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Section 1

MANE 6313

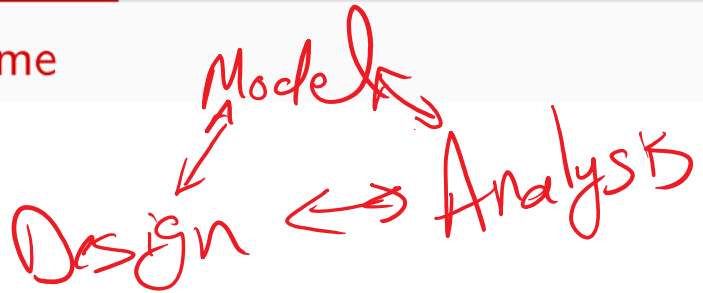
Subsection 1

Week 4, Module A

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome



Explain the model, parameters, strategy and tools used in an One-way Analysis of Variance (ANOVA).

One-way Analysis of Variance (ANOVA)

- Based upon the (fixed effects) model

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \quad \begin{cases} i = 1, 2, \dots, a \\ j = 1, 2, \dots, n \end{cases}$$

treatment effect
treatment replicates
random error

- There are a different factors. τ_i is the parameter associated with the i -th factor level
- There are n observations for each factor level
- Test the hypothesis

$$H_0 : \tau_1 = \tau_2 = \dots = \tau_a = 0$$

$$H_1 : \tau_i \neq 0 \text{ for at least one } i$$

model $y_{ij} = \mu + \varepsilon_{ij}$
→ all zero

with the requirement that $\sum_{i=1}^a \tau_i = 0$.

Important Points

- The statistical model is the *fixed effects model*
- The design used for the experimentation is called the completely randomized design
- A design is said to be *balanced* if the number of replicates for treatment level are equal.
- We will not deal with unbalanced designs in this course (but it is in your textbook for reference)
- Discuss notation

- Strategy is to partition variability.
 - Sum of Squares total

$$SS_T = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{..})^2$$

each observation
 $\bar{y}_{..}$ - overall mean

- Factor Sum of Squares:

Treatment

$$SS_{\text{Factor}} = n \sum_{i=1}^a (\bar{y}_{i.} - \bar{y}_{..})^2$$

$\bar{y}_{i.}$ - i^{th} treatment mean

- Error Sum of Squares:

$$SS_E = \sum_{i=1}^a \sum_{j=1}^n (y_{ij} - \bar{y}_{i.})^2$$

$$y_{ij} = \mu + \alpha_i + \epsilon_{ij}$$

- Note: $SS_T = SS_{\text{Factor}} + SS_E$

$$\rightarrow SS_E = SS_T - SS_{\text{Factor}}$$

- There are two method of calculating sum of squares for the one-way ANOVA.

- ① By hand using the convenience formulas:

$$SS_T = \sum_{i=1}^a \sum_{j=1}^n y_{ij}^2 - \frac{y_{..}^2}{an}$$

$$SS_{\text{Factor}} = \sum_{i=1}^a \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{an}$$

Computationally Superior

2. Using software such as MS Excel, or Minitab (there are otl

ANOVA Table

Analysis of Variance

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between Factors	SS_{Factor}	$a - 1$	$MS_{Factor} = \frac{SS_{Factor}}{a - 1}$	$F_0 = \frac{MS_{Factor}}{MS_E}$
Within (error)	SS_E	$a(n - 1)$ subtraction	$MS_E = \frac{SS_E}{a(n - 1)}$	
Total	SS_T	$an - 1$		

Reject H_0 if $F_0 > F_{\alpha, a-1, a(n-1)}$

Model Parameters

$$x_{ij} = \mu + \tau_i + \varepsilon_{ij} \quad \begin{cases} i = 1, \dots, a \\ j = 1, \dots, n \end{cases}$$

↑
unobservable

- Point estimators

$$\begin{aligned} \hat{\mu} &= \bar{y}_{..} \\ \hat{\tau}_i &= (\bar{y}_{i.} - \bar{y}_{..}) \quad i = 1, 2, \dots, a \\ \hat{\mu}_i &= \hat{\mu} + \hat{\tau}_i = \bar{y}_{i.} \end{aligned}$$

- Properties of ε_{ij}

- It is assumed that ε_{ij} is a $NID(0, \sigma^2)$ random variable

- Properties of the ANOVA

- $E(MS_E) = \sigma^2$ (noise)

- $E(MS_{\text{Treatments}}) = \sigma^2 + \frac{n \sum_{i=1}^a \tau_i^2}{a-1}$ (noise + signal)

$F_0 \approx \frac{\sigma^2 + \frac{n \sum \tau_i^2}{a-1}}{\sigma^2} \rightarrow \frac{\text{signal} + \text{noise}}{\text{noise}}$

Fixed vs. Random Factor?

- For the fixed model, the experimenter specifically chooses the a treatments.
- We estimate the parameters (μ, τ_i, σ^2)
- If the treatment levels were generated by randomly sampling from a population of treatments, the model is the random effects model
- In the random effects model, we generalize the sample results to the entire population of treatments.