Section 1

MANE 6313

Subsection 1

Week 13, Module G

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Compute stationary point for Box-Behnken example.

Stationary Points

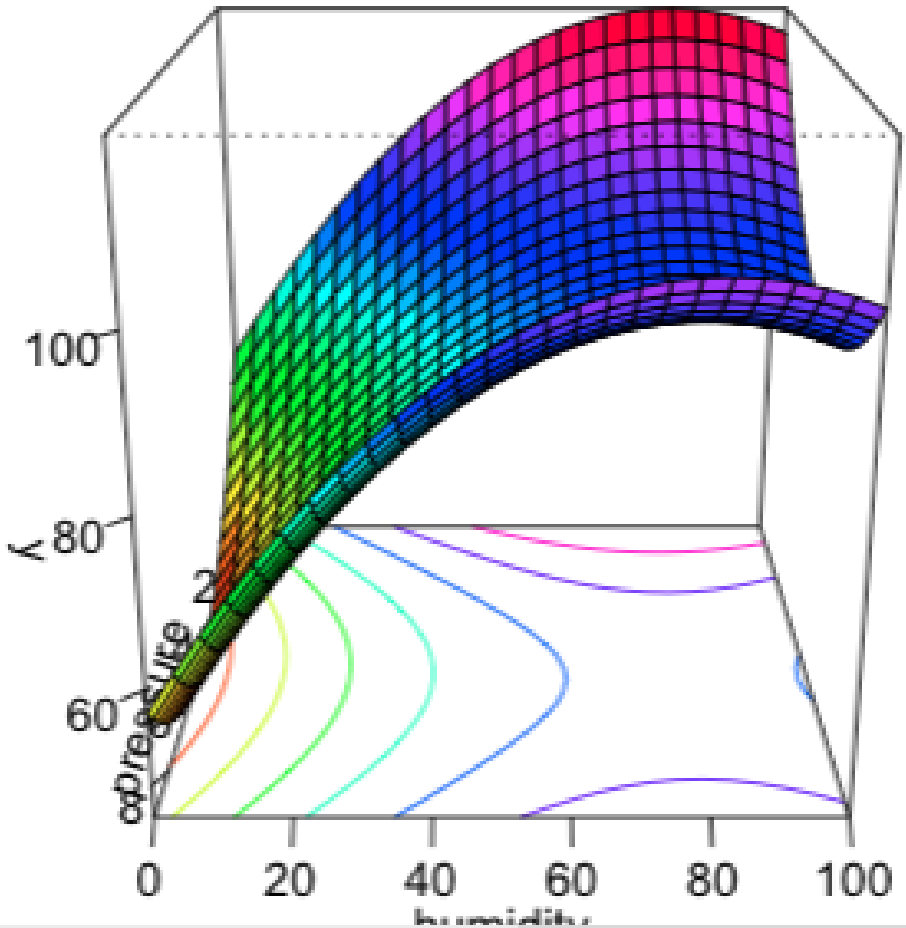
- Stationary points are points where $\frac{\partial \hat{y}}{\partial x_1} = \frac{\partial \hat{y}}{\partial x_2} = \cdots = \frac{\partial \hat{y}}{x_k} = 0$
- There are three types of stationary points: **maximum response**, **minimum response**, and **saddle point.**
- Note that stationary points are not optimal points, but could be optimal points
- Second derivative information is needed to determine if a stationary point is an optimal point,
- Contour plots and 3D plots can also be used to determine type of stationary point

Box-Benkhen Fitted Model

```
Call:
rsm(formula = y ~ FO(x2, x3) + PQ(x2, x3), data = bbd11.df)

             Estimate Std. Error t value Pr(>|t|)
(Intercept)  86.7368     7.4888 11.5822 7.17e-08 ***
x2           22.6250     6.6632  3.3955 0.005314 **
x3           -7.0000     6.6632 -1.0505 0.314166
x2^2        -19.1579     9.1719 -2.0888 0.058704 .
x3^2         19.0921     9.1719  2.0816 0.059456 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Multiple R-squared:  0.6349,    Adjusted R-squared:  0.5133
F-statistic: 5.218 on 4 and 12 DF,  p-value: 0.01138

Analysis of Variance Table

Response: y
             Df Sum Sq Mean Sq F value  Pr(>F)
FO(x2, x3)    2 4487.1 2243.56  6.3165 0.01337
PQ(x2, x3)    2 2926.1 1463.07  4.1191 0.04346
Residuals    12 4262.3  355.19
Lack of fit   4 1748.8  437.19  1.3915 0.31941
```

Box-Behnken Contour Image

```r
persp(bbd11.model2,x2~x3,zlab="y",theta=90, col=rainbow(50),contours="colors")
```



Stationary Points by Hand

From Second model

$$\hat{y} = 86.7368 + 22.625 x_2 - 7 x_3 - 19.1579 x_2^2 - 19.0921 x_3^2$$

$$\frac{\partial \hat{y}}{\partial x_2} = \frac{d\hat{y}}{dx_2} = 22.625 - 38.3158 x_2 \qquad (1)$$

$$\frac{\partial \hat{y}}{\partial x_3} = \frac{d\hat{y}}{dx_3} = -7 + 38.1420 x_3 \qquad (2)$$

To find stationary points, set equation (1) & (2) equal to zero and solve

From (1), $22.625 - 38.3158 x_2 = 0 \rightarrow x_2 = 0.5905$

From (2), $-7 + 38.142 x_3 = 0 \rightarrow x_3 = 0.1835$

Stationary Points from rsm()

- Check rsm() output
- Confirm graphically

```
Stationary point of response surface:
       x2        x3
0.5904876 0.1833218

Stationary point in original units:
 humidity  pressure
79.524382  5.733287
```

Figure 4: rsm() output containing stationary point