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Section 1

MANE 6313

Subsection 1

Week 13, Module C

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Describe designs for RSM.

Experimental Designs for Fitting Response Surfaces

A list of desirable features for an experimental design from section 11.4 in the textbook include:

- ① Provides a reasonable distribution of data points (and hence information) throughout the region of interest
- ② Allows model adequacy, including lack of fit, to be investigated
- ③ Allows experiments to be performed in blocks (if necessary)
- ④ Allows designs of higher order to be built up sequentially
- ⑤ Provides an internal estimate of error
- ⑥ Provides precise estimates of model coefficients (minimum variance)

- ⑦ Provides a good profile of the prediction variance throughout the experimental regions
- ⑧ Provides reasonable robustness against outliers or missing values
- ⑨ Does not require a large number of runs
- ⑩ Does not require too many levels of the independent variables
- ⑪ Ensures simplicity of calculation of the model parameters

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + \varepsilon$$

First-order Models

- We want to use only models which have a minimum variance for $\{\hat{\beta}_j\}$
- These designs are said to be *orthogonal first-order designs*. A first-order design is orthogonal if the off-diagonal elements of $(\mathbf{X}'\mathbf{X})$ are all zero.
- Orthogonal first-order designs include: 2^k factorial and fractional factorial designs and **simplex designs**.

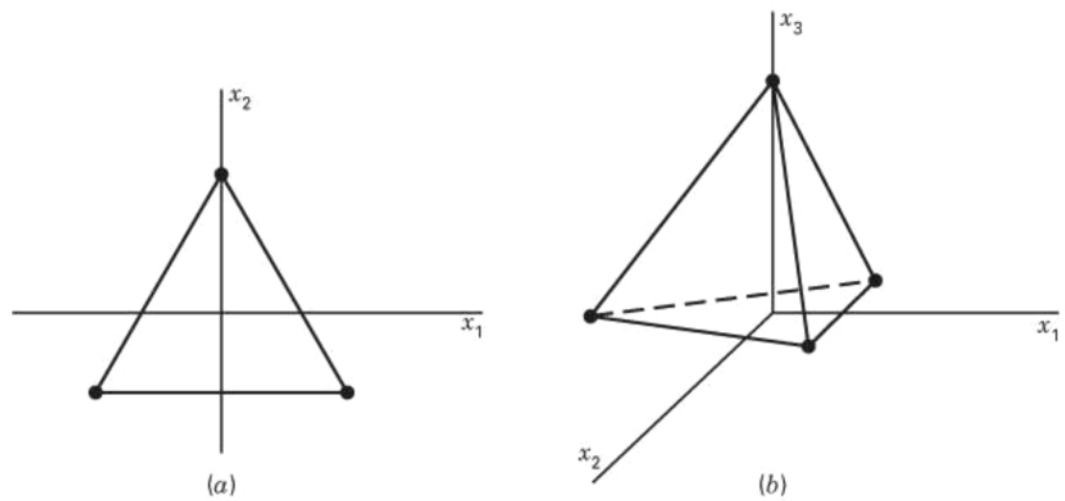


FIGURE 11.19 The simplex design for (a) $k = 2$ variables and (b) $k = 3$ variables

Figure 1: Textbook Figure 11.19

Second-order Models

- The most popular second-order design is the central composite design (CCD)
- Generally, the CCD is obtained sequentially
 - An initial factorial or resolution V fractional factorial was performed with center points. A lack of fit test indicated that quadratic terms should be added.
 - The axial points are added
- The design is composed of three components: the factorial component with n_f observations, n_c center points and $2k$ axial points
- Selection of α , the distance from the center point to the axial points
- Set $\alpha = (n_f)^{1/4}$. This design is said to be rotatable.
- Set $\alpha = \sqrt{k}$. This design is said to be a *spherical CCD*. It has a better prediction variance than a rotatable CCD and is typically more common.
 - Generally three to five center points are recommended

