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Section 1

MANE 6313

Subsection 1

Week 12, Module **B**

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Assessing linear regression assumptions.

Fitting Linear Regression Models

$y = \beta_0 + \beta_1 x + \varepsilon$

Model Assumptions and Residuals

- Least squares estimation requires that $E(\varepsilon) = 0$ and $V(\varepsilon) = \sigma^2$ and the $\{\varepsilon_i\}$ are uncorrelated
- To perform statistical hypothesis tests, we further assume that $\varepsilon \sim \text{NID}(0, \sigma^2)$
- These assumptions are validated by examining the residuals

constant variance

$\hat{\varepsilon}_i = e_i = y_i - \hat{y}_i$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \varepsilon$$

Test for Significance of Regression

- Test for significance of regression is a test to determine if there is a linear relationship between y and a subset of the regressors

$$H_0 : \beta_1 = \beta_2 = \cdots = \beta_k = 0$$
$$H_a : \beta_j \neq 0 \text{ for at least one } j$$

k factors or k slope terms

- The test statistic is

$$F_0 = \frac{SS_R/k}{SS_E/(n-k-1)} = \frac{MS_{Regression}}{MS_{Error}}$$

- Reject H_0 if $F_0 > F_{\alpha,k,n-k-1}$ or use p value

Example 12.8

```r
ex12_8.model <- lm(GrainRadius~PowderTemp+Extrusion+DieTemp,data=ex12_8.df)
summary(ex12_8.model)
```

```
Call:
lm(formula = GrainRadius ~ PowderTemp + Extrusion + DieTemp,
    data = ex12_8.df)

Residuals:
    1     2     3     4     5     6     7     8
-0.75  5.25  1.75 -2.25 -6.25 -2.25  1.25  3.25

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -284.50000   30.77729  -9.244 0.000761 ***
PowderTemp     0.12500    0.08501   1.470 0.215398
Extrusion      2.45833    0.28336   8.676 0.000972 ***
DieTemp        1.45000    0.11335  12.793 0.000215 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.809 on 4 degrees of freedom
Multiple R-squared:  0.9837,    Adjusted R-squared:  0.9714
F-statistic: 80.36 on 3 and 4 DF,  p-value: 0.0004967
```

k=3

1
2
3

Test for Significance of Regression

F_0, df for test (3,4)

p-value

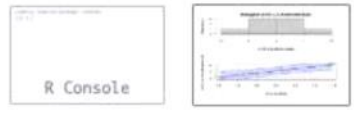
Conclude at least one term in model is statistically significant

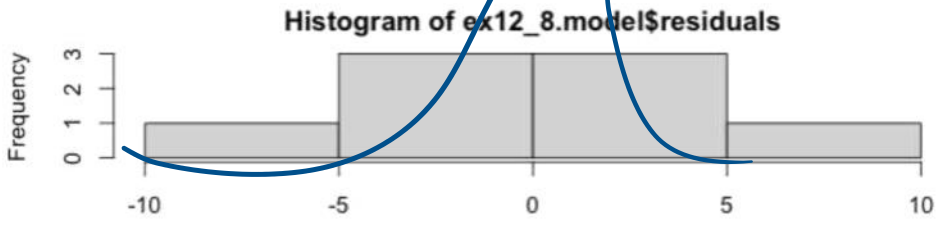
	Source	df
Regression		3
Error		7-3=4
Total		8-1=7

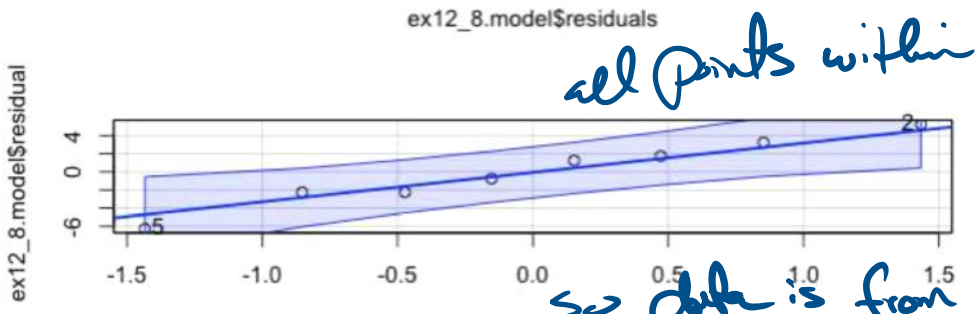
Residuals - Normality Assumption

```r
```{r}
par(mfrow=c(2,1))
hist(ex12_8.model$residuals)
library(car)
qqPlot(ex12_8.model$residuals)
```
```

raw residuals - unscaled







all points within bounds

so data is from normal distribution

Residuals vs.Fitted Values

```
54 ```{r}
55 ex12_8.stdres <- rstandard(ex12_8.model)
56 plot(ex12_8.model$fitted.values,ex12_8.stdres,xlab="Fitted Values",ylab="Standardized Residuals",main="Residuals vs. Fitted Values")
57 ```
```

