# Section 1

## MANE 6313

Subsection 1

Week 10, Module E

# Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

# Module Learning Outcome

*Analyze a one-quarter fraction manually.*

## The quarter-fraction of the $2^k$ Design

- You must select two generators, $I = P$ and $I = Q$
- Don't forget the generalized interaction. The *complete defining relation* is $I = P = Q = PQ$
- There are four possible fractions formed by the combinations of $(\pm P, \pm Q)$
- The principal fraction is defined by $I = P = Q$
- The complementary fractions are $I = -P = Q$, $I = P = -Q$, $I = -P = -Q$

## Quarter Fraction Example – Problem 8.11 (Textbook 9th edition)

**8.11**    An article in *Industrial and Engineering Chemistry* ("More on Planning Experiments to Increase Research Efficiency," 1970, pp. 60–65) uses a $2^{5-2}$ design to investigate the effect of $A$ = condensation temperature, $B$ = amount of material 1, $C$ = solvent volume, $D$ = condensation time, and $E$ = amount of material 2 on yield. The results obtained are as follows:

$$e = 23.2 \quad ad = 16.9 \quad cd = 23.8 \quad bde = 16.8$$
$$ab = 15.5 \quad bc = 16.2 \quad ace = 23.4 \quad abcde = 18.1$$

**(a)** Verify that the design generators used were $I = ACE$ and $I = BDE$.

**(b)** Write down the complete defining relation and the aliases for this design.

**(c)** Estimate the main effects.

**(d)** Prepare an analysis of variance table. Verify that the $AB$ and $AD$ interactions are available to use as error.

**(e)** Plot the residuals versus the fitted values. Also ~~~~

# Design Verification

Design Generators

$$I = ACE \qquad \rightarrow A = A^2CE \rightarrow A = CE$$

$$I = BDE \qquad \rightarrow B = B^2DE \rightarrow B = DE$$

Create full factorial in CDE and A & B using design generators

| C | D | E | A = CE | B = DE | trt |
|---|---|---|--------|--------|-----|
| − | − | − | + | + | a b |
| + | − | − | − | + | b c |
| − | + | − | + | − | a d |
| + | + | − | − | − | c d |
| − | − | + | − | − | e |
| + | − | + | + | − | a c e |
| − | + | + | − | + | b d e |

## Defining Relation

Full    Defining  Relation

$$I = ACE = BDE = ABCDE^2$$
$$= ACE = BDE = ABCD$$

$\ell_A = A(I = ACE = BDE = ABCD) = AI = A^2CE = ABDE = A^2BCD$
$$= A = CE = ABDE = BCD$$

$\ell_B = B(I = ACE = BDE = ABCD) \Rightarrow B = ABCE = DE = ACD$

$\ell_C = C(I = ACE = BDE = ABCD) \rightarrow C = AE = BCDE = ABD$

$\ell_D = D(I = ACE = BDE = ABCD) \rightarrow D = ACDE = BE = ABC$

## Aliasing for Interaction Effects

<u>2 - Factor Interactions</u>

There are $\binom{4}{3} = 6$ two-factor interactions:

$AB, AC, AD, BC, BD, CD$

$\ell_{AB}:$  $AB(I = ACE = BDE = ABCD)$

  $:$  $AB = A^2BCE = AB^2DE = CD$

  $:$  $AB = BCE = ADE = CD$

$\ell_{AC}:$  $AC(I = ACE = BDE = ABCD)$

  $:$  $AC = E = ABCDE = BD$

$\ell_{AD}:$  $AD(I = ACE = BDE = ABCD)$

  $:$  $AD = CDE = ABE = BC$