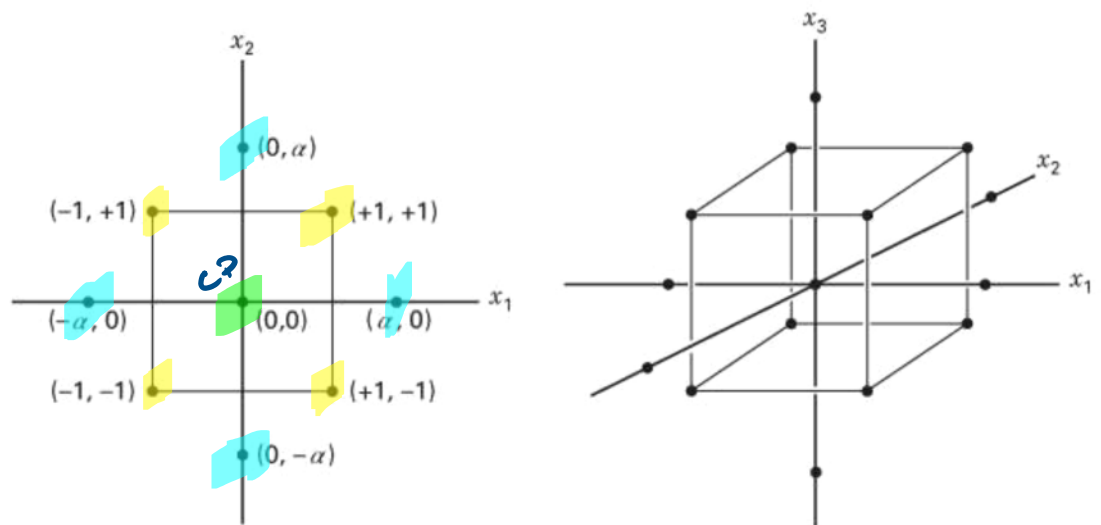


FIGURE 11.20 Central composite designs for $k = 2$ and $k = 3$

Figure 2: Textbook Figure 11.20

Box-Behnken Design

- Box and Behnken proposed some three-level designs for fitting response models
- These designs are typically very efficient in terms of number of runs required
- Some of the designs are spherical
- Not requiring the extreme points of the design space may be an advantage

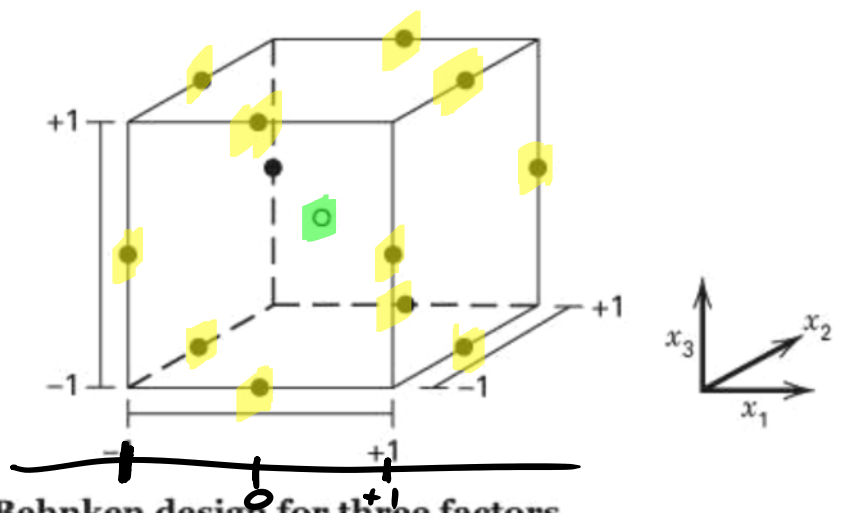


FIGURE 11.22 A Box-Behnken design for three factors

Figure 3: Textbook Figure 11.22

Face-centered CCD

- If the area of interest is a cube instead of a sphere, we set the axial points at the surface of the cube.
- That is, $\alpha = 1$
- This design is not rotatable
- The number of center points required is less and typically is set to 2

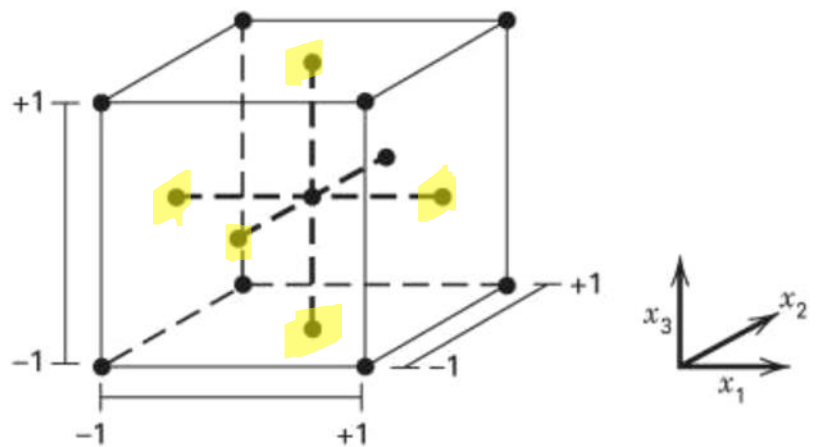


FIGURE 11.23 A face-centered central composite design for $k = 3$

Figure 4: Textbook Figure 11.23