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Sunday, August 29, 2021 2:44 PM

Section 1

MANE 6313

Subsection 1

Week 2, Module H

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome

Analyze a two-sample t-test with unknown and equal variances.

Example: Two-sample t-test[1]

10-33. The overall distance traveled by a golf ball is tested by hitting the ball with Iron Byron, a mechanical golfer with a swing that is said to emulate the distance hit by the legendary champion, Byron Nelson. Ten randomly selected balls of two different brands are tested and the overall distance measured. The data follow:

Brand 1: 275, 286, 287, 271, 283, 271, 279, 275, 263, 267
Brand 2: 258, 244, 260, 265, 273, 281, 271, 270, 263, 268

(a) Is there evidence that overall distance is approximately normally distributed? Is an assumption of equal variances justified?
(b) Test the hypothesis that both brands of ball have equal mean overall distance. Use $\alpha = 0.05$. What is the P-value?
(c) Construct a 95% two-sided CI on the mean difference in overall distance for the two brands of golf balls.
(d) What is the power of the statistical test in part (b) to detect a true difference in mean overall distance of 5 yards?
(e) What sample size would be required to detect a true difference in mean overall distance of 3 yards with power of approximately 0.75?

Classical Approach

$H_0: \mu_1 - \mu_2 = 0$

$H_1: \mu_1 - \mu_2 \neq 0 \quad (\alpha = .05)$

WORKSHEET 2

Descriptive Statistics: Brand 1, Brand 2

Statistics

Variable	N	N*	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
Brand 1	10	0	275.70	2.54	8.03	263.00	270.00	275.00	283.75	287.00
Brand 2	10	0	265.30	3.18	10.04	244.00	259.50	266.50	271.50	281.00

TS: $t_0 = \dfrac{\bar{x}_1 - \bar{x}_2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$

$s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} = \dfrac{(10-1)8.03^2 + (10-1)(10.04)^2}{10+10-2} = 82.643 \quad 9.0907$

$s_p = \sqrt{s_p^2} = \sqrt{82.643} = 9.0907$

$t_0 = \dfrac{275.7 - 265.3}{9.0907\sqrt{\frac{1}{10}+\frac{1}{10}}} = 2.5581$

Rejection Region & Conclusion[a]

[a]Montgomery and Runger (2014). *Applied Statistics and Probability for Engineers, 6th edition.* John Wiley & Sons.

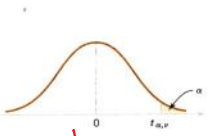

TABLE • V Percentage Points $t_{\alpha,\nu}$ of the t Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674

Rejection Region

Reject H_0 if

$|t_0| > t_{\alpha/2, n_1+n_2-2}$

$|t_0| > t_{.025,18} = 2.101$

$t_0 = 2.5581$

Conclusion: Reject H_0

Two-sample t-test using Minitab

WORKSHEET 2

Two-Sample T-Test and CI: Brand 1, Brand 2

Method

μ_1: population mean of Brand 1
μ_2: population mean of Brand 2
Difference: $\mu_1 - \mu_2$

Equal variances are assumed for this analysis.

Descriptive Statistics

Sample	N	Mean	StDev	SE Mean
Brand 1	10	275.70	8.03	2.5
Brand 2	10	265.3	10.0	3.2

Estimation for Difference

Difference	Pooled StDev	95% CI for Difference
10.40	9.09	(1.86, 18.94)

Sp

Test

Null hypothesis H_0: $\mu_1 - \mu_2 = 0$
Alternative hypothesis H_1: $\mu_1 - \mu_2 \neq 0$

P-value Approach

- Use same approach as demonstrated in Week 2, Module D