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Section 1

MANE 6313

Subsection 1

Week 14, Module E

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Apply method of steepest ascent (descent)

Sequential Nature

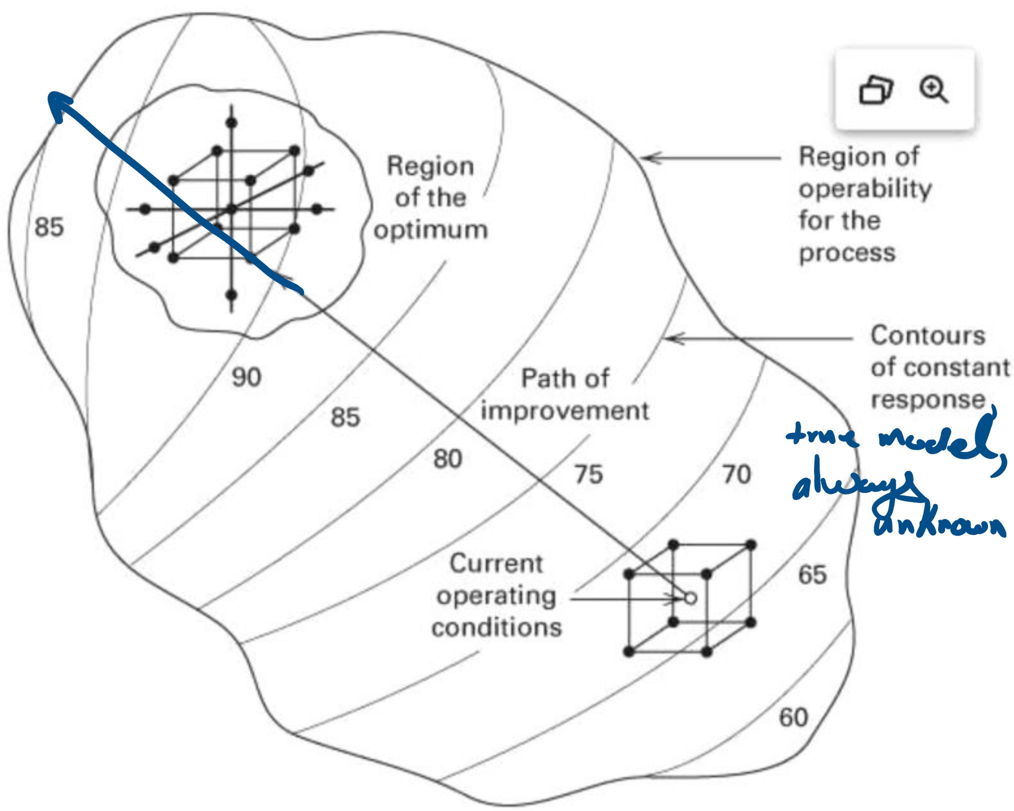
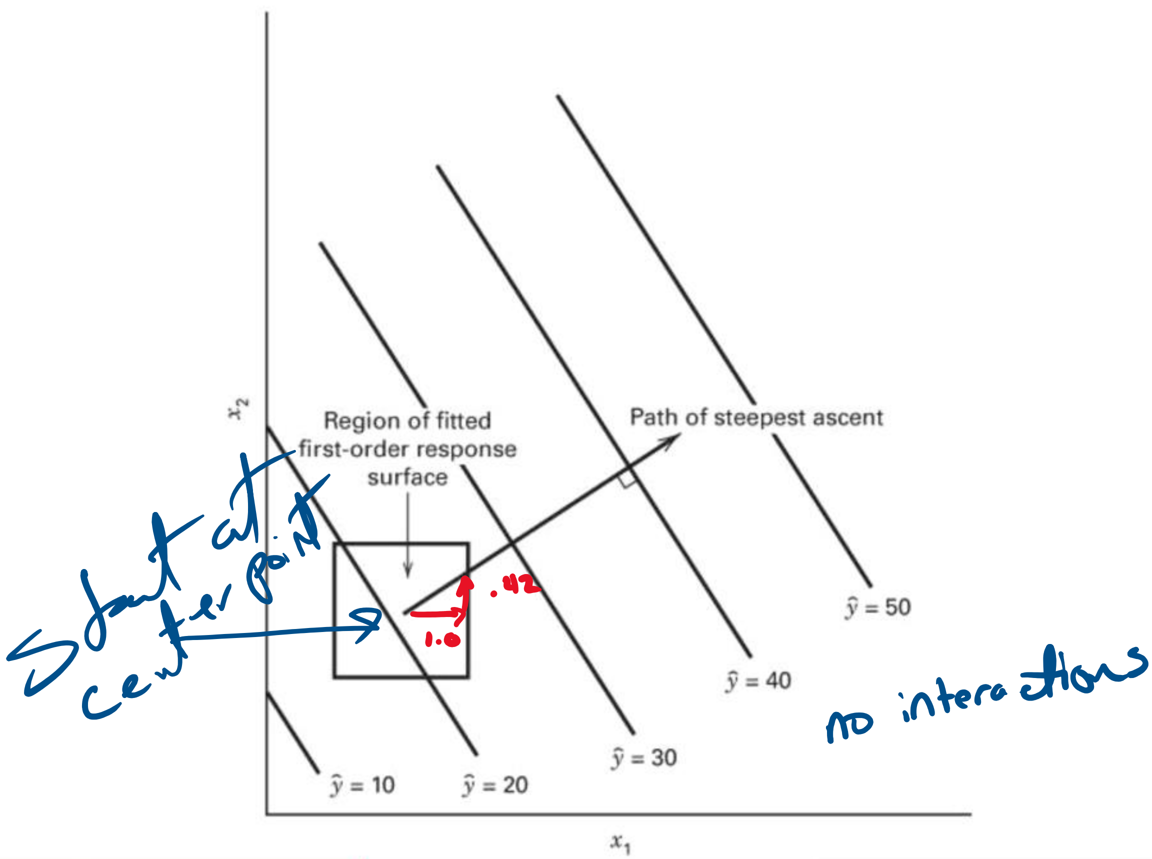


