Section 1

# MANE 6313

Subsection 1

## Week 11, Module C

# Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

# Module Learning Outcome

*Appraise minimum aberration designs.*

## Minimum Aberration Designs

- Concept introduced by Fries and Hunter in 1980
- "The purpose of this paper is to provide a method for selecting a best subset of designs from the set of 2^(k-p) fractional factorial designs of highest resolution"
- Minimum aberration is used as measure for best designs
- Fries and Hunter define a design to be a minimum aberration design if "...the design which minimizes the number of words in the defining relation that are of minimum length."

# Fries and Hunter Example



TABLE 1—*Three choices for a* $2_{IV}^{7-2}$ *fractional factorial design for seven variables* **1, 2, 3, 4, 5, 6, 7** *in 32 runs. Entries underlined with a tilde (~) are to be regarded as boldfaced characters.*

| Design | (a) | (b) | (c) |
|---|---|---|---|
| Generators | 6=123,7=234 | 6=123,7=145 | 6=1234,7=1235 |
| Defining relation | I=1236=2347=1467 | I=1236=1457=234567 | I=12346=12357=4567 |
| Strings of aliased two-factor interactions (assuming three-factor and higher-order interactions are negligible) | 12+36<br>13+26<br>14+67<br>17+46<br>24+37<br>27+34<br>16+23+47 | 12+36<br>13+26<br>14+57<br>15+47<br>16+23<br>17+45 | 45+67<br>46+57<br>47+56 |

Figure 1: Minimum Aberration Design Example

# FrF2

- FrF2 provides minimum aberration (ma) designs
- FrF2 also considers word length pattern (wlp) and clear 2 factor interactions (2fis)

number of runs

| number of factors | 8 | 16 | 32 | 64 | 128 | 256 | 512 | 1024 | 2048 | 4096 |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | *only the MA design* | | | | |
| 3 | full | | | | | | | | | |
| 4 | IV | full | | | | | | | | |
| 5 | III | V | full | | | | | | | |
| 6 | III | IV | VI | full | | | | | | |
| 7 | III | IV | IV | VII | full | | | | | |
| 8 | | IV | IV | V | VIII | full | | | | |
| 9 | | III | IV | IV | VI | IX | full | | | |
| 10 | | III | IV | IV | V | VI | X | full | | |
| 11 | | III | IV | IV | V | VI | VII | XI | full | |
| 12 | | III | IV | IV | IV | VI | VI | VIII | XII | full |
| 13 | | III | IV | IV | IV | V | VI | VII | VIII | XIII |
| 14 | | III | IV | IV | IV | V | VI | VII | VIII | IX |
| 15 | | III | IV | IV | IV | V | VI | VII | VIII | VIII |
| 16 | | | IV | IV | IV | V | VI | VI | VIII | VIII |
| 17 | | | III | IV | IV | V | VI | VI | VII | VIII |
| 18 | | | III | IV | IV | IV | VI | VI | VII | VIII |
| 19 | | | III | IV | IV | IV | V | VI | VII | VIII |
| 20 | | | III | IV | IV | IV | V | VI | VII | VIII |
| 21 | | | III | IV | IV | IV | V | VI | VII | VIII |
| 22 | | | III | IV | IV | IV | V | VI | VII | VIII |
| 23 | | | III | IV | IV | IV | V | VI | VII | VIII |
| 24 | | | III | IV | IV | IV | IV | VI | VI | VIII |

