Section 1

MANE 6313

Subsection 1

Week 10, Module F

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Analyze a one-quarter fraction using R.

Quarter Fraction Example – Problem 8.11 (Textbook 9th edition)

8.11 An article in *Industrial and Engineering Chemistry* ("More on Planning Experiments to Increase Research Efficiency," 1970, pp. 60–65) uses a 2^{5-2} design to investigate the effect of A = condensation temperature, B = amount of material 1, C = solvent volume, D = condensation time, and E = amount of material 2 on yield. The results obtained are as follows:

$e = 23.2$	$ad = 16.9$	$cd = 23.8$	$bde = 16.8$
$ab = 15.5$	$bc = 16.2$	$ace = 23.4$	$abcde = 18.1$

(a) Verify that the design generators used were $I = ACE$ and $I = BDE$.

(b) Write down the complete defining relation and the aliases for this design.

(c) Estimate the main effects.

(d) Prepare an analysis of variance table. Verify that the AB and AD interactions are available to use as error.

(e) Plot the residuals versus the fitted values

FrF2 Generators

generators

There are `log2(nruns)` base factors the full factorial of which spans the design (e.g. 3 for 8 runs). The generators specify how the remaining factors are to be allocated to interactions of these.

`generators` can be

a list of vectors with position numbers of base factors (e.g. c(1,3,4) stands for the interaction between first, third and fourth base factor)

a vector of character representations of these interactions, e.g. "ACD" stands for the same interaction as above

a vector of columns numbers in Yates order (e.g. 13 stands for ACD). Note that the columns 1, 2, 4, 8, etc., i.e. all powers of 2, are reserved for the base factors and cannot be used for assigning additional factors, because the design would become a resolution II design. For looking up which column number stands for which interaction, type e.g. `names(Yates)[1:15]` for a 16 run design.

In all cases, preceding the respective entry with a minus sign (e.g. -c(1,3,4), "-ACD", -13) implies that the levels of the respective column are reversed. WARNING: Minus signs do not cause an error, but neither have an effect in case of automatic assignment of split-plot designs or hard-to-change columns.

Figure 2: FrF2 Generators Documentation

Incorrect Generators

```
105 ```{r}
106 library(FrF2)
107 p8_11.design2 <- FrF2(8,gen=c("ACE","BDE"),randomize=FALSE)
108 summary(p8_11.design2)
109 ```
```

```
Error in gen.check(k, generators) :
 All generators must contain integer numbers from 1 to 3
 or letters from A to C only.
```

Show Traceback

Figure 3: Incorrect Generators

Change of Variables

Design Generators

$I = ACE \rightarrow A = A^2CE \rightarrow A = CE$

$I = BDE \rightarrow B = B^2DE \rightarrow B = DE$

Create full factorial in CDE and AB using design generators

$a' \uparrow \quad b' \uparrow \quad c' \uparrow \quad d' = a'c' \quad e' = b'c'$

C	D	E	A = CE	B = DE	trt
-	-	-	+	+	a b
+	-	-	-	+	b c
-	+	-	+	-	a d
+	+	-	-	-	c d
-	-	+	-	-	e
+	-	+	+	-	a c e
-	+	+	-	+	b d e

Correct FrF2 Design

```
112 ```{r}
113 library(FrF2)
114 p8_11.design3 <- FrF2(8,factor.names=list(Solvent=c("-","+"),condTime=c("-","+"),Material2=c("-","+"),
    condTemp=c("-","+"),Material1=c("-","+")),generators=c("AC","BC"),randomize=FALSE)
115 summary(p8_11.design3)
116 ```
```

R Console | data.frame 2 x 5 | data.frame 8 x 5

	Solvent (C) <fctr>	condTime (D) <fctr>	Material2 (E) <fctr>	condTemp (A) <fctr>	Material1 (B) <fctr>
1	-	-	-	+	+
2	+	-	-	-	+
3	-	+	-	+	-
4	+	+	-	-	-
5	-	-	+	-	-
6	+	-	+	+	-
7	-	+	+	-	+
8	+	+	+	+	+

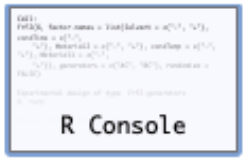
8 rows

trt	Y
ab	15.5
bc	16.2
ad	16.9
cd	23.8
e	23.2
ace	23.4
bde	16.8
abcde	18.1

Figure 5: Correct FrF2 Design

Design Details

```r
library(FrF2)
p8_11.design3 <- FrF2(8,factor.names=list(Solvent=c("-","+"),condTime=c("-","+"),Material2=c("-","+"),
condTemp=c("-","+"),Material1=c("-","+")),generators=c("AC","BC"),randomize=FALSE)
summary(p8_11.design3)
```




```
    "+")), generators = c("AC", "BC"), randomize = FALSE)

Experimental design of type  FrF2.generators
8  runs

Factor settings (scale ends):

Design generating information:
$legend
[1] A=Solvent   B=condTime  C=Material2 D=condTemp  E=Material1

$generators
[1] D=AC E=BC


Alias structure:
$main
[1] A=CD    B=CE    C=AD=BE D=AC    E=BC

$fi2
[1] AB=DE AE=BD
```

