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MANE 6313

Section 1

MANE 6313

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Subsection 1

Week 13, Module A

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Describe response surface methodology.

Introduction to RSM

- *Response Surface Methodology* is a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables and the objective is to optimize this response
- Consider a two-variable function where \rightarrow ~~highly~~ *highly accurate model*

$$y = f(x_1, x_2) + \epsilon$$

- The expected response function is

$$E(y) = f(x_1, x_2) = \eta$$

- Thus the function η is often called the response surface.
- The response surface is often shown graphically

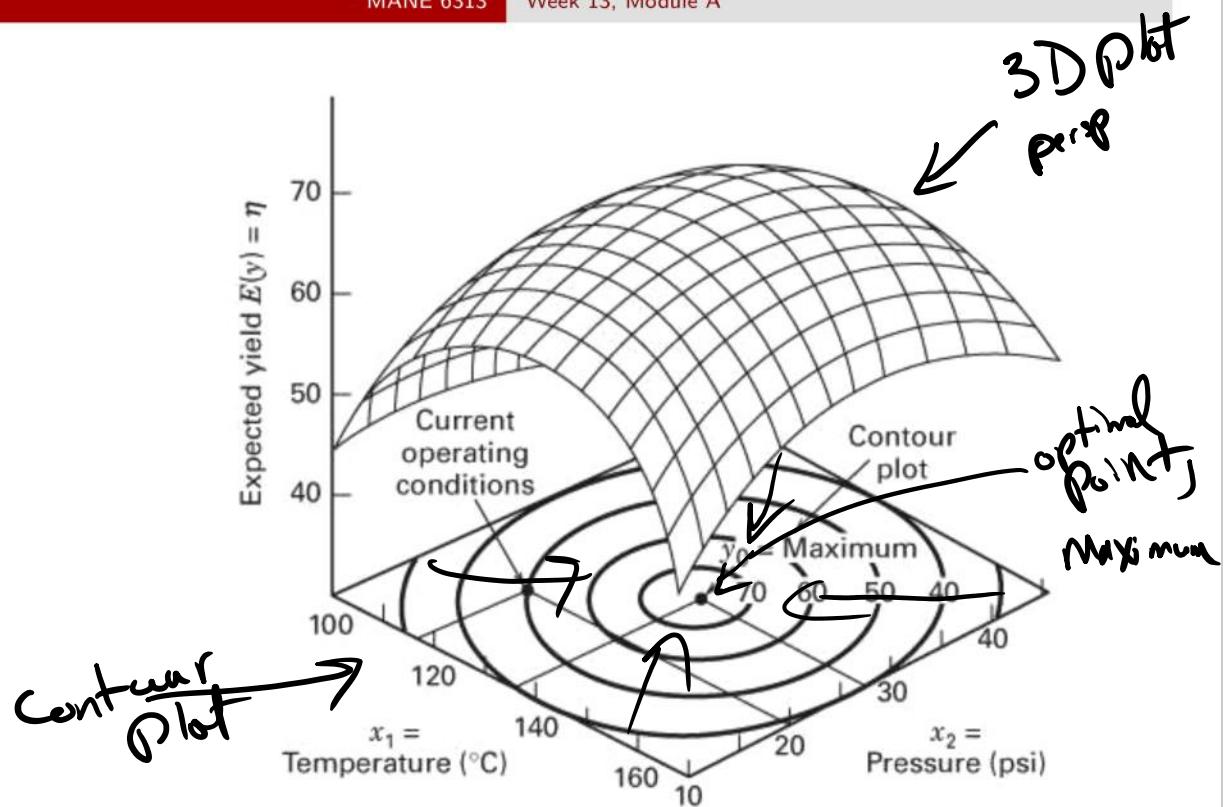


FIGURE 11.2 A contour plot of a response surface

Figure 11.2 is a contour plot of a response surface, showing the relationship between two independent variables (Temperature and Pressure) and a dependent variable (Expected yield). The surface is a dome-shaped paraboloid, representing a quadratic response surface. The contour plot shows the level curves of the response surface, which are concentric circles centered at the peak. The peak of the surface represents the maximum yield, and the point on the surface corresponding to the peak is labeled $y_0 = \text{Maximum}$. The current operating conditions are indicated by a point on the surface, which is labeled "Current operating conditions". The axes are labeled $x_1 = \text{Temperature } (\text{°C})$ and $x_2 = \text{Pressure } (\text{psi})$.