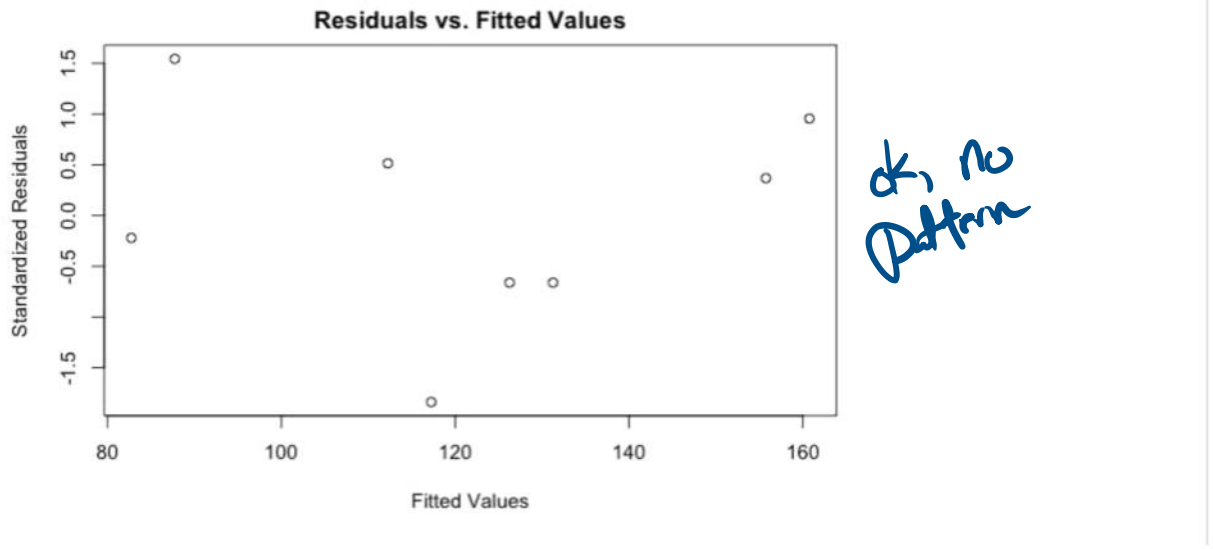
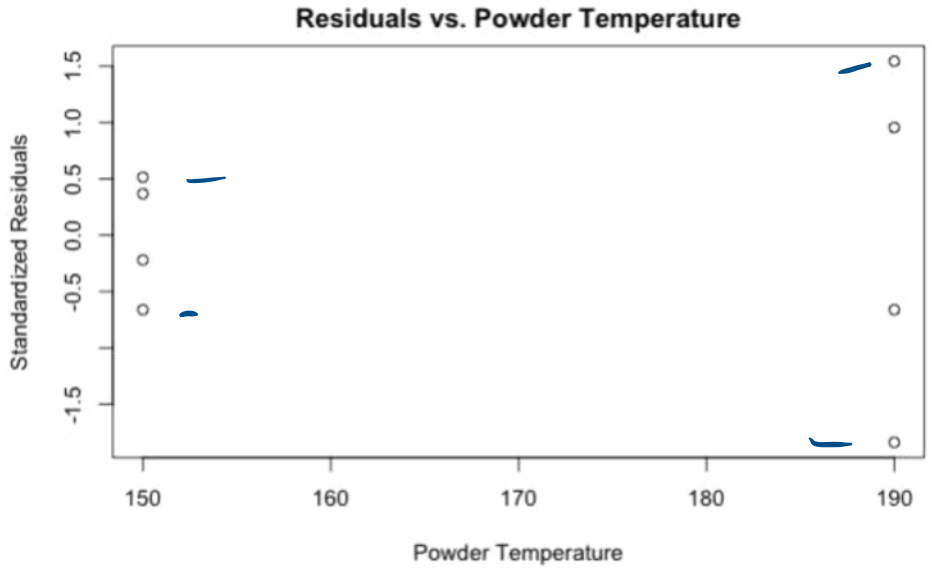


Figure 3: Ex. 12.8 Residuals vs. Fitted Values

Residuals vs. Powder Temperature

```
41 ```{r warning=FALSE,message=FALSE}
42 ex12_8.stdres <- rstandard(ex12_8.model)
43 plot(ex12_8.df$PowderTemp,ex12_8.stdres,xlab="Powder Temperature",ylab="Standardized Residuals",main="Residuals vs.
   Powder Temperature")
44 ```
```



