

Printout

Sunday, October 10, 2021 11:39 AM

Section 1

MANE 6313

Subsection 1

Week 8, Module C

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Creating design with more than two blocks.

Confounding the 2^k design in 4 blocks

- You must define 2 linear combinations L_1 and L_2
- For each treatment effect, we construct the ordered pair $(L_1 \text{ mod } 2, L_2 \text{ mod } 2)$
- This will result in four blocks having values $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$
- Work a 2^4 example in 4 blocks

$$\begin{aligned}L_1 &= A \oplus C = 1x_1 + 1x_2 + 1x_3 + 0x_4 \\L_2 &= A \oplus D = 1x_1 + 0x_2 + 1x_3 + 0x_4\end{aligned}$$

$$L_1 = 1x_1 + 1x_2 + 1x_3 + 0x_4$$

$$L_2 = 1x_1 + 0x_2 + 1x_3 + 1x_4$$

Four Block Example (L_1, L_2)

| $t \setminus t$ | x_1 | x_2 | x_3 | x_4 | $L_1 \text{ Mod } 2$ | $L_2 \text{ Mod } 2$ | $(0,0)$ | $(1,0)$ | $(0,1)$ | $(1,1)$ |
|-----------------|-------|-------|-------|-------|----------------------|----------------------|---------|---------|---------|---------|
| (1) | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 |
| b | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| ab | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| ac | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| bc | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| abc | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| bd | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| ad | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| cd | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| abd | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| acd | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| bc | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| abcd | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| abcd | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

$$\begin{array}{c}
 \text{Anova} \\
 \hline
 \text{Source} \quad d_f \\
 \hline
 \text{Block} \quad 4-1=3
 \end{array}$$

Generalized Interactions

- Examining the example problem, we see three degrees of freedom for blocks (4-1). One degree can be associated with ABC and the second with BCD . There is a missing effect. *or more*
- There is a generalized interaction that occurs when two linear combinations are used.
- $GI = ABC(ACD) = \cancel{A^2} \cancel{B^2} \cancel{C^2} D = BD$
- Care must be exercised in selecting L_1 and L_2 because the GI might contain important information.

Confounding the 2^k Design in 2^p Blocks

- Select p independent effects to be confounded.
- Independent effects means that no effect chosen is the generalized interaction of the others.
- The p defining contrasts L_1, L_2, \dots, L_p are used to define the 2^p blocks
- In addition, there are exactly $2^p - p - 1$ other effects confounded with these blocks.
- Often people look to tables such as Table 7.9 on page 322 for helping in selecting the effects used to generate blocks

all combinations of
blocking generators

Table 7.9

■ TABLE 7.9
Suggested Blocking Arrangements for the 2^k Factorial Design

| Number of Factors, k | Number of Blocks, 2^p | Block Size, 2^{k-p} | Effects Chosen to Generate the Blocks | Interactions Confounded with Blocks |
|------------------------|-------------------------|-----------------------|---------------------------------------|--|
| 3 | 2 | 4 | ABC | ABC |
| | 4 | 2 | AB, AC | AB, AC, BC |
| 4 | 2 | 8 | ABCD | ABCD |
| | 4 | 4 | ABC, ACD | ABC, ACD, BD |
| | 8 | 2 | AB, BC, CD | AB, BC, CD, AC, BD, AD, ABCD |
| 5 | 2 | 16 | ABCDE | ABCDE |
| | 4 | 8 | ABC, CDE | ABC, CDE, ABDE |
| | 8 | 4 | ABE, BCE, CDE | ABE, BCE, CDE, AC, ABCD, BD, ADE |
| | 16 | 2 | AB, AC, CD, DE | All two- and four-factor interactions (15 effects) |
| 6 | 2 | 32 | ABCDEF | ABCDEF |
| | 4 | 16 | ABCF, CDEF | ABCF, CDEF, ABDE |
| | 8 | 8 | ABEF, ABCD, ACE | ABEF, ABCD, ACE, BCF, BDE, CDEF, ADF |
| | 16 | 4 | ABF, ACF, BDF, DEF | ABF, ACF, BDF, DEF, BC, ABCD, ABDE, AD, ACDE, CE, CDF, BCDEF, ABCEF, AEF, BE |
| 7 | 32 | 2 | AB, BC, CD, DE, EF | All two-, four-, and six-factor interactions (31 effects) |
| | 2 | 64 | ABCDEF | ABCDEF |
| | 4 | 32 | ABCFG, CDEFG | ABCFG, CDEFG, ABDE |
| | 8 | 16 | ABCD, CDEF, ADFG | ABC, DEF, AFG, ABCDEF, BCFG, ADEG, BCDEG |
| | 16 | 8 | ABCD, EFG, CDE, ADG | ABCD, EFG, CDE, ADG, ABCDEFG, ABE, BCG, CDFG, ADEF, ACEG, ABFG, BCEF, BDEG, ACF, BDF |
| 32 | 32 | 4 | ABG, BCG, CDG, DEG, EFG | ABG, BCG, CDG, DEG, EFG, AC, BD, CE, DF, AE, BF, ABCD, ABDE, ABEF, BCDE, BCEF, CDEF, ABCDEFG, ADG, ACDEG, ACEFG, ABDFG, ABCEG, BEG, BDEFG, CFG, ADEF, ACDF, ABCF, AFG, BCDFG |