*)

Degrees of freedom for the denominator (ν_2)

Degrees of freedom for the numerator (ν_1)

ν_2	ν_1 = 1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.76
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48



Chi-squared Test using Minitab

WORKSHEET 2

Test and CI for Two Variances: Brand 1, Brand 2

Method

σ_1: standard deviation of Brand 1
σ_2: standard deviation of Brand 2
Ratio: σ_1/σ_2
F method was used. This method is accurate for normal data only.

Descriptive Statistics

Variable	N	StDev	Variance	95% CI for σ^2
Brand 1	10	8.028	64.456	(30.495, 214.821)
Brand 2	10	10.045	100.900	(47.738, 336.285)

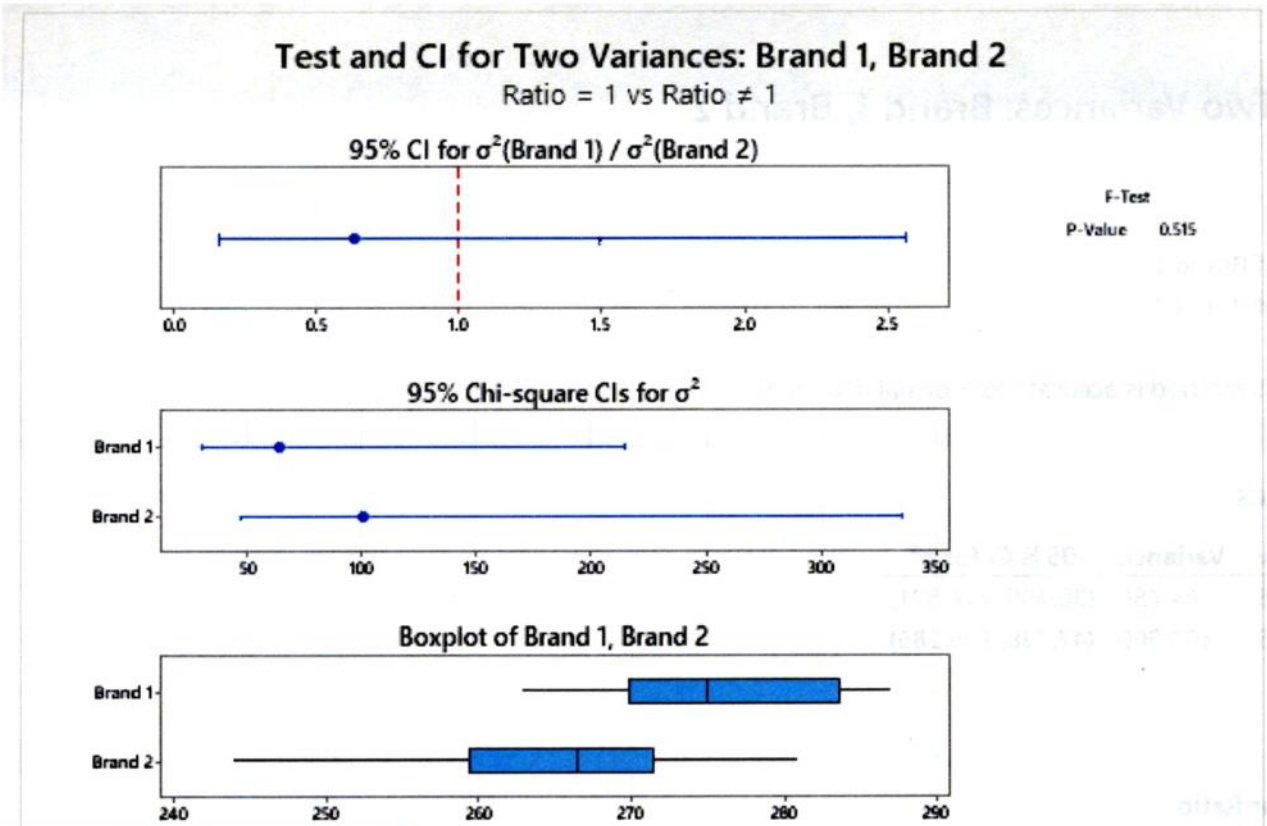
Ratio of Variances

Estimated Ratio	95% CI for Ratio using F
0.638806	(0.159, 2.572)

Test

Null hypothesis	H_0: $\sigma_1^2 / \sigma_2^2 = 1$
Alternative hypothesis	H_1: $\sigma_1^2 / \sigma_2^2 \neq 1$
Significance level	$\alpha = 0.05$

Chi-squared Test using Minitab



P-value Approach

WORKSHEET 2

Descriptive Statistics: Brand 1, Brand 2

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Brand 1	10	0	275.70	2.54	8.03	263.00	270.00	275.00	283.75	287.00
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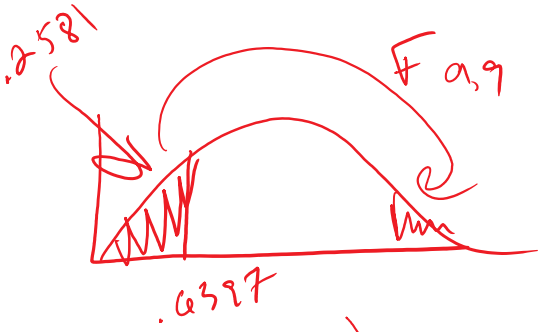

Test

Null hypothesis	$H_0: \sigma_1^2 / \sigma_2^2 = 1$
Alternative hypothesis	$H_1: \sigma_1^2 / \sigma_2^2 \neq 1$
Significance level	$\alpha = 0.05$

Method	Test Statistic	DF1	DF2	P-Value
F	0.64	9	9	0.515

$F_{9,9}(.6397)$

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P-value = 2(.2581) = .5162