Add Response Variable

```
118 ```{r}
119 library(FrF2)
120 y2 <- c(15.5,16.2,16.9,23.8,23.2,23.4,16.8,18.1)
121 p8_11.design3 <- add.response(p8_11.design3,y2)
122 summary(p8_11.design3)
123 ```
```

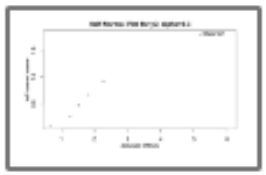
	Solvent <fctr>	condTime <fctr>	Material2 <fctr>	condTemp <fctr>	Material1 <fctr>	y2 <dbl>
1	-	-	-	+	+	15.5
2	+	-	-	-	+	16.2
3	-	+	-	+	-	16.9
4	+	+	-	-	-	23.8
5	-	-	+	-	-	23.2
6	+	-	+	+	-	23.4
7	-	+	+	-	+	16.8
8	+	+	+	+	+	18.1

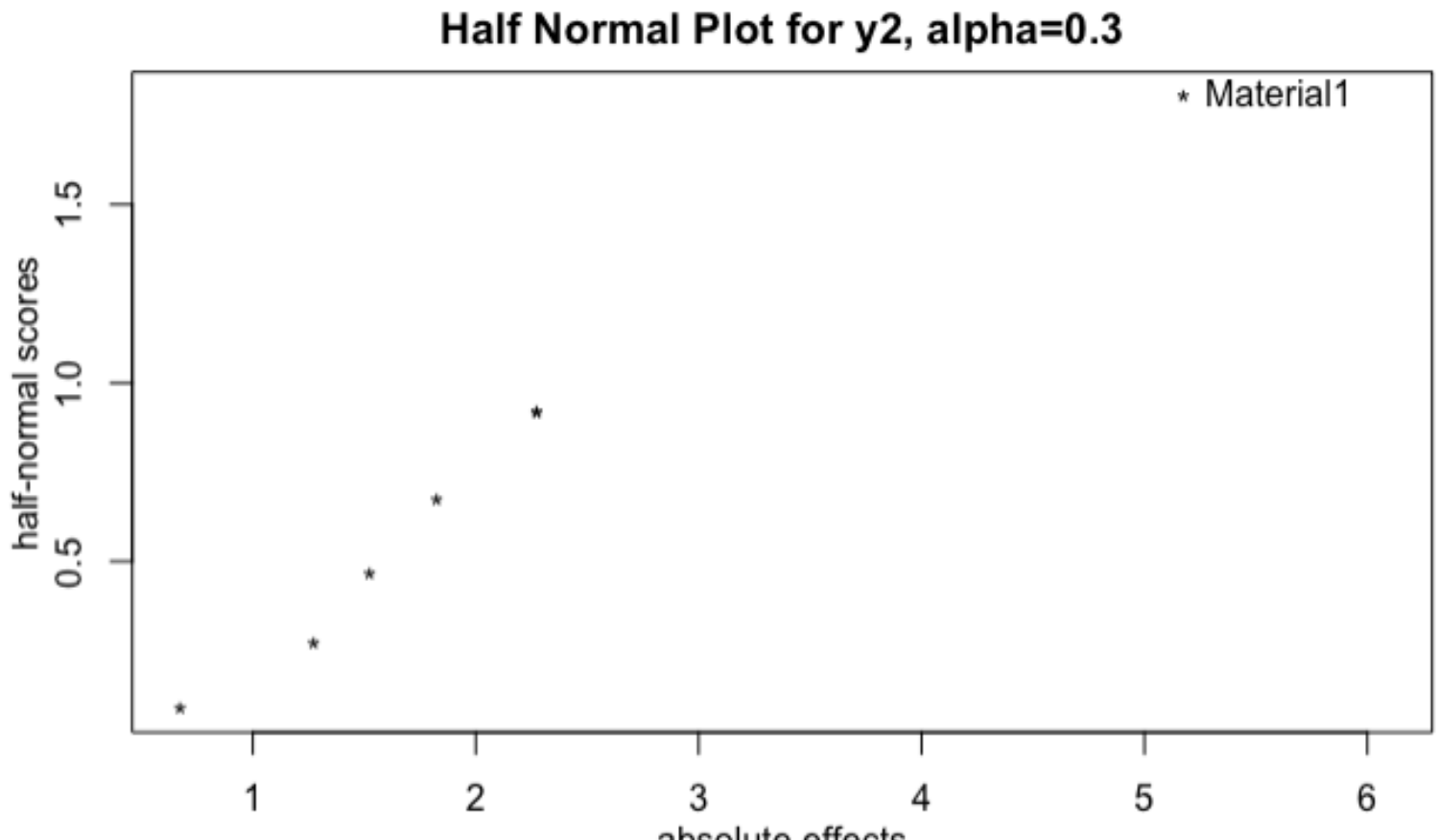
8 rows

Figure 7: Adding Response Variable

Half Normal Plot

```r
# Daniel Plot
DanielPlot(p8_11.design3,half=TRUE,response='y2',alpha=0.3)
```





