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Section 1

MANE 6313

Subsection 1

Week 9, Module A

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Describe fractional factorial designs, generators, defining relation and aliasing schemes for a one-half fraction.

Fractional Factorial Designs

- As the number of factors grows in a 2^k experiment, the number of runs often exceeds our ability to conduct the experiments. e.g. 2^6 experiment requires 64 runs.
- Often we are only interested in a few effects. E.g. in the 2^6 experiment, there are 6 main effects and 15 two-factor interactions. $\binom{6}{2}$ The remaining 42 degrees of freedom are associated with three-factor or higher interactions.
- Often we can get the information we need by running only a fraction of the factorial experiment
- Fractional factorials are often used as *screening experiments*.

Key Ideas for Fractional Factorial Experiments

- *The sparsity of effects principle.* The system or process is likely to be primarily driven by some of the main effects and low-order interactions.
- *The projection property.* When we identify unimportant variables and remove them from the model, the resulting model is stronger (larger) designs.
- *Sequential Experimentation.* It is possible to combine the runs of two (or more) fractional factorials to assemble sequentially a larger design to estimate the factor effects and interactions of interest.

The one-half fraction of a 2^k design

- This design results in a 2^{k-1} experiment, a half-fraction.
- You must select an effect to generate the two fractions. This effect is called the generator. E.g. in a 2^3 design select ABC as the *generator*. The other fraction is $-ABC$.
- We always associate I with the positive fraction. Thus, $I = ABC$ and we call this quantity the *defining relation* for the fractional factorial experiment.
- The fraction containing the positive generator is called the *principal fraction*. The other fraction is called the alternate or complementary fraction.

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The Two Fractions

$2^{3-1} \rightarrow \frac{1}{2}$ Fraction (4 runs)

generator ABC, defining relation $I = ABC$

$C(I) = ABC^2$

$C = AB$

A	B	C = AB	trt
−	−	+	c
+	−	−	a
−	+	−	b
+	+	+	abc

$C(I) = -ABC^2$

$C = -AB$

A	B	C = −AB	trt
−	−	−	(1)
+	−	+	ac
−	+	+	bc
+	+	−	ab

Aliasing Scheme from ~~Principal~~ Complementary Fraction

$I = ABC$

$l_A : A(I + ABC), \quad A + A^2BC, \quad A + BC$

$l_B : B(I + ABC), \quad B + AB^2C, \quad B + AC$

$l_C : \quad C + AB$

Aliasing Scheme from the ~~Complementary~~ Principal Fraction

$I = -ABC$

$l'_A : A - BC$

$l'_B : B - AC$

$l'_C : C - AB$

Combining Aliasing Schemes

$$\frac{l_A + l'_A}{2} = \frac{A + BC + (A - BC)}{2} = A$$

$$\frac{l_A - l'_A}{2} = \frac{A + BC - (A - BC)}{2} = BC$$