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Section 1

MANE 6313

Subsection 1

Week 10, Module A

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Describe fractional factorial designs, generators, defining relation and aliasing schemes for a one-half fraction.

Fractional Factorial Designs

- As the number of factors grows in a 2^k experiment, the number of runs often exceeds our ability to conduct the experiments. e.g. 2^6 experiment requires 64 runs.

$\binom{6}{2} = \frac{6!}{2!(6-2)!}$

- Often we are only interested in a few effects. E.g. in the 2^6 experiment, there are 6 main effects and 15 two-factor interactions. The remaining 42 degrees of freedom are associated with three-factor or higher interactions.
- Often we can get the information we need by running only a fraction of the factorial experiment
- Fractional factorials are often used as *screening experiments*.

2-factor interactions

Key Ideas for Fractional Factorial Experiments

- *The sparsity of effects principle.* The system or process is likely to be primarily driven by some of the main effects and low-order interactions.
- *The projection property.* When we identify unimportant variables and remove them from the model, the resulting model is stronger (larger) designs.
- *Sequential Experimentation.* It is possible to combine the runs of two (or more) fractional factorials to assemble sequentially a larger design to estimate the factor effects and interactions of interest.

The one-half fraction of a 2^k design

2^{-1} – ½ fraction

- This design results in a 2^{k-1} experiment, a half-fraction.
- You must select an effect to generate the two fractions. This effect is called the generator. E.g. in a 2^3 design select ABC as the *generator*. The other fraction is $-ABC$.
- We always associate I with the positive fraction. Thus, $I = ABC$ and we call this quantity the *defining relation* for the fractional factorial experiment.

  generator
- The fraction containing the positive generator is called the *principal fraction*. The other fraction is called the alternate or complementary fraction.

2^{3-1}

$I = ABC \longrightarrow IC = ABC^2$

$C = AB$

$2^{3-1} = 2^2$

Fraction Generators

- The defining relationship can also to generate the fractions
- One-half fraction of a 3 factor design
  - Definining relation is I=ABC
  - First fraction generator: C=AB
  - Second fraction generator: C=-AB

trts	A	B	C = AB
c	−	−	+
a	+	−	−
b	−	+	−
abc	+	+	+

1/2 fraction

2^(3-1) Example

$I = -ABC \Rightarrow IC = -ABC^2 \Rightarrow C = -AB$

A	B	C = -AB	trt
-	-	-	(1)
+	-	+	ac
-	+	+	bc
+	+	-	ab

2^{3-1}

Aliasing Scheme from Principal Fraction

1/2 fraction: Defining relations $I = ABC$

ℓ_A : $A(I = ABC) \rightarrow A = A^2BC \rightarrow A = BC$

ℓ_B : $B(I = ABC) \rightarrow B = AC \; [B + AC]$

ℓ_C : $C(I = ABC) \rightarrow C = AB$

ℓ_{AB} : $AB(I = ABC) \rightarrow AB = A^2B^2C \rightarrow AB = C$

ℓ_{AC} : $AC = B$

ℓ_{BC} : $BC = A$

trt	A	B	C = AB	BC
c	−	−	+	−
a	+	−	−	+
b	−	+	−	−
abc	+	+	+	+

Main Effect A: $\frac{a+ab}{2n} - \frac{c+abc}{2n}$ BC Effect: $\frac{a+abc}{2n} - \frac{c+abc}{2n}$

$I = -ABC$

Aliasing Scheme from the Complementary Fraction

$\ell'_A : A(I = -ABC) \rightarrow A = -A^2BC \rightarrow A = -BC$

$\ell'_B : B(I = -ABC) \rightarrow B = -AC$

$\ell'_C : C(I = -ABC) \rightarrow C = -AB$

Combining Aliasing Schemes

ℓ_A: $A = BC$

ℓ'_A: $A = -BC$

$$\frac{\ell_A + \ell'_A}{2} = \frac{(A+BC) + (A-BC)}{2} = \frac{2A+0}{2} = A$$

$$\frac{\ell_A - \ell'_A}{2} = \frac{(A+BC) - (A-BC)}{2} = \frac{2BC}{2} = BC$$