First Model

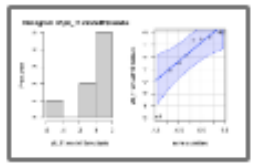
```r
p8_11.model1 <- aov(y2~Material1,data=p8_11.design3)
summary(p8_11.model1)
```

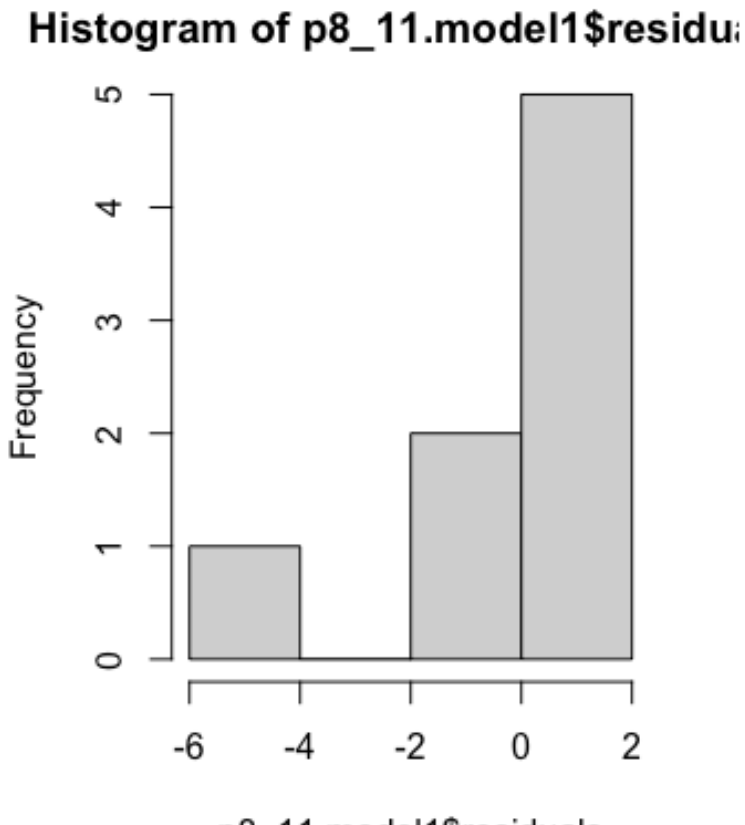
```
            Df Sum Sq Mean Sq F value Pr(>F)
Material1    1  53.56   53.56   8.883 0.0246 *
Residuals    6  36.18    6.03
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

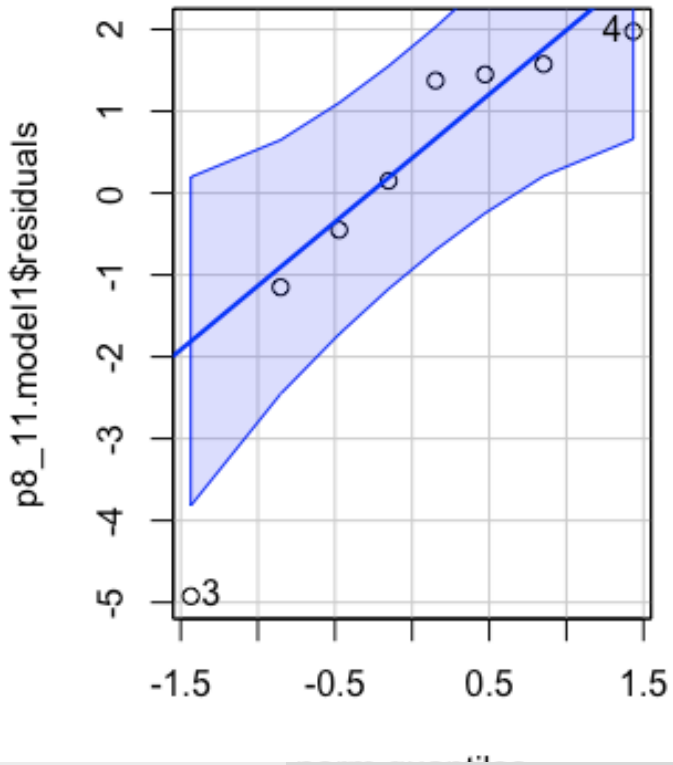
Figure 9: First Model

Residual Analysis: Normality Assumption