FIGURE 11.3 The sequential nature of RSM

Method of Steepest Ascent

- Search method to find a local maximum
- Non-linear optimization techniques usually require:
  - Search direction, and
  - Step size
- Method of Steepest Descent is used to find a local minimum

First-order Example



Textbook Procedure

Starts with a first-order model with coded data

$$\hat{y} = \widehat{\beta_0} + \sum_{i=1}^{k} \widehat{\beta}_i x_i$$

Assumes that the point $x_1 = x_2 = \cdots = x_k = 0$ is the base point or origin point

Textbook Procedure, continued

$\to x_1$

1. Choose a step size in one of the process variables, say Δx_j. Usually, we would select the variable we know the most about, or we would select the variable that has the largest absolute regression coefficient $|\widehat{\beta}_j|$.
2. The step size in the other variables is

$$\Delta_i = \frac{\widehat{\beta}_i}{\widehat{\beta}_j / \Delta_j} \quad i = 1, 2, \ldots, k \;\; i \neq j$$

3. Convert the Δx_i from coded variables to the natural variables.

Example 11.1

The fitted, coded regression model is

$$\widehat{y} = 40.44 + 0.775x_1 + 0.325x_2$$

The coded variables are

$$x_1 = \frac{\xi - 35}{5}$$
$$x_2 = \frac{\xi - 155}{5}$$

Example 11.1, Step 1

$\Delta = 1$

- x_1 is selected as the variable to base the steepest ascent upon

Example 11.1, Step 2

$\Delta_i = \frac{\beta_i}{\beta_j / \Delta_j}$ $i = 2$, $j = 1$

- Find Δ_2

$$\Delta_2 = \frac{0.325}{0.775/1.0} = 0.42$$

Example 11.1, Step 3

- Find x_1 and x_2 for origin plus 1Δ

$$x_1 = 0 + 1 = 1$$
$$x_2 = 0 + 0.42 = 0.42$$

Example 11.1, Step 4

- Convert to Natural (uncoded) Variables

$$x_1 = \frac{\xi_1 - 35}{5} \longrightarrow 1.0 = \frac{\xi_1 - 35}{5} \longrightarrow \xi_1 = 40$$

$$x_2 \frac{\xi_2 - 155}{5} \longrightarrow 0.42 = \frac{\xi_2 - 155}{5} \longrightarrow \xi_2 = 157.1$$

