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Saturday, April 1, 2023   10:59 AM

Section 1

MANE 6313

Subsection 1

Week 12, Module D

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Employing test of hypothesis test on groups of variables.

Applying Tests of Hypothesis for Groups of Variables

- There is a concern regarding the overall error rate for multiple decisions
- Very similar to family-wise error rate in multiple comparisons
- In Ex 12.8, when examining the slope terms, three decisions are made at some value of alpha (usually 0.05). The overall error rate will be larger

$$\alpha_{overall} = 1 - (1 - \alpha_{individual})^n$$

- For Ex 12.8 using $\alpha_{individual} = 0.05$,

$$\alpha_{overall} = 1 - (1 - .05)^3 = 0.1426$$

Tests for groups of regression coefficients

- We can examine the contribution of the regression sum of squares for a particular variable, say x_j, given that other variables $x_i (i \neq j)$ are included in the model
- We can also determine if the subset of regressor variables $x_1, x_2, \ldots, x_r (r < k)$ contribute to the model
- This test is necessary to control the overall error rate
- The full model is

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon$$

- We partition the regressors in two groups

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

where β_1 is $(r \times 1)$ and β_2 is $[(p - r) + 1]$

- We can rewrite the **full model** as

want → to remove

$$\mathbf{y} = \mathbf{X_1}\beta_1 + \mathbf{X_2}\beta_2 + \varepsilon$$

- The regression sum of squares for the full model is denoted $SS_R(\beta)$ with p degrees of freedom

- We define the reduced model to be *Keepers*

$$\mathbf{y} = \mathbf{X_2}\beta_2\varepsilon$$

- The regression sum of squares for the reduced model is $SS_r(\beta_2)$ with $p - r$ degrees of freedom.

- The regression sum of squares due to β_1 given that β_2 is already in the model is

$$SS_R(\beta_1|\beta_2) = SS_R(\beta) - SS_R(\beta_2)$$

- The null hypothesis $\beta_1 = \mathbf{0}$ can be tested with the statistic

[handwritten: → all members of this group]

$$F_0 = \frac{SS_R(\beta_1|\beta_2)/r}{MS_E}$$

[handwritten: → # of elements in β_1]

- Reject H_0 if $F_0 > F_{\alpha, r, n-p}$

- This is sometimes called the *partial F test* or extra sum of squares method

Step 1 - Fit Full Model

```r
95 ```{r}
96 ex12_8.full <- lm(GrainRadius~PowderTemp+Extrusion+DieTemp,data=ex12_8.df)
97 anova(ex12_8.full)
98 ```
```

```
Analysis of Variance Table

Response: GrainRadius
           Df Sum Sq Mean Sq  F value    Pr(>F)
PowderTemp  1   50.0    50.0   2.1622 0.2153975
Extrusion   1 1740.5  1740.5  75.2649 0.0009715 ***
DieTemp     1 3784.5  3784.5 163.6541 0.0002152 ***
Residuals   4   92.5    23.1
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 1: Full Model

Step 2 - Fit Reduced Model

```r
84 ▾  ```{r}
85    ex12_8.reduced <- lm(GrainRadius~Extrusion+DieTemp,data=ex12_8.df)
86    anova(ex12_8.reduced)
87 ▴  ```
```

```
Analysis of Variance Table

Response: GrainRadius
          Df Sum Sq Mean Sq F value    Pr(>F)
Extrusion  1 1740.5  1740.5   61.07 0.0005501 ***
DieTemp    1 3784.5  3784.5  132.79 8.629e-05 ***
Residuals  5  142.5    28.5
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Figure 2: Reducedl Model

Step 3 - Analysis

Full Model	Reduced Model
$SS_R(\beta) = 50 + 1740.5 + 3784.5$	$SS_R(\beta_2) = 1740.5 + 3784.5$
$= 5575$	$= 5525$
$df_R = 3$	$df_R = 2$

$MS_E = 23.1$

$df_E = 4$

$$F_0 = \frac{(5575 - 5525)/(3-2)}{23.1} = 2.164$$

numerator

Reject H_0 if $F_0 > F_{.05, 1, 4} = 7.71$

Conclusion: Fail to reject H_0, all terms (Powder Temp) in β_2 have slopes of zero.

Partial F-test in R

```r
84 ▾  ```{r}
85    anova(ex12_8.reduced,ex12_8.model)
86 ▴  ```
```

```
Analysis of Variance Table

Model 1: GrainRadius ~ Extrusion + DieTemp
Model 2: GrainRadius ~ PowderTemp + Extrusion + DieTemp
  Res.Df    RSS Df Sum of Sq        F Pr(>F)
1      5 142.5
2      4  92.5  1           50 2.1622 0.2154
```

(handwritten annotations: "F₀" pointing to 2.1622, "P-value" pointing to 0.2154)

Source:
https://www.statology.org/partial-f-test/#:~:text=A%20partial%20F-test%20is%20used%20to%20determine%20whether,the%20predictor%20variables%20in%20the%20overall%20regression%20model.