

Printout

Friday, March 24, 2023 2:57 PM

Section 1

MANE 6313

Subsection 1

Week 11, Module C

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Appraise minimum aberration designs.

Minimum Aberration Designs

- Concept introduced by Fries and Hunter in 1980
- “The purpose of this paper is to provide a method for selecting a best subset of designs from the set of $2^{(k-p)}$ fractional factorial designs of highest resolution”
- Minimum aberration is used as measure for best designs
- Fries and Hunter define a design to be a minimum aberration design if “... the design which minimizes the number of words in the defining relation that are of minimum length.”

Fries and Hunter Example

602

ARTHUR FRIES AND WILLIAM G. HUNTER

TABLE 1—Three choices for a 2^{7-2} fractional factorial design for seven variables 1, 2, 3, 4, 5, 6, 7 in 32 runs. Entries underlined with a tilde ($\sim$) are to be regarded as boldfaced characters.

Design	(a)	(b)	(c)
Generators	$\underline{6=123}, \underline{7=234}$	$\underline{6=123}, \underline{7=145}$	$\underline{6=1234}, \underline{7=1235}$
Defining relation	$\underline{I=1236=2347=1467}$	$\underline{I=1236=1457=234567}$	$\underline{I=12346=12357=4567}$
Strings of aliased two-factor interactions (assuming three-factor and higher-order interactions are negligible)	$\underline{12+36}$ $\underline{13+26}$ $\underline{14+67}$ $\underline{17+46}$ $\underline{24+37}$ $\underline{27+34}$ $\underline{16+23+47}$	$\underline{12+36}$ $\underline{13+26}$ $\underline{14+57}$ $\underline{15+47}$ $\underline{16+23}$ $\underline{17+45}$	$\underline{45+67}$ $\underline{46+57}$ $\underline{47+56}$

Figure 1: Minimum Aberration Design Example

FrF2

- FrF2 provides minimum aberration (ma) designs
- FrF2 also considers word length pattern (wlp) and clear 2 factor interactions (2fis)

		number of runs									
		8	16	32	64	128	256	512	1024	2048	4096
		<i>only the MA design</i>									
number of factors	3	full									
	4	IV	full								
	5	III	V	full							
	6	III	IV	VI	full						
	7	III	IV	IV	VII	full					
	8		IV	IV	V	VIII	full				
	9		III	IV	IV	VI	IX	full			
	10		III	IV	IV	V	VI	X	full		
	11		III	IV	IV	V	VI	VII	XI	full	
	12		III	IV	IV	IV	VI	VI	VIII	XII	full
	13		III	IV	IV	IV	V	VI	VII	VIII	XIII
	14		III	IV	IV	IV	V	VI	VII	VIII	IX
	15		III	IV	IV	IV	V	VI	VII	VIII	VIII
	16			IV	IV	IV	V	VI	VI	VII	VII
	17			III	IV	IV	V	VI	VI	VII	VII
	18			III	IV	IV	IV	VI	VI	VII	VII
	19			III	IV	IV	IV	V	VI	VII	VII
	20			III	IV	IV	IV	V	VI	VII	VII
	21			III	IV	IV	IV	V	VI	VII	VII
	22			III	IV	IV	IV	V	VI	VII	VII
	23			III	IV	IV	IV	V	VI	VII	VII
	24			III	IV	IV	IV	IV	VI	VI	VII

Design Info, part 1

```

66 library(FrF2)
67 design.info(FrF2(nruns=32,nfactors=9))
68

```

```

$type
[1] "FrF2"

$nruns
[1] 32

$nfactors
[1] 9

$factor.names
$factor.names$A
[1] -1 1

$factor.names$B
[1] -1 1

$factor.names$C
[1] -1 1

$factor.names$D
[1] -1 1

$factor.names$E
[1] -1 1

$factor.names$F
[1] -1 1

$factor.names$G
[1] -1 1

$factor.names$H
[1] -1 1

$factor.names$I
[1] -1 1

```

32

$2^9 = 512$
 $2^{9.4}$
 $\frac{1}{2} \rightarrow 256$
 $\frac{1}{4} \rightarrow 128$
 $\frac{1}{8} \rightarrow 64$
 $\frac{1}{32} \rightarrow 32$

← skips I

Design Info, part 2

```

$catlg.name
[1] "catlg"

$catlg.entry
Design: 9-4.1
  32 runs, 9 factors,
  Resolution IV
  Generating columns: 7 11 19 29
  WLP (3plus): 0 6 8 0 0 , 8 clear 2fis
  Factors with all 2fis clear: J

$aliased
$aliased$legend
[1] "A=A" "B=B" "C=C" "D=D" "E=E" "F=F" "G=G" "H=H" "J=J"

$aliased$main
character(0)

$aliased$fi2
[1] "AB=CF=DG=EH" "AC=BF" "AD=BG" "AE=BH" "AF=BC" "AG=BD" "AH=BE" "CD=FG"
[9] "CE=FH" "CG=DF" "CH=EF" "DE=GH" "DH=EG"

$FrF2.version
[1] "2.2-3"

$replications
[1] 1

$repeat.only
[1] FALSE

$randomize
[1] TRUE

$seed
NULL

```

2⁹⁻⁴
9-4.1

($\frac{1}{2}$) = 36 2-factor interactions
8 clear 2fis
AJ - clear

Generators

- List effects in standard order and match generating columns from catalog entry

$2^{9-4} \rightarrow$ Full factorial in 5 factors (A, B, C, D, E), generate 4 columns (F, G, H, J) [note I skipped]

Generating columns: 7, 11, 19, 29

column #	effect	column #	effect
1	A	23	ABCE
2	B	24	DE
3	AB	25	ADE
4	C	26	BDE
5	AC	27	ABDE
6	BC	28	CDE
7	ABC ✓	29	ACDE ✓
8	D	:	:
9	AD		
10	BD		
11	ABD		
12	CD		
13	ACD		
14	BCD		
15	ABCD		
16	DE		
17	AE		
18	BE		

Handwritten notes:

- $F = ABC$ or $I = ABC$
- $G = ABD$ or $I = ABD$
- column 19: ABE
- $H = ABE$ or $I = ABEH$
- $J = ACDE$ or $I = ACDES$

Generators

1st Generator (factor F) from column 7

$$F = ABC \rightarrow \tau = ABCF$$

2nd Generator (factor G) from column 11

$$G = ABD$$

3rd Generator (factor H) from column 19

$$H = ABE$$

4th Generator (factor J) from column 29

$$J = ACDE$$

Figure 6. Example of a Generator

Generators from FrF2

```

70 {r}
71 library(FrF2)
72 maDesign <- FrF2(design = "9-4.1", randomize=FALSE)
73 summary(maDesign)
74

```

R Console

data.frame
2 x 9

data.frame
32 x 9

Call:
FrF2(design = "9-4.1", randomize = FALSE)

Experimental design of type FrF2
32 runs

Factor settings (scale ends):

Design generating information:

\$legend

[1] A=A B=B C=C D=D E=E F=F G=G H=H J=J

\$generators

[1] F=ABC G=ABD H=ABE J=ACDE

Alias structure:

\$fi2

[1] AB=CF=DG=EH AC=BF AD=BG AE=BH AF=BC AG=BD AH=BE CD=FG

CE=EH

[10] CG=DF CH=EF DE=GH DH=EG