# Design Info, part 1

```
66 library(FrF2)
67 design.info(FrF2(nruns=32,nfactors=9))
68 ```
```

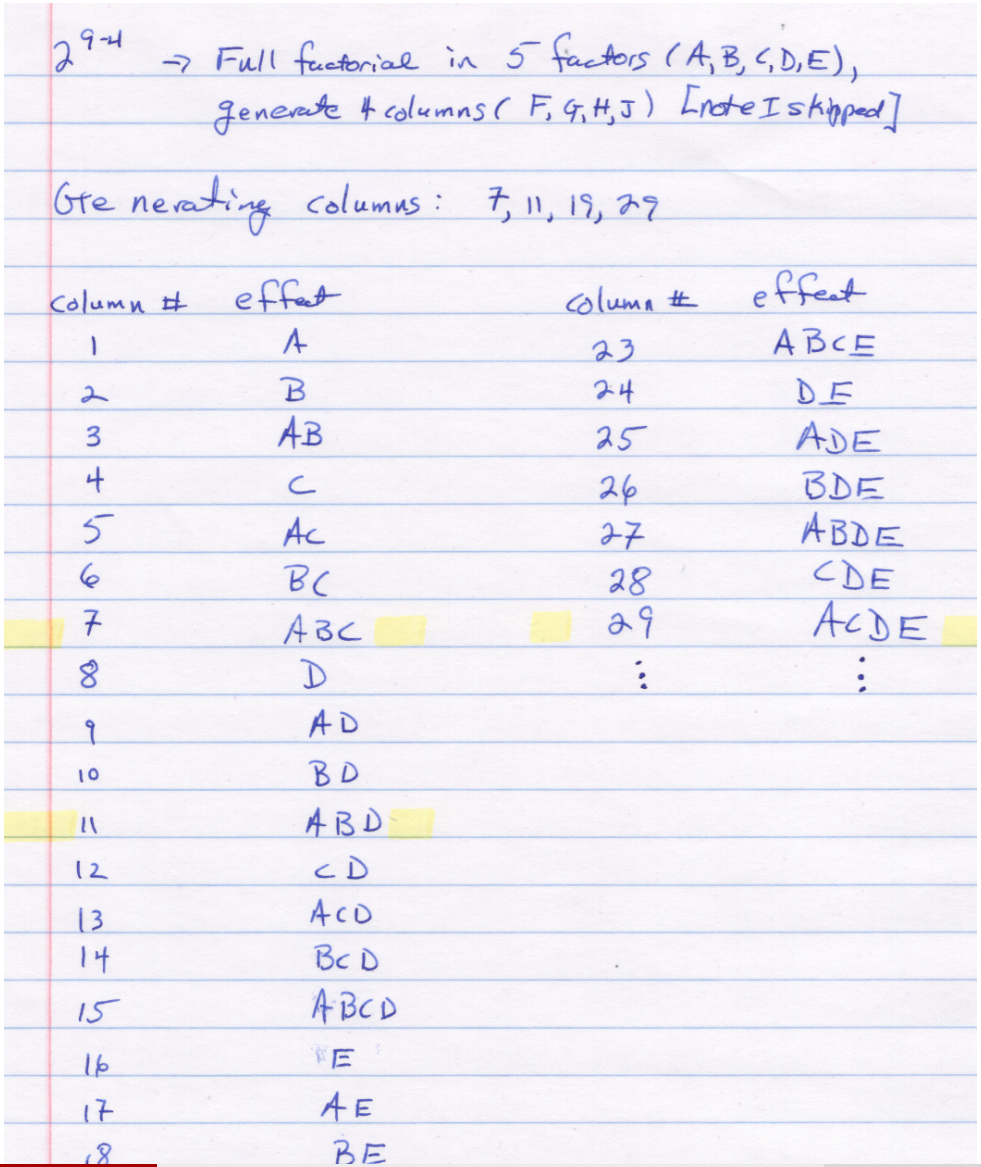
```
$type
[1] "FrF2"

$nruns
[1] 32

$nfactors
[1] 9

$factor.names
$factor.names$A
[1] -1  1

$factor.names$B
[1] -1  1

$factor.names$C
[1] -1  1

$factor.names$D
[1] -1  1

$factor.names$E
[1] -1  1

$factor.names$F
[1] -1  1

$factor.names$G
[1] -1  1

$factor.names$H
[1] -1  1

$factor.names$J
```

# Design Info, part 2

```
$catlg.name
[1] "catlg"

$catlg.entry
Design:  9-4.1
   32  runs,  9  factors,
   Resolution  IV
   Generating columns:  7 11 19 29
   WLP (3plus):  0 6 8 0 0 ,  8  clear 2fis
 Factors with all 2fis clear:  J

$aliased
$aliased$legend
[1] "A=A" "B=B" "C=C" "D=D" "E=E" "F=F" "G=G" "H=H" "J=J"

$aliased$main
character(0)

$aliased$fi2
 [1] "AB=CF=DG=EH" "AC=BF"       "AD=BG"       "AE=BH"       "AF=BC"       "AG=BD"       "AH=BE"       "CD=FG"
 [9] "CE=FH"       "CG=DF"       "CH=EF"       "DE=GH"       "DH=EG"


$FrF2.version
[1] "2.2-3"

$replications
[1] 1

$repeat.only
[1] FALSE

$randomize
[1] TRUE

$seed
NULL
```

# Generators

- List effects in standard order and match generating columns from catalog entry



$2^{9-4}$ → Full factorial in 5 factors (A, B, C, D, E), generate 4 columns (F, G, H, J) [note I skipped]

Generating columns: 7, 11, 19, 29

| column # | effect | column # | effect |
|---|---|---|---|
| 1 | A | 23 | ABCE |
| 2 | B | 24 | DE |
| 3 | AB | 25 | ADE |
| 4 | C | 26 | BDE |
| 5 | AC | 27 | ABDE |
| 6 | BC | 28 | CDE |
| 7 | ABC | 29 | ACDE |
| 8 | D | ⋮ | ⋮ |
| 9 | AD | | |
| 10 | BD | | |
| 11 | ABD | | |
| 12 | CD | | |
| 13 | ACD | | |
| 14 | BCD | | |
| 15 | ABCD | | |
| 16 | E | | |
| 17 | AE | | |
| 18 | BE | | |

# Generators

1st Generator (factor F) from column 7

$F = ABC$

2nd Generator (factor G) from column 11

$G = ABD$

3rd Generator (factor H) from column 19

$H = ABE$

4th Generator (factor J) from column 29

$J = ACDE$

Figure 6: Example Problem: Generators

# Generators from FrF2

```r
library(FrF2)
maDesign <- FrF2(design = "9-4.1",randomize=FALSE)
summary(maDesign)
```




```
Call:
FrF2(design = "9-4.1", randomize = FALSE)

Experimental design of type  FrF2
32  runs

Factor settings (scale ends):

Design generating information:
$legend
[1] A=A B=B C=C D=D E=E F=F G=G H=H J=J

$generators
[1] F=ABC  G=ABD  H=ABE  J=ACDE


Alias structure:
$fi2
 [1] AB=CF=DG=EH AC=BF       AD=BG       AE=BH       AF=BC       AG=BD       AH=BE       CD=FG
CE=FH
[10] CG=DF       CH=EF       DE=GH       DH=EG
```