

# Printout

Sunday, October 10, 2021 11:39 AM

## Section 1

MANE 6313

## Subsection 1

### Week 8, Module C

## Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

## Module Learning Outcome

*Creating design with more than two blocks.*

### Confounding the $2^k$ design in 4 blocks

- You must define 2 linear combinations  $L_1$  and  $L_2$
- For each treatment effect, we construct the ordered pair  $(L_1 \bmod 2, L_2 \bmod 2)$
- This will result in four blocks having values (0,0), (0,1), (1,0) and (1,1)
- Work a  $2^4$  example in 4 blocks

$$\begin{aligned} L_1 &= A B C = 1x_1 + 1x_2 + 1x_3 + 0x_4 \\ L_2 &= A C D = 1x_1 + 0x_2 + 1x_3 + 0x_4 \end{aligned}$$

$$L_1 = 1x_1 + 1x_2 + 1x_3 + 0x_4$$

$$L_2 = 1x_1 + 0x_2 + 1x_3 + 1x_4$$

## Four Block Example $(L_1, L_2)$

	$x_1$	$x_2$	$x_3$	$x_4$	$L_1 \text{ mod } 2$	$L_2 \text{ Mod } 2$	$(0,0)$	$(1,0)$	$(0,1)$	$(1,1)$
$t, t$	0	0	0	0	0	0	(1)	b	ab	a
$c$	1	0	0	0	1	1	ac	abc	bc	c
$a$	0	1	0	0	0	0	abd	acd	d	bcd
$b$	1	0	0	0	1	1	bcd	cd	ad	abcd
$a, b$	0	1	0	0	0	0				
$a, c$	1	0	0	0	1	1				
$a, b, c$	0	1	0	0	0	0				
$a, d$	1	0	0	0	1	1				
$a, b, d$	0	1	0	0	0	0				
$a, b, c, d$	1	0	0	0	1	1				
$a, b, c, d$	0	1	0	0	0	0				
$a, b, c, d$	1	0	0	0	1	1				
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$a, b, c, d$	0	1	0	0	0					

$$\begin{array}{r}
 \text{ANOVA} \\
 \hline
 \text{Source} \quad d.f. \\
 \hline
 \text{Block} \quad 4-1=3
 \end{array}$$

### Generalized Interactions

- Examining the example problem, we see three degrees of freedom for blocks (4-1). One degree can be associated with  $ABC$  and the second with  $BCD$ . There is a missing effect.
- There is a generalized interaction that occurs when two <sup>or more</sup> linear combinations are used.
- $GI = ABC(ACD) = \cancel{A^2} \cancel{BC^2} D = BD$
- Care must be exercised in selecting  $L_1$  and  $L_2$  because the GI might contain important information.



### Confounding the $2^k$ Design in $2^p$ Blocks

- Select  $p$  independent effects to be confounded.
- Independent effects means that no effect chosen is the generalized interaction of the others.
- The  $p$  defining contrasts  $L_1, L_2, \dots, L_p$  are used to define the  $2^p$  blocks
- In addition, there are exactly  $2^p - p - 1$  other effects confounded with these blocks.
- Often people look to tables such as Table 7.9 on page 322 for helping in selecting the effects used to generate blocks

→ all combinations of blocking generators

Table 7.9

■ TABLE 7.9

Suggested Blocking Arrangements for the  $2^k$  Factorial Design

Number of Factors, $k$	Number of Blocks, $2^p$	Block Size, $2^{k-p}$	Effects Chosen to Generate the Blocks	Interactions Confounded with Blocks
3	2	4	$ABC$	$ABC$
	4	2	$AB, AC$	$AB, AC, BC$
4	2	8	$ABCD$	$ABCD$
	4	4	$ABC, ACD$	$ABC, ACD, BD$
	8	2	$AB, BC, CD$	$AB, BC, CD, AC, BD, AD, ABCD$
5	2	16	$ABCDE$	$ABCDE$
	4	8	$ABC, CDE$	$ABC, CDE, ABDE$
	8	4	$ABE, BCE, CDE$	$ABE, BCE, CDE, AC, ABCD, BD, ADE$
	16	2	$AB, AC, CD, DE$	All two- and four-factor interactions (15 effects)
6	2	32	$ABCDEF$	$ABCDEF$
	4	16	$ABCF, CDEF$	$ABCF, CDEF, ABDE$
	8	8	$ABEF, ABCD, ACE$	$ABEF, ABCD, ACE, BCF, BDE, CDEF, ADF$
	16	4	$ABF, ACF, BDF, DEF$	$ABF, ACF, BDF, DEF, BC, ABCD, ABDE, AD, ACDE, CE, CDF, BCDEF, ABCEF, AEF, BE$
	32	2	$AB, BC, CD, DE, EF$	All two-, four-, and six-factor interactions (31 effects)
7	2	64	$ABCDEFG$	$ABCDEFG$
	4	32	$ABCFG, CDEFG$	$ABCFG, CDEFG, ABDE$
	8	16	$ABCD, CDEF, ADFG$	$ABC, DEF, AFG, ABCDEF, BCFG, ADEG, BCDEG$
	16	8	$ABCD, EFG, CDE, ADG$	$ABCD, EFG, CDE, ADG, ABCDEFG, ABE, BCG, CDFG, ADEF, ACEG, ABFG, BCEF, BDEG, ACF, BDF$
	32	4	$ABG, BCG, CDG, DEG, EFG$	$ABG, BCG, CDG, DEG, EFG, AC, BD, CE, DF, AE, BF, ABCD, ABDE, ABFG, BCDE, BCEF, CDEF, ABCDEFG, ADG, ACDEG, ACEFG, ABDFG, ABCEG, BEG, BDEFG, CFG, ADEF, ACDF, ABCF, AFG, BCDFG$