```
135 ```{r}
136 par(mfrow=c(1,2))
137 hist(p8_11.model1$residuals)
138 library(car)
139 qqPlot(p8_11.model1$residuals)
140 ```
```

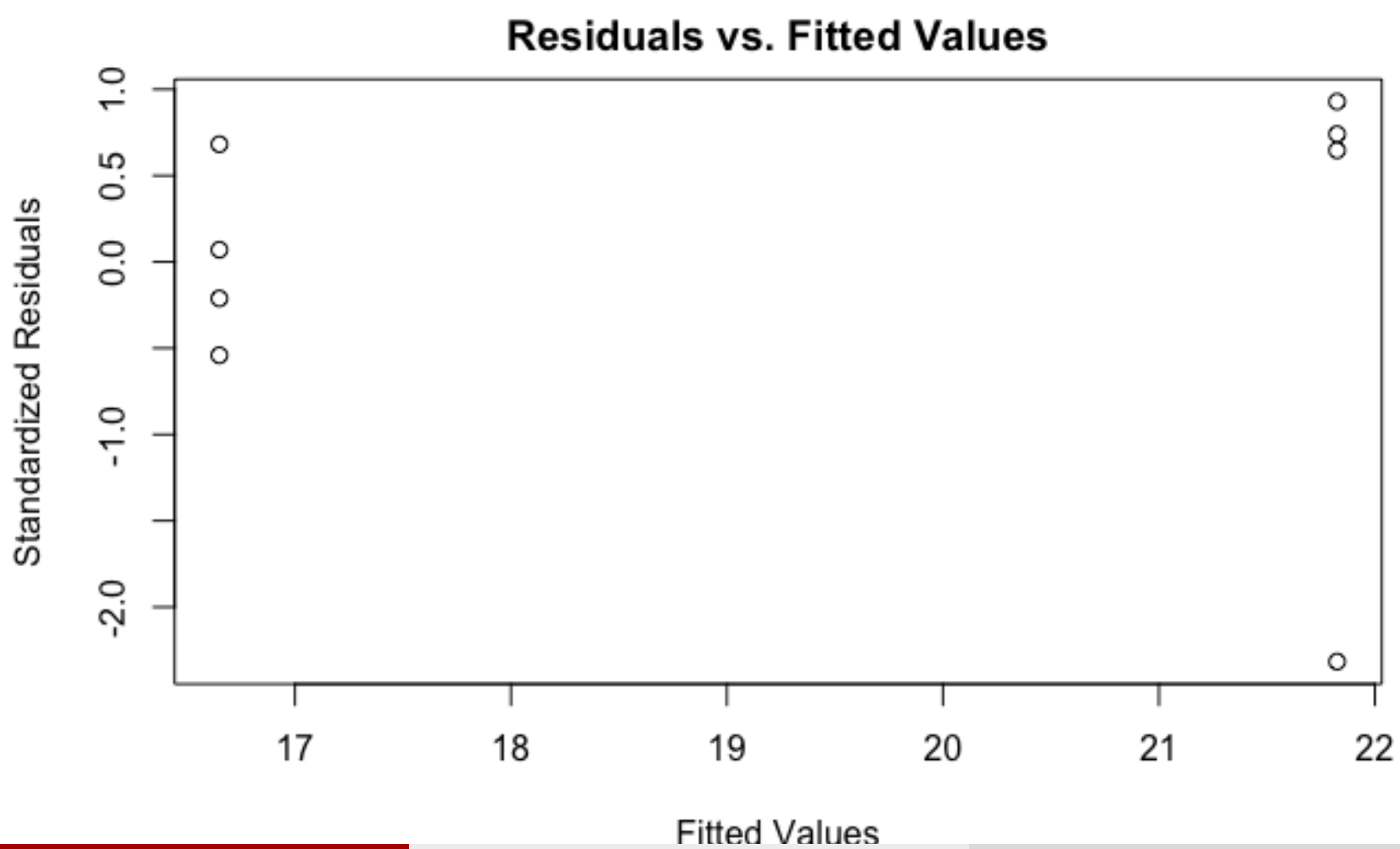






Residuals vs. Fitted Values

```r
```{r}
p8_11.stdRes <- rstandard(p8_11.model1)
plot(p8_11.model1$fitted.values,p8_11.stdRes,xlab="Fitted Values",ylab="Standardized
Residuals", main="Residuals vs. Fitted Values")
```
```



Residuals vs. Factor A

