Printout

Monday, September 13, 2021 7:27 AM

Section 1

MANE 6313

Subsection 1

Week 4, Module A

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome

Model

Design $\leftrightarrow$ Analysis

Explain the model, parameters, strategy and tools used in an One-way Analysis of Variance (ANOVA).

One-way Analysis of Variance (ANOVA)

treatment effect

- Based upon the (fixed effects) model

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij} \begin{cases} i = 1, 2, \ldots, a \\ j = 1, 2, \ldots, n \end{cases}$$

treatments
replicates
random error

- There are a different factors. τ_i is the parameter associated with the i -th factor level
- There are n observations for each factor level

model $y_{ij} = \mu + \varepsilon_{ij}$

- Test the hypothesis

$$H_0: \quad \tau_1 = \tau_2 = \cdots = \tau_a = 0$$
$$H_1: \quad \tau_i \neq 0 \text{ for at least one } i$$

→ all zero

with the requirement that $\sum_{i=1}^{a} \tau_i = 0$.

Important Points

- The statistical model is the *fixed effects model*
- The design used for the experimentation is called the *completely randomized design*
- A design is said to be *balanced* if the number of replicates for treatment level are equal.
- We will not deal with unbalanced designs in this course (but it is in your textbook for reference)
- Discuss notation

- Strategy is to partition variability.
  - Sum of Squares total

each observation

$\bar{y}_{..}$ - overall mean

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{..})^2$$

- Factor Sum of Squares:

$\bar{y}_{i.}$ - i^{th} treatment mean

Treatment

$$SS_{\text{Factor}} = n\sum_{i=1}^{a}(\bar{y}_{i.} - \bar{y}_{..})^2$$

- Error Sum of Squares:

$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$

$$SS_E = \sum_{i=1}^{a}\sum_{j=1}^{n}(y_{ij} - \bar{y}_{i.})^2$$

- Note: $SS_T = SS_{\text{Factor}} + SS_E$ $\Rightarrow SS_E = SS_T - SS_{\text{Factor}}$

- There are two method of calculating sum of squares for the one-way ANOVA.
  1. By hand using the convenience formulas:

$$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{n} y_{ij}^2 - \frac{y_{..}^2}{an}$$

$$SS_{\text{Factor}} = \sum_{i=1}^{a} \frac{y_{i.}^2}{n} - \frac{y^2..}{an}$$

Computationally Superior

2. Using software such as MS Excel, or Minitab (there are otl

ANOVA Table

Analysis of Variance

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between Factors	SS_{Factor}	$a-1$	$MS_{Factor} = \frac{SS_{Factor}}{a-1}$	$F_0 = \frac{MS_{Factor}}{MS_E}$
Within (error)	SS_E	$a(n-1)$ subtraction	$MS_E = \frac{SS_E}{a(n-1)}$	
Total	SS_T	$an-1$		

Reject H_0 if $F_0 > F_{\alpha,a-1,a(n-1)}$

Model Parameters

$x_{ij} = \mu + \tau_i + \varepsilon_{ij}$ $\begin{cases} i = 1, \ldots, a \\ j = 1, \ldots, n \end{cases}$

unobservable

- Point estimators

$$\begin{aligned} \hat{\mu} &= \bar{y}_{..} \\ \hat{\tau}_i &= (\bar{y}_{i.} - \bar{y}_{..}) \quad i = 1, 2, \ldots, a \\ \hat{\mu}_i &= \hat{\mu} + \hat{\tau}_i = \bar{y}_{i.} \end{aligned}$$

- Properties of ε_{ij}
  - It is assumed that ε_{ij} is a NID$(0, \sigma^2)$ random variable
- Properties of the ANOVA
  - $E(MS_{\mathrm{E}}) = \sigma^2$ (noise)
  - $E(MS_{\mathrm{Treatments}}) = \sigma^2 + \frac{n \sum_{i=1}^{a} \tau_i^2}{a-1}$ (noise + signal)

$F_0 \approx n \quad \dfrac{\sigma^2 + r \sum \tau_i^2 / (c-1)}{\sigma^2} \rightarrow \dfrac{\text{signal} + \text{noise}}{\text{noise}}$

Fixed vs. Random Factor?

- For the fixed model, the experimenter specifically chooses the *a* treatments.
- We estimate the parameters (μ, τ_i, σ^2)
- If the treatment levels were generated by randomly sampling from a population of treatments, the model is the random effects model
- In the random effects model, we generalize the sample results to the entire population of treatments.