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Section 1

MANE 6313

Subsection 1

Week 10, Module D

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Describe resolution of an experimental design.

Resolution of Experimental Design

- Definition. A design is of resolution R if no p-factor effect is aliased with another effect containing less than $R - p$ factors.
- The three most common design resolutions are:
  - *Resolution III designs.* No main effect is aliased with any other main effect, but main effects are aliased with two-factor interactions and two-factor interactions may be aliased with each other.
  - *Resolution IV designs.* No main effect is aliased with any other main effect *or* with any two-factor interaction, but two interactions are aliased with other two-factor interactions
  - *Resolution V designs.* No main effect or two-factor interactions is aliased with any other main effect or two-factor interaction, but two-factor interactions are aliased with three-factor interactions.
- In general, the resolution of a two-level fractional factorial design is equal to the smallest number of letters in the defining relation.

Projection of Fractions into Factorials

- Any fractional factorial design of resolution R contains complete factorial designs (possibly replicated factorials) in any subset of $R-1$ factors
- Very useful result in screening experiments
- If we can eliminate variables as being non-significant, the fractional factorial design may become a (replicated) factorial design

- See figure 8.2 on page 278 of your textbook.

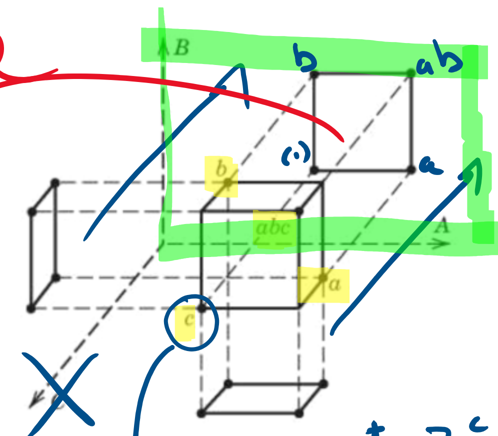

FIGURE 8.2 Projection of a 2^{3-1}_{III} design into three 2^2 designs

Figure 1: Figure 8.2

Sources of Design Resolution

- Textbook
  - Table 8.14
  - Table VIII (appendix a-16)
- R
  - FrF2

Textbook Table 8.14

TABLE 8.14

Selected 2^{k-p} Fractional Factorial Designs

Number of Factors, k	Fraction	Number of Runs	Design Generators
3	2_{III}^{3-1}	4	$C = \pm AB$
4	2_{IV}^{4-1}	8	$D = \pm ABC$
5	2_{V}^{5-1}	16	$E = \pm ABCD$
	2_{III}^{5-2}	8	$D = \pm AB$
			$E = \pm AC$
6	2_{VI}^{6-1}	32	$F = \pm ABCDE$
	2_{IV}^{6-2}	16	$E = \pm ABC$
			$F = \pm BCD$
	2_{III}^{6-3}	8	$D = \pm AB$
			$E = \pm AC$
			$F = \pm BC$
7	2_{VII}^{7-1}	64	$G = \pm ABCDEF$
	2_{IV}^{7-2}	32	$F = \pm ABCD$
			$G = \pm ABDE$
	2_{IV}^{7-3}	16	$E = \pm ABC$
			$F = \pm BCD$
			$G = \pm ACD$
	2_{III}^{7-4}	8	$D = \pm AB$
			$E = \pm AC$
			$F = \pm BC$
			$G = \pm ABC$
8	2_{V}^{8-2}	64	$G = \pm ABCD$
			$H = \pm ABEF$
	2_{IV}^{8-3}	32	$F = \pm ABC$
			$G = \pm ABD$
			$H = \pm BCDE$
	2_{IV}^{8-4}	16	$E = \pm BCD$
			$F = \pm ACD$

2^{4-1} Resolution

Full factorial in 4-1 columns, Start with A. Create 1 column from others

A B C D=ABC

2^{k-1} design it is 2^{-1} or $\frac{1}{2}$ fraction

Textbook Table VIII (A-16)

TABLE VIII

Alias Relationships for 2^{k-p} Fractional Factorial Designs with $k \le 15$ and $n \le 64$

Designs with 3 Factors

(a) 2^{3-1}; 1/2 fraction of 3 factors in 4 runs — **Resolution III**

Design Generators

$C = AB$

Defining relation: $I = ABC$

Aliases

$A = BC$

$B = AC$

$C = AB$

Designs with 4 Factors

(b) 2^{4-1}; 1/2 fraction of 4 factors in 8 runs — **Resolution IV**

Design Generators

$D = ABC$

Defining relation: $I = ABCD$

Aliases

$A = BCD$

$B = ACD$

$C = ABD$

$D = ABC$

$AB = CD$

$AC = BD$

FrF2 Design Catalog

Source: Grompig, U. (2014). R Package FrF2 for Creating and Analyzing Fractional Factorial 2-Level Designs. *Journal of Statistical Software*, 56(1), 1-56.

number of runs

number of factors	8	16	32	64	128	256	512	1024	2048	4096
						only the MA design				
3	full									
4	IV	full								
5	III	V	full							
6	III	IV	VI	full						
7	III	IV	IV	VII	full					
8		IV	IV	V	VIII	full				
9		III	IV	IV	VI	IX	full			
10		III	IV	IV	V	VI	X	full		
11		III	IV	IV	V	VI	VII	XI	full	
12		III	IV	IV	IV	VI	VI	VIII	XII	full
13		III	IV	IV	IV	V	VI	VII	VIII	XIII
14		III	IV	IV	IV	V	VI	VII	VIII	IX
15		III	IV	IV	IV	V	VI	VII	VIII	VIII
16			IV	IV	IV	V	VI	VI	VIII	VIII
17			III	IV	IV	V	VI	VI	VII	VIII
18			III	IV	IV	IV	VI	VI	VII	VIII
19			III	IV	IV	IV	V	VI	VII	VIII
20			III	IV	IV	IV	V	VI	VII	VIII
21			III	IV	IV	IV	V	VI	VII	VIII
22			III	IV	IV	IV	V	VI	VII	VIII
23			III	IV	IV	IV	V	VI	VII	VIII
24			III	IV	IV	IV	IV	VI	VI	VIII

Resolution III up to	31	63	127			factors.			
Resolution IV up to		32	64	80	160	factors.			
Resolution V up to number of factors:						33	47	65	
Resolution VI up to number of factors:						24	34	48	
First design is MA up to number of factors:									
	31	63	127	36	29	28	32	26	

FrF2 Design Resolution

```{r}
library(FrF2)
summary(FrF2(nfactors=8,resolution=4))
```

R Console | data.frame 2 x 8 | data.frame 16 x 8

```
Call:
FrF2(nfactors = 8, resolution = 4)

Experimental design of type  FrF2
16  runs

Factor settings (scale ends):

Design generating information:
$legend
[1] A=A B=B C=C D=D E=E F=F G=G H=H

$generators
[1] E=ABC F=ABD G=ACD H=BCD


Alias structure:
$fi2
[1] AB=CE=DF=GH AC=BE=DG=FH AD=BF=CG=EH AE=BC=DH=FG AF=BD=CH=EG AG=BH=CD=EF AH=BG=CF=DE


The design itself:
class=design, type= FrF2
```

(4)

$2^8 = 256$ ↓ 16

4 generators $2^{8-4} = 16$

2^{8-4}

1) Full factorial in 4 factors

2) create additional 4 columns using generators