```r
# Create Data Frame with all residual information
residuals.df <- data.frame(p8_11.stdRes,p8_11.model1$fitted.values,p8_11.design3$Solvent,p8_11.design3$condTime,p8_11.design3$Material2,p8_11.design3$condTemp,p8_11.design3$Material1)
summary(residuals.df)
#
library(ggplot2)
ggplot(residuals.df,aes(x=p8_11.design3$Solvent,y=p8_11.stdRes)) +
  geom_dotplot(binaxis='y',stackdir='center') +
  labs(x="Solvent",y="Standard Residuals",title="Solvent vs. Residuals Plot")
```

R Console

[38;5;232mBin width defaults to 1/30 of the range of the data. Pick better value with `binwidth`. [39m

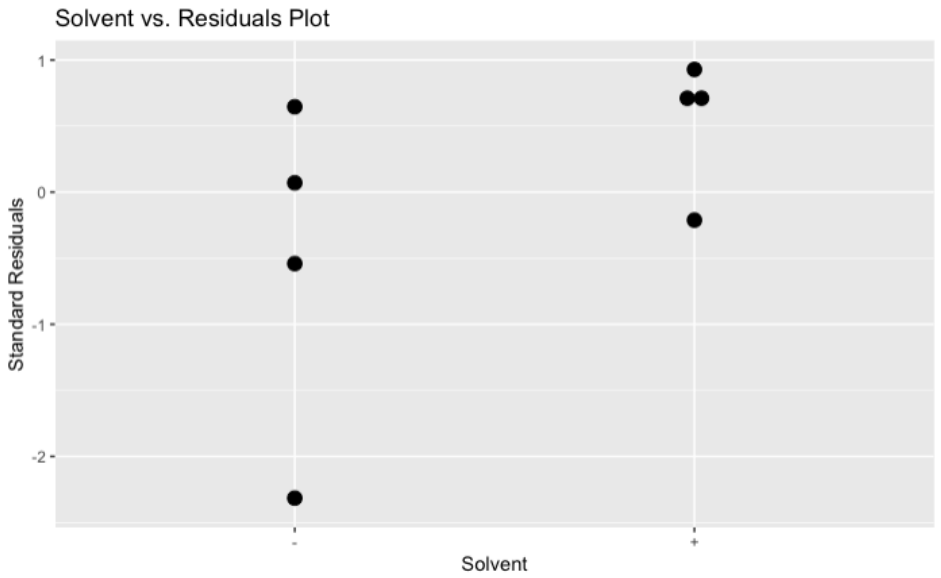


Figure 12: Residuals vs. Factor A

Residuals vs. Factor B

