# Section 1

## MANE 6313

# Subsection 1

## Week 12, Module D

# Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

# Module Learning Outcome

*Employing test of hypothesis test on groups of variables.*

# Applying Tests of Hypothesis for Groups of Variables

- There is a concern regarding the overall error rate for multiple decisions
- Very similar to family-wise error rate in multiple comparisons
- In Ex 12.8, when examining the slope terms, three decisions are made at some value of alpha (usually 0.05). The overall error rate will be larger

$$\alpha_{overall} = 1 - (1 - \alpha_{individual})^n$$

- For Ex 12.8 using $\alpha_{individual} = 0.05$,

$$\alpha_{overall} = 1 - (1 - .05)^3 = 0.1426$$

## Tests for groups of regression coefficients

- We can examine the contribution of the regression sum of squares for a particular variable, say $x_j$, given that other variables $x_i (i \neq j)$ are included in the model
- We can also determine if the subset of regressor variables $x_1, x_2, \ldots, x_r (r < k)$ contribute to the model
- This test is necessary to control the overall error rate
- The full model is

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon$$

- We partition the regressors in two groups

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

where $\beta_\mathbf{1}$ is $(r \times 1)$ and $\beta_\mathbf{2}$ is $[(p - r) + 1]$

- We can rewrite the **full model** as

$$\mathbf{y} = \mathbf{X_1}\beta_\mathbf{1} + \mathbf{X_2}\beta_\mathbf{2} + \varepsilon$$

- The regression sum of squares for the full model is denoted $SS_R(\beta)$ with $p$ degrees of freedom
- We define the reduced model to be

$$\mathbf{y} = \mathbf{X_2}\beta_\mathbf{2}\varepsilon$$

- The regression sum of squares for the reduced model is $SS_r(\beta_\mathbf{2})$ with $p - r$ degrees of freedom.
- The regression sum of squares due to $\beta_\mathbf{1}$ given that $\beta_2$ is already in the model is

$$SS_R(\beta_\mathbf{1}|\beta_\mathbf{2}) = SS_R(\beta) - SS_R(\beta_2)$$

- The null hypothesis $\beta_{\mathbf{1}} = \mathbf{0}$ can be tested with the statistic

$$F_0 = \frac{SS_R(\beta_{\mathbf{1}}|\beta_{\mathbf{2}})/r}{MS_E}$$

- Reject $H_0$ if $F_0 > F_{\alpha,r,n-p}$
- This is sometimes called the *partial F test* or extra sum of squares method

# Step 1 - Fit Full Model

```
```{r}
ex12_8.full <- lm(GrainRadius~PowderTemp+Extrusion+DieTemp,data=ex12_8.df)
anova(ex12_8.full)
```
```

```
Analysis of Variance Table

Response: GrainRadius
           Df Sum Sq Mean Sq  F value    Pr(>F)
PowderTemp  1   50.0    50.0   2.1622 0.2153975
Extrusion   1 1740.5  1740.5  75.2649 0.0009715 ***
DieTemp     1 3784.5  3784.5 163.6541 0.0002152 ***
Residuals   4   92.5    23.1
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 1: Full Model

## Step 2 - Fit Reduced Model

```
```{r}
ex12_8.reduced <- lm(GrainRadius~Extrusion+DieTemp,data=ex12_8.df)
anova(ex12_8.reduced)
```
```

```
Analysis of Variance Table

Response: GrainRadius
          Df Sum Sq Mean Sq F value    Pr(>F)    
Extrusion  1 1740.5  1740.5   61.07 0.0005501 ***
DieTemp    1 3784.5  3784.5  132.79 8.629e-05 ***
Residuals  5  142.5    28.5                      
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 2: Reducedl Model

## Step 3 - Analysis

**Full Model**

$SS_R(\beta) = 50 + 1740.5 + 3784.5$
$= 5575$

$df_R = 3$

**Reduced Model**

$SS_R(\beta_2) = 1740.5 + 3784.5$
$= 5525$

$df_R = 2$

$MS_E = 23.1$

$df_E = 4$

$$F_0 = \frac{(5575 - 5525)/(3-2)}{23.1} = 2.164$$

Reject $H_0$ if $F_0 > F_{.05,1,4} = 7.71$

Conclusion: Fail to reject $H_0$, all terms (PowderTemp) in $\beta_2$ have slopes of zero.

## Partial F-test in R

```
```{r}
anova(ex12_8.reduced,ex12_8.model)
```
```

```
Analysis of Variance Table

Model 1: GrainRadius ~ Extrusion + DieTemp
Model 2: GrainRadius ~ PowderTemp + Extrusion + DieTemp
  Res.Df   RSS Df Sum of Sq      F Pr(>F)
1      5 142.5
2      4  92.5  1        50 2.1622 0.2154
```

Source:
https://www.statology.org/partial-f-test/#:~:text=A%20partial%20F-test%20is%20used%20to%20determine%20whether,the%20predictor%20variables%20in%20the%20overall%20regression%20model.