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Section 1

MANE 6313

Subsection 1

Week 4, Module D

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome

Assess the assumption of equal variances in one-way analysis of variance.

Bartlett's Test

$$H_0 : \sigma_1^2 = \sigma_2^2 = \cdots = \sigma_a^2$$

$$H_1 : \text{above not true for at least one } \sigma_i^2$$

- TS: $\chi_0^2 = 2.3026 \frac{q}{c}$ where

$$q = (N - a) \log_{10} S_p^2 - \sum_{i=1}^a (n_i - 1) \log_{10} S_i^2$$

$$c = 1 + \frac{1}{3(a-1)} \left(\sum_{i=1}^a (n_i - 1)^{-1} - (N - a)^{-1} \right)$$

$$S_p^2 = \frac{\sum_{i=1}^a (n_i - 1) S_i^2}{N - 1}$$

- Reject H_0 if $\chi_0^2 > \chi_{\alpha, a-1}^2$
- Very sensitive to normality assumption

Bartlett's Test Example Problem

- 12.30 Ott (1993: 787, 788) considers an experiment in which 10 specimens were obtained from the centers of each of three lakes at a depth of 1 foot. The mean dissolved oxygen contents (in ppm) were then calculated for each of the 30 specimens and the following data were obtained.

A		Lake		C	
		B			
0	2	1	7	14	22
2	3	3	5	26	21
1	4	4	3	25	16
3	1	6	4	18	20
1	5	8	5	19	30

Source: Data from R. Lyman Ott, 1993, *An Introduction to Statistical Methods and Data Analysis*, 4th ed. Belmont, CA: Wadsworth. Reprinted with permission of the publisher.

- 12.10 Twenty-five (relatively homogeneous) plots of ground were prepared in an identical fashion and planted with the same grain. Randomly selected plots received fertilizer treatments (in pounds per plot) as shown in the accompanying table. Tabled values indicate plot yields (in bushels per acre).

Control	Fertilizer Levels				
0	10	20	30	40	
20	25	36	35	43	
25	29	37	39	40	
23	31	29	31	36	
27	30	40	42	48	
19	27	33	44	47	

Analysis of Variance by Hand

Problem 12.30

Descriptive Statistics

Variable	lake	N	Mean	Median	TrMean	StDev
ppm	A	10	2.200	2.000	2.125	1.549
	B	10	4.600	4.500	4.625	2.066
	C	10	21.10	20.50	20.88	4.84

Variable	lake	SE Mean	Minimum	Maximum	Q1	Q3
ppm	A	0.490	0.000	5.000	1.000	3.250
	B	0.653	1.000	8.000	3.000	6.250
	C	1.53	14.00	30.00	17.50	25.25

To Construct Anova

$$\sum \sum y_{ij} = 279$$

$$\sum \sum y_{ij}^2 = 4,983$$

$$SS_T = \sum y_{ij}^2 - \frac{y_{..}^2}{n} = 4983 - \frac{279^2}{30} = 2388.3$$

$$SS_{\text{Treatment}} = \sum \frac{y_{i.}^2}{n} - \frac{y_{..}^2}{n} = \left(\frac{[10(2.20)]^2}{10} + \frac{[10(4.60)]^2}{10} + \frac{[10(21.10)]^2}{10} \right) - \frac{279^2}{30} = 2117.40$$

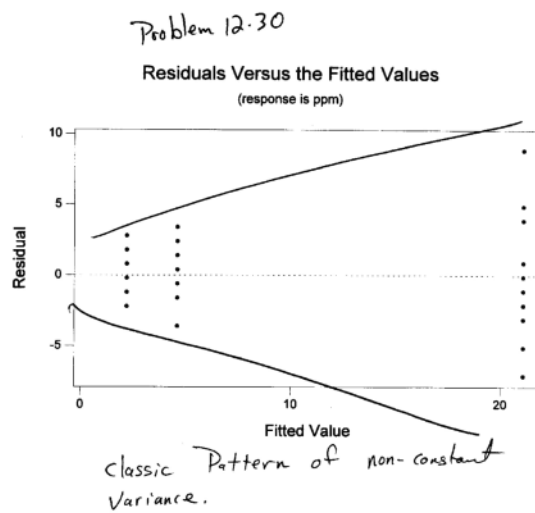
Anova	df	SS	MS	F
Treatment	2	2117.4	1058.7	105.87
Error	27	270.9	10.0	
Total	29	2388.3		

Rejection Region reject H_0 if $F_0 > F_{\alpha, 2, 27}$
at $\alpha = 0.05$

$$F_{0.05, 2, 27} = 3.328$$

Conclusion: reject H_0 and conclude that at least one of the lakes has a different dissolved oxygen content.

Residual Plot



Bartlett's Test by Hand

Bartlett's

$$S_A^2 = 2.399 \quad S_B^2 = 4.268 \quad S_C^2 = 23.426$$

$$S_p^2 = 10.031$$

$$q = (30-3) \log_{10}(10.031) - \left[9 \log_{10} 2.399 + 9 \log_{10} 4.268 + 9 \log_{10} 23.426 \right]$$

$$= 5.617$$

$$C = 1 + \frac{1}{3(2)} \left(\frac{3}{27} - \frac{1}{27} \right)$$

$$= 1.0494$$

$$\frac{q}{C} = 5.353$$

$$\chi^2_0 = 23026(5.353) = 12.325$$

$$\chi^2_{0.05, 2} = 5.99$$

Conclusion: Unequal Variances

Minitab Demonstration

- Bartlett's Test

