

MANE 6313

Section 1

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Subsection 1

Week 9, Module E

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

$$2^{k-p}$$

Describe a general $2^{(k-p)}$ fractional factorial design.

The general 2^{k-p} Fractional Factorial Design

- A 2^k fractional factorial design containing 2^{k-p} runs is called a $1/2^p$ fraction of the 2^k design
- These designs requires p independent generators (same definition from last week).
- There are $2^p - p - 1$ generalized interactions included
- There is an "art" to selecting the correct generators. Look to table 8.14 (page 353) for suggestions.

$$\begin{aligned} p=1 & \text{, } 1/2 \\ p=2 & \text{, } 1/4 \\ p=3 & \text{, } 1/8 \\ p=4 & \text{, } 1/16 \end{aligned}$$

$$\begin{aligned} p=2 & \rightarrow 2^2 - 2 - 1 = 1 \text{ G.I.} \\ p=3 & \rightarrow 2^3 - 3 - 1 = 4 \text{ G.I.} \end{aligned}$$

Table 8.14

TABLE 8.14 Selected 2^{k-p} Fractional Factorial Designs							
Number of Factors, k	Fraction	Number of Runs	Design Generators	Number of Factors, k	Fraction	Number of Runs	Design Generators
4	2^{4-1}_{IV}	8	$D = \pm ABC$				$F = \pm BCD$
5	2^{5-1}_{V}	16	$E = \pm ABCD$				$G = \pm ACD$
	2^{5-2}_{III}	8	$D = \pm AB$				$H = \pm ABD$
6			$E = \pm AC$				$J = \pm ABCD$
	2^{6-1}_{VI}	32	$F = \pm ABCDE$ ✓	10	2^{10-3}_{V}	128	$H = \pm ABCG$
	2^{6-2}_{IV}	16	$E = \pm ABC$				$J = \pm BCDE$
			$F = \pm BCD$				$K = \pm ACFD$
2^{6-3}_{III}	8	$D = \pm AB$	$G = \pm BCDF$				
7			$E = \pm AC$		2^{10-4}_{IV}	64	$H = \pm ACFD$
			$F = \pm BC$				$J = \pm ABDE$
	2^{7-1}_{VII}	64	$G = \pm ABCDEF$				$K = \pm ABCE$
	2^{7-2}_{IV}	32	$F = \pm ABCD$		2^{10-5}_{IV}	32	$F = \pm ABCD$
			$G = \pm ABDE$				$G = \pm ABCE$
			$E = \pm ABC$				$H = \pm ABDE$
8			$F = \pm BCD$				$J = \pm ACDE$
			$G = \pm ACD$				$K = \pm BCDE$
	2^{7-3}_{IV}	16	$D = \pm AB$		2^{10-6}_{III}	16	$E = \pm ABC$
			$E = \pm AC$				$F = \pm BCD$
			$F = \pm BC$				$G = \pm ACD$
			$G = \pm ABC$				$H = \pm ABD$
			$H = \pm ABCD$				$J = \pm ABCD$
			$I = \pm ABCE$				$K = \pm AB$
9	2^{8-2}_{V}	64	$G = \pm ABCD$	11	2^{11-5}_{IV}	64	$G = \pm CDE$
			$H = \pm ABEF$				$H = \pm ABCD$
	2^{8-3}_{IV}	32	$F = \pm ABC$				$J = \pm ABF$
			$G = \pm ABD$				$K = \pm BDEF$
10			$H = \pm BCDE$				$L = \pm ADEF$
			$I = \pm BCDE$				$F = \pm ABC$
			$E = \pm BCD$		2^{11-6}_{IV}	32	$G = \pm BCD$
			$F = \pm ACD$				$H = \pm CDE$
			$G = \pm ABC$				$J = \pm ACD$
			$H = \pm ABD$				$K = \pm ADE$
11	2^{9-2}_{VI}	128	$H = \pm ACDFG$				$L = \pm BDE$
			$J = \pm BCEFG$				$E = \pm ABC$
	2^{9-3}_{IV}	64	$G = \pm ABCD$		2^{11-7}_{III}	16	$F = \pm BCD$
			$H = \pm ACEF$				$G = \pm ACD$
12			$J = \pm CDEF$				$H = \pm ABD$
			$F = \pm BCDE$				$I = \pm ABCD$
	2^{9-4}_{IV}	32	$G = \pm ACDE$				
			$H = \pm ABDE$				

Appendix X

X Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \leq 15$ and $n \leq 64$ (Continued)

Designs with 6 Factors

(e) 2^{6-3} ; 1/8 fraction of
6 factors in 8 runs

Resolution III

Design Generators

$$D = AB \quad E = AC \quad F = BC$$

$$\text{Defining relation: } I = ABD = ACE = BCDE = BCF = ACDF = ABEF = DEF$$

Aliases

$$A = BD = CE = CDF = BEF \quad E = AC = DF = BCD = ABF$$

$$B = AD = CF = CDE = AEF \quad F = BC = DE = ACD = ABE$$

$$C = AE = BF = BDE = ADF \quad CD = BE = AF = ABC = ADE = BDF = CEF$$

$$D = AB = EF = BCE = ACF$$

Resolution III Designs

- It is possible to construct resolution III designs for investigating up to $k = N - 1$ factors in N runs when N is a multiple of 4
- These experiments are said to be *saturated*
- Pay particular attention to Sequential assembly of fractions to separate effects.

Minitab

- Example problems demonstrate the use of Minitab for fractional factorials
- Minitab provides excellent support