- In general the function η is unknown.
- We will approximate η with low-order polynomial functions.
(limited to 2nd order)
- A first-order model is

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k + \epsilon$$

- A second-order model is

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i < j} \sum \beta_{ij} x_i x_j + \epsilon$$

FQ *TWI* *PQ*

- The method of least squares will be used to estimate the parameters, β

SO - Second order, contains FQ, TWI, PQ

Sequential Approach

- The use of RSM often requires sequential analysis
- Most of the time, you will not be operating at (or possibly near) an optimal region
- Perform an initial experiment, often first-order design *(Screening)*
- Determine direction towards optimum point
- Conduct another experiment nearer to the optimum point
- Repeat until in the neighborhood of the optimum
- Conduct an experiment using a second-order design

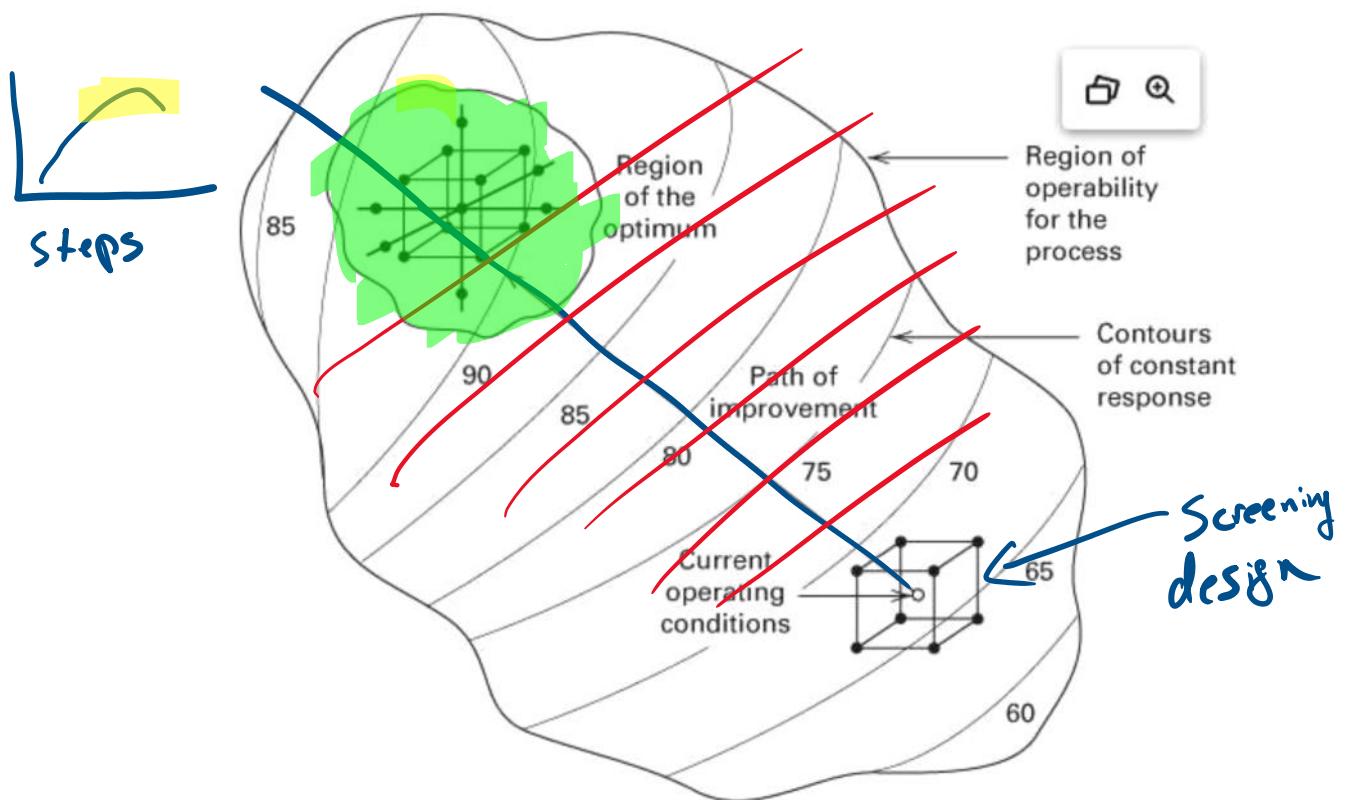


FIGURE 11.3 The sequential nature of RSM

Analysis of the 2nd-order Response Surface

- Suppose that we wish to find the levels of x_1, x_2, \dots, x_k that optimize the predicted response
- The point, if it exists, will be the set of x_1, x_2, \dots, x_k for which $\partial \hat{y} / \partial x_1 = \partial \hat{y} / \partial x_2 = \dots = \partial \hat{y} / \partial x_k = 0$
- This point is called **the stationary point**.
- Based upon our knowledge of calculus what are the possible types of stationary points?
- How do we determine if a stationary point is an optimal point?

