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Monday, April 10, 2023 9:17 AM

Section 1

MANE 6313

Subsection 1

Week 13, Module G

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Compute stationary point for Box-Behnken example.

Stationary Points

$$\nabla \frac{dy}{dx_1}$$

- Stationary points are points where $\frac{\partial \hat{y}}{\partial x_1} = \frac{\partial \hat{y}}{\partial x_2} = \dots = \frac{\partial \hat{y}}{\partial x_k} = 0$
- There are three types of stationary points: **maximum response**, **minimum response**, and **saddle point**.
- Note that stationary points are not optimal points, but could be optimal points
- Second derivative information is needed to determine if a stationary point is an optimal point,
- Contour plots and 3D plots can also be used to determine type of stationary point

Box-Benkhen Fitted Model

$$\hat{y} = 86.7368 + 22.625 x_2$$

Call:

rsm(formula = y ~ F0(x2, x3) + PQ(x2, x3), data = bbd11.df)

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	86.7368	7.4888	11.5822	7.17e-08 ***
x2	22.6250	6.6632	3.3955	0.005314 **
x3	-7.0000	6.6632	-1.0505	0.314166
x2^2	-19.1579	9.1719	-2.0888	0.058704 .
x3^2	19.0921	9.1719	2.0816	0.059456 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared: 0.6349, Adjusted R-squared: 0.5133

F-statistic: 5.218 on 4 and 12 DF, p-value: 0.01138

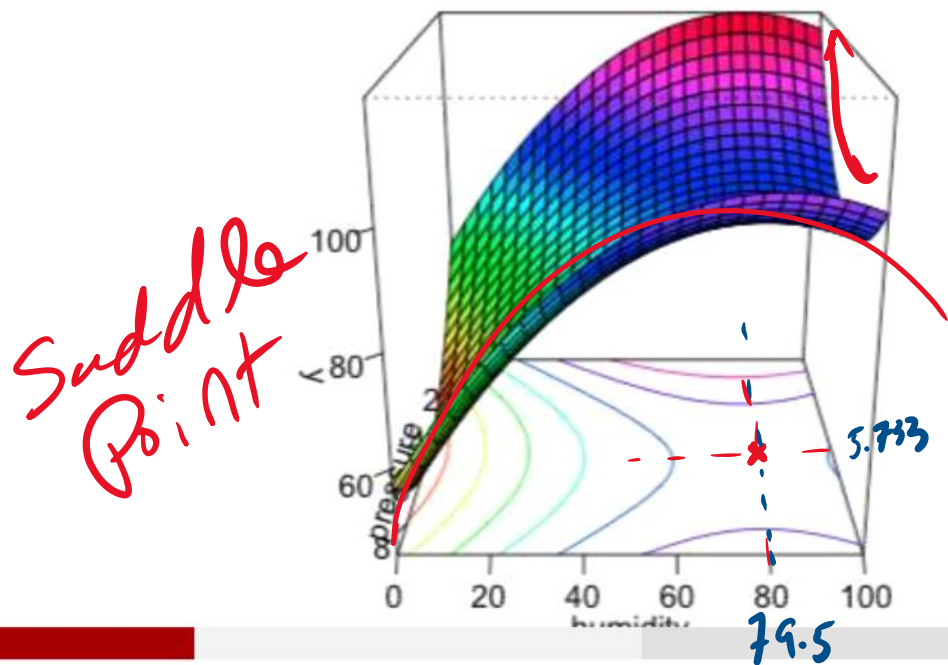
Analysis of Variance Table

Response: y

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
F0(x2, x3)	2	4487.1	2243.56	6.3165	0.01337
PQ(x2, x3)	2	2926.1	1463.07	4.1191	0.04346
Residuals	12	4262.3	355.19		
Lack of fit	4	1748.8	437.19	1.3915	0.31941

Box-Behnken Contour Image

```
87 > ""{r}  
88 persp(bbd11.model2,x2~x3,zlab="y",theta=90, col=rainbow(50),contours="colors")  
89 > ""
```



Stationary Points by Hand

From second model

$$\hat{y} = 86.7368 + 22.625x_2 - 7x_3 - 19.1579x_2^2 - 19.0921x_3^2$$

$$\frac{\partial \hat{y}}{\partial x_2} = \frac{d\hat{y}}{dx_2} = 22.625 - 38.3158x_2 \quad (1)$$

$$\frac{\partial \hat{y}}{\partial x_3} = \frac{d\hat{y}}{dx_3} = -7 + 38.1420x_3 \quad (2)$$

To find stationary points, set equation (1) & (2) equal to zero and solve

$$\text{From (1), } 22.625 - 38.3158x_2 = 0 \rightarrow x_2 = 0.5905$$

$$\text{From (2), } -7 + 38.142x_3 = 0 \rightarrow x_3 = 0.1835$$

Stationary Points from rsm()

- Check rsm() output
- Confirm graphically

Stationary point of response surface:

x2	x3
0.5904876	0.1833218

Stationary point in original units:

humidity	pressure
79.524382	5.733287

$$x_2 = (\text{humidity} - 50) / 50$$

$$0.5904876 = (H - 50) / 50$$

$$H - 50 = 50(0.5904876)$$

$$H = 50 + 50(0.5904876) = 79.5238$$

Figure 4: rsm() output containing stationary point