```
166 ```{r warning=FALSE,message=FALSE}
167 ggplot(residuals.df,aes(x=p8_11.design3$condTime,y=p8_11.stdRes)) +
168   geom_dotplot(binaxis='y',stackdir='center') +
169   labs(x="Condensation Time",y="Standard Residuals",title="Condensation Time vs. Residuals Plot")
170 ```
```

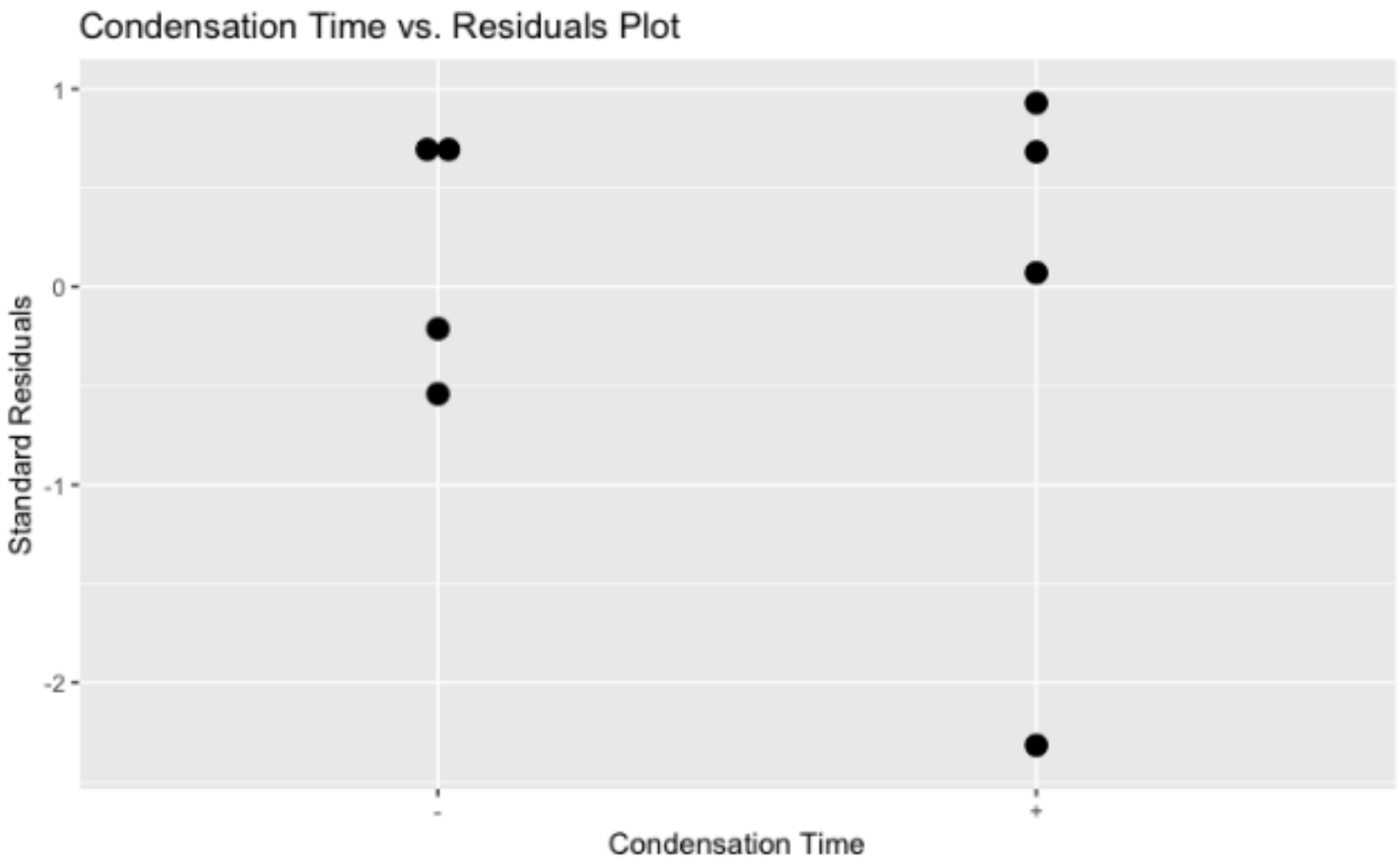
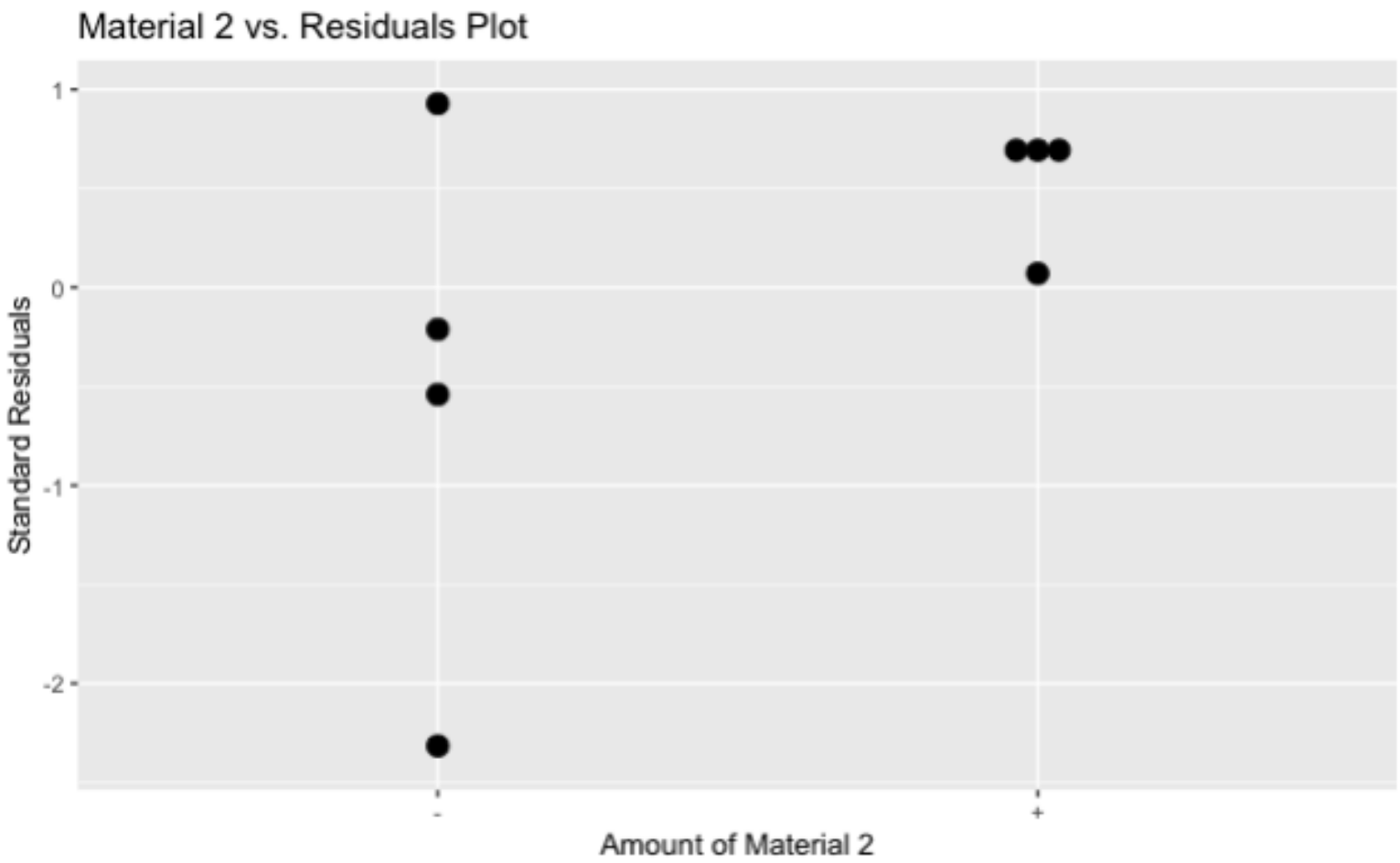


Figure 13: Residuals vs. Factor B

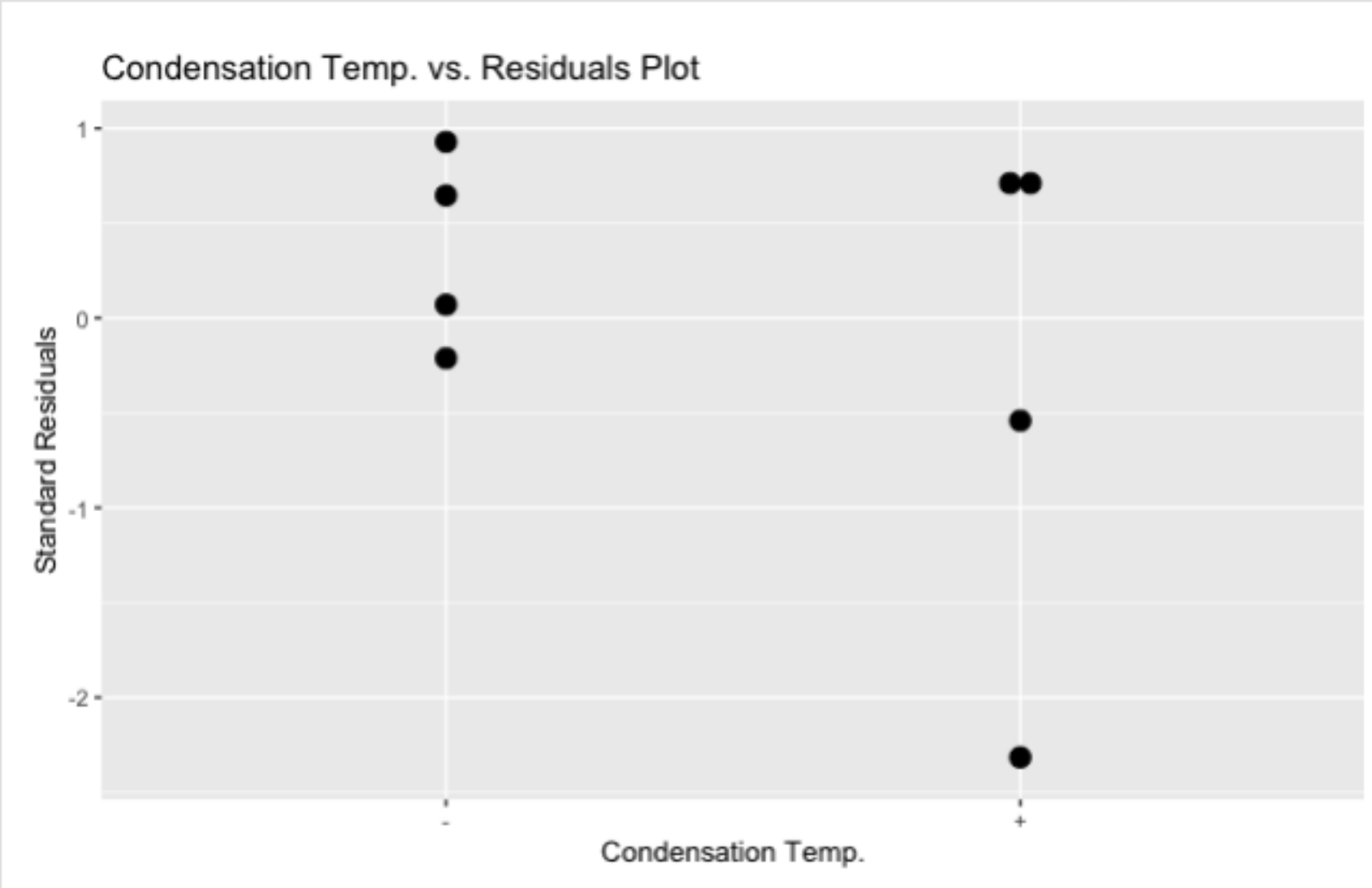
Residuals vs. Factor C

