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Section 1

MANE 6313

Subsection 1

Week 5, Module C

Student Learning Outcome

Analyze simple comparative experiments and experiments with a single factor.

Module Learning Outcome

Evaluate Latin Square designs.

Latin Square Design

blocking variables

- Design that allows the elimination of two nuisance sources of variability.
- Each row and column represents a restriction on randomization (number of rows=number of columns=number of treatments) p
- The statistical model is

$$y_{ijk} = \mu + \alpha_i + \tau_j + \beta_k + \varepsilon_{ijk} \quad \begin{cases} i = 1, 2, \dots, p \\ j = 1, 2, \dots, p \\ k = 1, 2, \dots, p \end{cases}$$

Handwritten annotations:
 - An arrow points from "row" to α_i .
 - An arrow points from "treatment" to τ_j .
 - An arrow points from "column" to β_k .
 - An arrow points from "error" to ε_{ijk} .

where α_i is the i th row effect, τ_j is the j th treatment effect, and β_k is the k th column effect

- ANOVA and SS formulas given on pg. 155.

Problem 4.28

4.28 An industrial engineer is investigating the effect of four assembly methods (A, B, C, D) on the assembly time for a color television component. Four operators are selected for the study. Furthermore, the engineer knows that each assembly method produces such fatigue that the time required for the last assembly may be greater than the time required for the first, regardless of the method. That is, a trend develops in the required assembly time. To account for this source of variability, the engineer uses the Latin square design that follows. Analyze the data from this experiment ($\alpha = 0.05$) and draw appropriate conclusions.

Order of Assembly	Operator			
	1	2	3	4
1	C = 10	D = 14	A = 7	B = 8
2	B = 7	C = 18	D = 11	A = 8
3	A = 5	B = 10	C = 11	D = 9
4	D = 10	A = 10	B = 12	C = 14

→ each treatment appears once in every row

each treatment appears once in every column

Problem 4.28, continued

Problem 4-28 -

Total Sum of Squares

$$\sum \sum \sum y_{ijk}^2 = 1834 \quad y_{...} = 164$$

$$SS_{\text{total}} = 1834 - \frac{164^2}{16}$$

$$= 153$$

Treatments

$$y_{A.} = (5+10+7+8) = 30$$

$$y_{B.} = (7+10+12+8) = 37$$

$$y_{C.} = (10+18+11+14) = 53$$

$$y_{D.} = (10+14+11+5) = 44$$

$$SS_{\text{treatments}} = \frac{1}{4}(30^2 + 37^2 + 53^2 + 44^2) - \frac{164^2}{16}$$

$$= 72.5$$

Rows

$$y_{...} = (10+14+7+8) = 39$$

$$y_{..} = (7+18+11+8) = 44$$

$$y_{3..} = (5+10+11+9) = 35$$

$$y_{4..} = (10+10+12+14) = 46$$

$$SS_{\text{rows}} = \frac{1}{4}(39^2 + 44^2 + 35^2 + 46^2) - \frac{164^2}{16}$$

$$= 18.5$$

Problem 4.28, continued

Problem 4.28, page 2

Columns

$$y_{.1} = (10 + 7 + 5 + 10) = 32$$

$$y_{.2} = (14 + 18 + 10 + 10) = 52$$

$$y_{.3} = (7 + 11 + 11 + 12) = 41$$

$$y_{.4} = (8 + 8 + 9 + 14) = 39$$

$$SS_{\text{columns}} = \frac{1}{4}(32^2 + 52^2 + 41^2 + 39^2) - \frac{424^2}{16}$$

$$= 51.5$$

ANOVA

Source	df	SS	MS	F ₀
Treatment	3	72.5	24.167	13.810 *
Rows	3	18.5	6.167	3.544
Columns	3	51.5	17.167	9.810 *
Error	6	10.5	1.75	
Total	15	153		

$F_{3,6,05} = 4.76$

Problem 4.28, continued

- Minitab Demonstration

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↓	C1	C2	C3	C4-T	C
	time	operator	order	treatment	
1	10	1	1	C	
2	7	1	2	B	
3	5	1	3	A	
4	10	1	4	D	
5	14	2	1	D	

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Factor Information

Factor	Type	Levels	Values
operator	Random	4	1, 2, 3, 4
order	Random	4	1, 2, 3, 4
treatment	Fixed	4	A, B, C, D

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
operator	3	51.50	17.167	9.81	0.010
order	3	18.50	6.167	3.52	0.089
treatment	3	72.50	24.167	13.81	0.004
Error	6	10.50	1.750		
Total	15	153.00			

$\alpha = 0.05$

Sign
not sign.
Sign.

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
1.32288	93.14%	82.84%	51.20%

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