Example 11.1 Step 5

- Run experiment using ξ_1 and ξ_2 to get reponse value

Example 11.1, Step 6

- Update table of results (see Table 11.1) and optionally create graph

Example 11.1, Step 7

- If local stationary point, stop
- Otherwise, increase Δ and repeat cycle starting at step 2

Table of Results

TABLE 11.3

Steepest Ascent Experiment for Example 11.1

	Coded Variables		Natural Variables		Response
Steps	x_1	x_2	ξ_1	ξ_2	y
Origin	0	0	35	155	
Δ	1.00	0.42	5	2	
Origin + Δ	1.00	0.42	40	157	41.0
Origin + 2Δ	2.00	0.84	45	159	42.9
Origin + 3Δ	3.00	1.26	50	161	47.1
Origin + 4Δ	4.00	1.68	55	163	49.7
Origin + 5Δ	5.00	2.10	60	165	53.8
Origin + 6Δ	6.00	2.52	65	167	59.9
Origin + 7Δ	7.00	2.94	70	169	65.0
Origin + 8Δ	8.00	3.36	75	171	70.4
Origin + 9Δ	9.00	3.78	80	173	77.6
Origin + 10Δ	10.00	4.20	85	175	80.3
Origin + 11Δ	11.00	4.62	90	179	76.2

Graph of Results

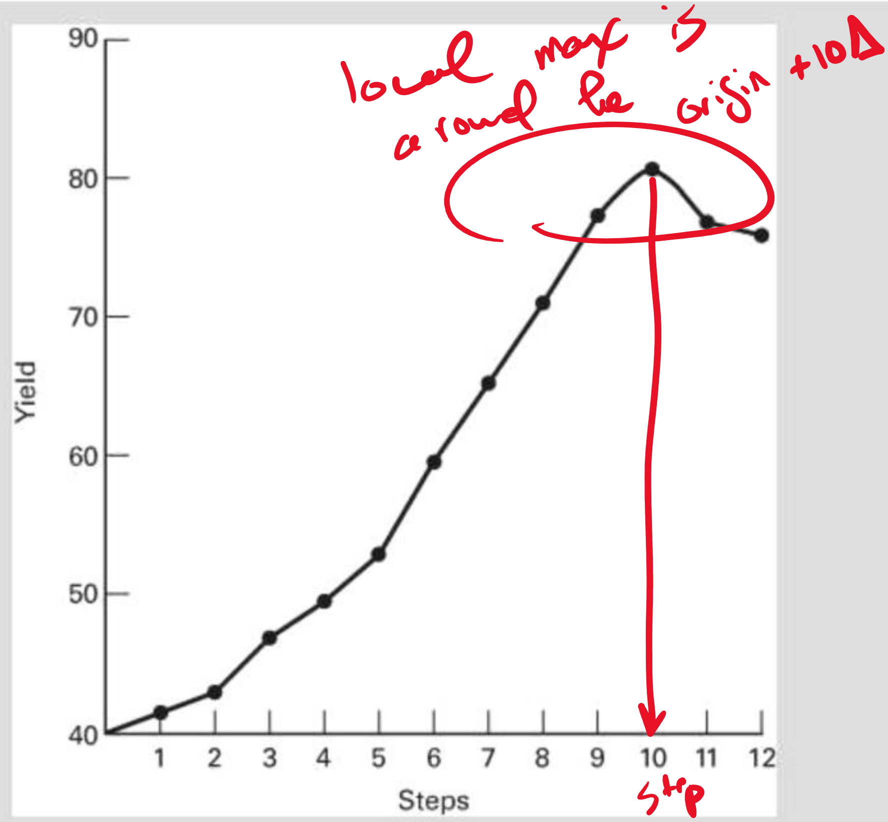


FIGURE 11.5 Yield versus steps along the path of steepest ascent for Example 11.1

Method of Steepest Descent

- You are searching in the opposite direction
- Therefore, you must change the sign of Δ_i

Steepest Descent - local min

gradient descent

Non-First Order Models

- Note that Δ_i is $\frac{\partial \widehat{y}}{\partial x_i} = \widehat{\beta}_i$
- Therefore, if a non-first order is used then $\Delta_i = \frac{\partial \widehat{y}}{\partial x_i}$

steepest() in R

- Do not use!
- Is entirely model-based
  - Any mismatch between "real world" and model will result in inaccuracies
  - Does not require additional experiments to be run so results could be incorrect