```
```{r message=FALSE}
ggplot(residuals.df,aes(x=p8_11.design3$Material2,y=p8_11.stdRes)) +
  geom_dotplot(binaxis='y',stackdir='center') +
  labs(x="Amount of Material 2",y="Standard Residuals",title="Material 2 vs. Residuals Plot")
```
```



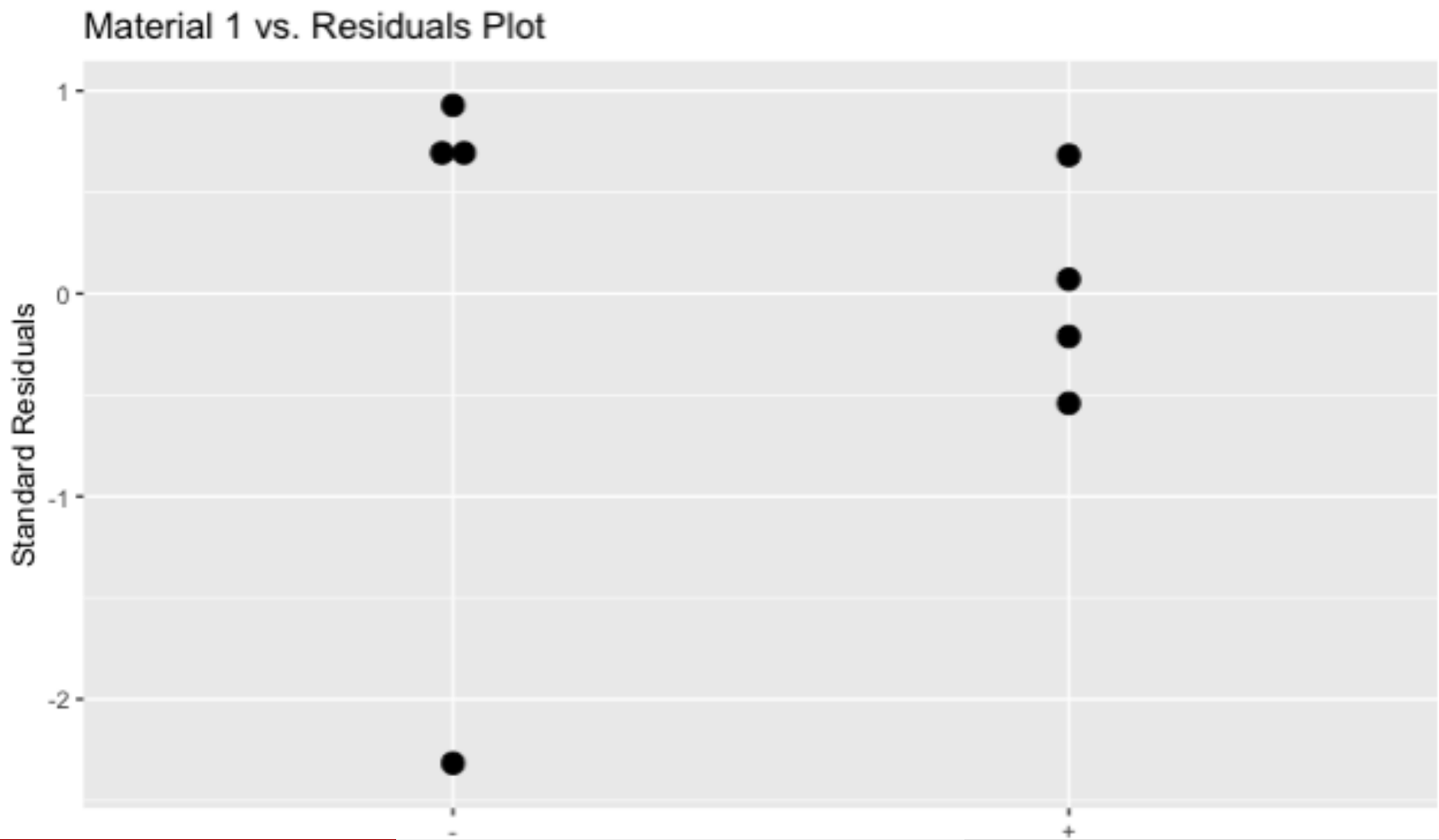
Residuals vs. Factor D

```
178 ```{r message=FALSE}
179 ggplot(residuals.df,aes(x=p8_11.design3$condTemp,y=p8_11.stdRes)) +
180   geom_dotplot(binaxis='y',stackdir='center') +
181   labs(x="Condensation Temp.",y="Standard Residuals",title="Condensation Temp. vs. Residuals Plot")
182 ```
```



Residuals vs. Factor E

```
184 ```{r message=FALSE}
185 p_e <- ggplot(residuals.df,aes(x=p8_11.design3$Material1,y=p8_11.stdRes)) +
186   geom_dotplot(binaxis='y',stackdir='center') +
187   labs(x="Amount of Material 1",y="Standard Residuals",title="Material 1 vs. Residuals Plot")
188 p_e
189 ```
```



Standardized Residuals

```
191 ```{r}
192 p8_11.stdRes
193 ```
```

```
        1         2         3         4         5         6         7         8
-0.5407837 -0.2116110 -2.3159648  0.9287371  0.6465892  0.7406385  0.0705370  0.6818577
```

Figure 17: Standardized Residuals