Slight difference in ranges for Powder Temp at 170 & 190

Figure 4: Residuals vs. Powder Temperature

Residuals vs. Extrusion

```
46 ```{r warning=FALSE,message=FALSE}
47 plot(ex12_8.df$Extrusion,ex12_8.stdres,xlab="Extrusion",ylab="Standardized Residuals",main="Residuals vs. Extrusion")
48 ```
```

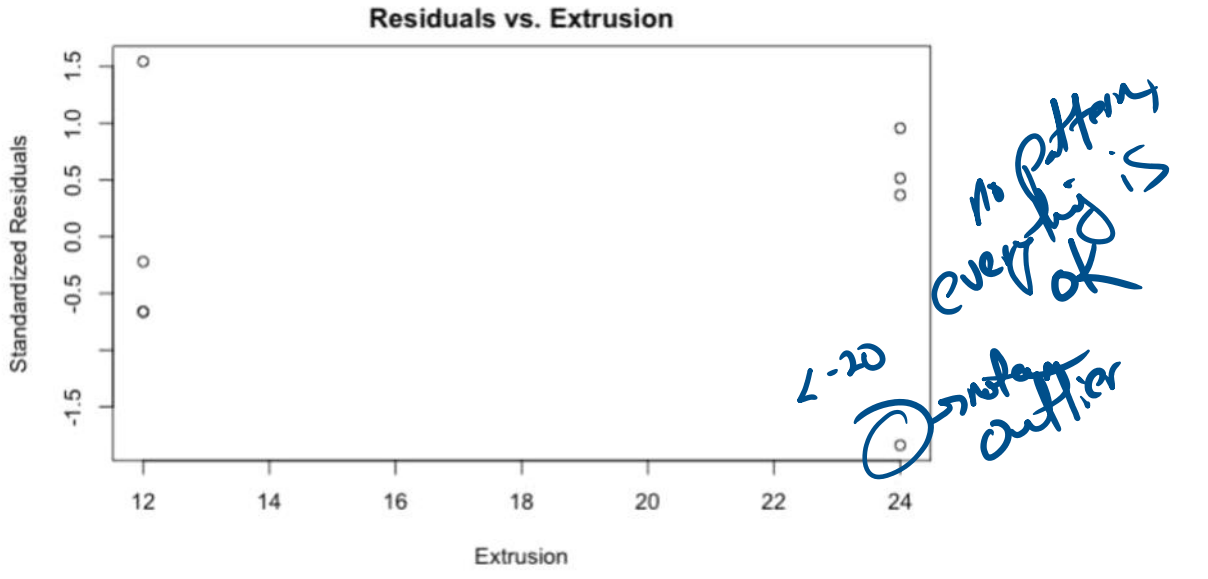
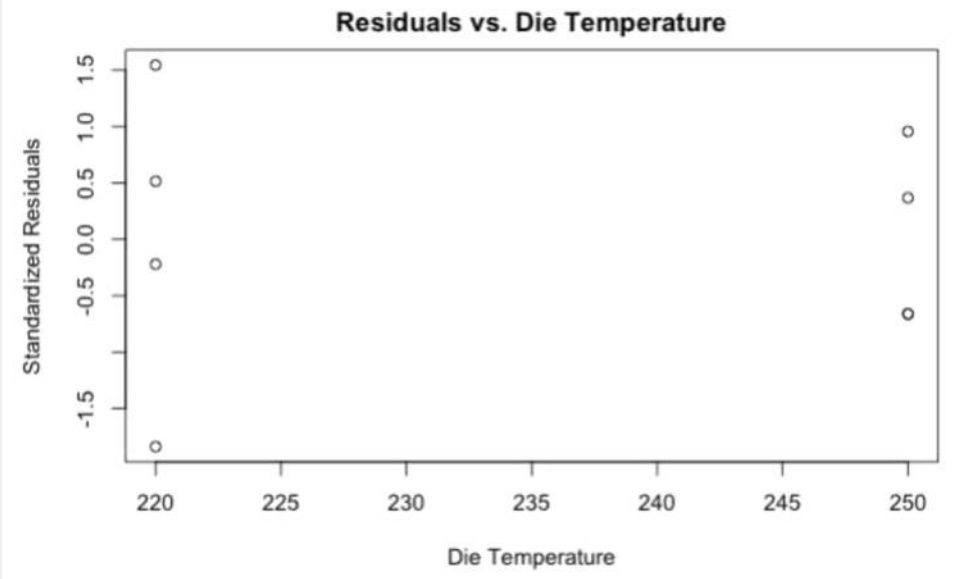


Figure 5: Residuals vs. Extrusion

Residuals vs. Die Temperature

```
50 ```{r warning=FALSE,message=FALSE}
51 plot(ex12_8.df$DieTemp,ex12_8.stdres,xlab="Die Temperature",ylab="Standardized Residuals",main="Residuals vs. Die
   Temperature")
52 ```
```



No pattern
So everything
is OK

Figure 6: Residuals vs. Die Temperature