Section 1

MANE 6313

Subsection 1

Week 11, Module F

Student Learning Outcome

- Select an appropriate experimental design with one or more factors,
- Select an appropriate model with one or more factors,
- Evaluate statistical analyses of experimental designs,
- Assess the model adequacy of any experimental design, and
- Interpret model results.

Module Learning Outcome

Describe sequential analysis and addition of center points.

Center Points for Fractional Factorial

- Same concept as introduced in Chapter 6.8 - The Addition of Center Points to the 2^k Design
- R documentation provides the following reasons to add center points to fractional factorial designs
  - "Provide a repeated benchmark run that can alert the experiment to unplanned changes in experimental conditions,"
  - "Provide an independent estimate of experimental error," and
  - "Provide a possibility for checking whether a first order model is sufficient"

Source:
https://search.r-project.org/CRAN/refmans/FrF2/html/add.center.html

Example Problem

- Devore, Chang and Sutherland provided an example
  - One-eight fraction of a seven factor design (16 runs)
  - 7 center points are added

Base Design

```r
library(FrF2)
base.df <- FrF2(nruns=16,nfactors=7,randomize=FALSE)
y.base <- c(458,323,355,353,404,360,376,314,404,361,361,388,377,393,427,312)
base.df <- add.response(base.df,y.base)
summary(base.df)
```

R Console | data.frame 2 x 7 | data.frame 16 x 8

	A <fctr>	B <fctr>	C <fctr>	D <fctr>	E <fctr>	F <fctr>	G <fctr>	y.base <dbl>
1	-1	-1	-1	-1	-1	-1	-1	458
2	1	-1	-1	-1	1	1	1	323
3	-1	1	-1	-1	1	1	-1	355
4	1	1	-1	-1	-1	-1	1	353
5	-1	-1	1	-1	1	-1	1	404
6	1	-1	1	-1	-1	1	-1	360
7	-1	1	1	-1	-1	1	1	376
8	1	1	1	-1	1	-1	-1	314
9	-1	-1	-1	1	-1	1	1	404
10	1	-1	-1	1	1	-1	-1	361

1–10 of 16 rows Previous 1 2 Next

Figure 1: Example Problem - Base Design

Combined Design

```r
library(FrF2)
combined.df <- add.center(base.df,7)
y.combined <- c(458,323,355,353,404,360,376,314,404,361,361,388,377,393,427,312,548,532,526,541,518,537,516)
combined.df <- add.response(combined.df,y.combined)
summary(combined.df)
```

R Console | data.frame 2 x 7 | data.frame 23 x 9

	A <dbl>	B <dbl>	C <dbl>	D <dbl>	E <dbl>	F <dbl>	G <dbl>	y.base <dbl>	y.combined <dbl>
11	-1	1	-1	1	1	-1	1	361	361
12	1	1	-1	1	-1	1	-1	388	388
13	-1	-1	1	1	1	1	-1	377	377
14	1	-1	1	1	-1	-1	1	393	393
15	-1	1	1	1	-1	-1	-1	427	427
16	1	1	1	1	1	1	1	312	312
17	0	0	0	0	0	0	0	NA	548
18	0	0	0	0	0	0	0	NA	532
19	0	0	0	0	0	0	0	NA	526
20	0	0	0	0	0	0	0	NA	541

11–20 of 23 rows Previous 1 2 3 Next

Figure 2: Combined Design

Design Details for Combined Design

```r
library(FrF2)
combined.df <- add.center(base.df,7)
y.combined <- c(458,323,355,353,404,360,376,314,404,361,361,388,377,393,427,312,548,532,526,541,518,537,516)
combined.df <- add.response(combined.df,y.combined)
summary(combined.df)
```


```
FrF2(nruns = 16, nfactors = 7, randomize = FALSE)

Experimental design of type  FrF2.center
23  runs

Factor settings (scale ends):

Responses:
[1] y.base     y.combined

Design generating information:
$legend
[1] A=A B=B C=C D=D E=E F=F G=G

$generators
[1] E=ABC F=ABD G=ACD


Alias structure:
$fi2
[1] AB=CE=DF AC=BE=DG AD=BF=CG AE=BC=FG AF=BD=EG AG=CD=EF BG=CF=DE
```

Half Normal Plot

```r
library(FrF2)
DanielPlot(combined.df,response="y.combined",half=TRUE,alpha=0.1)
```

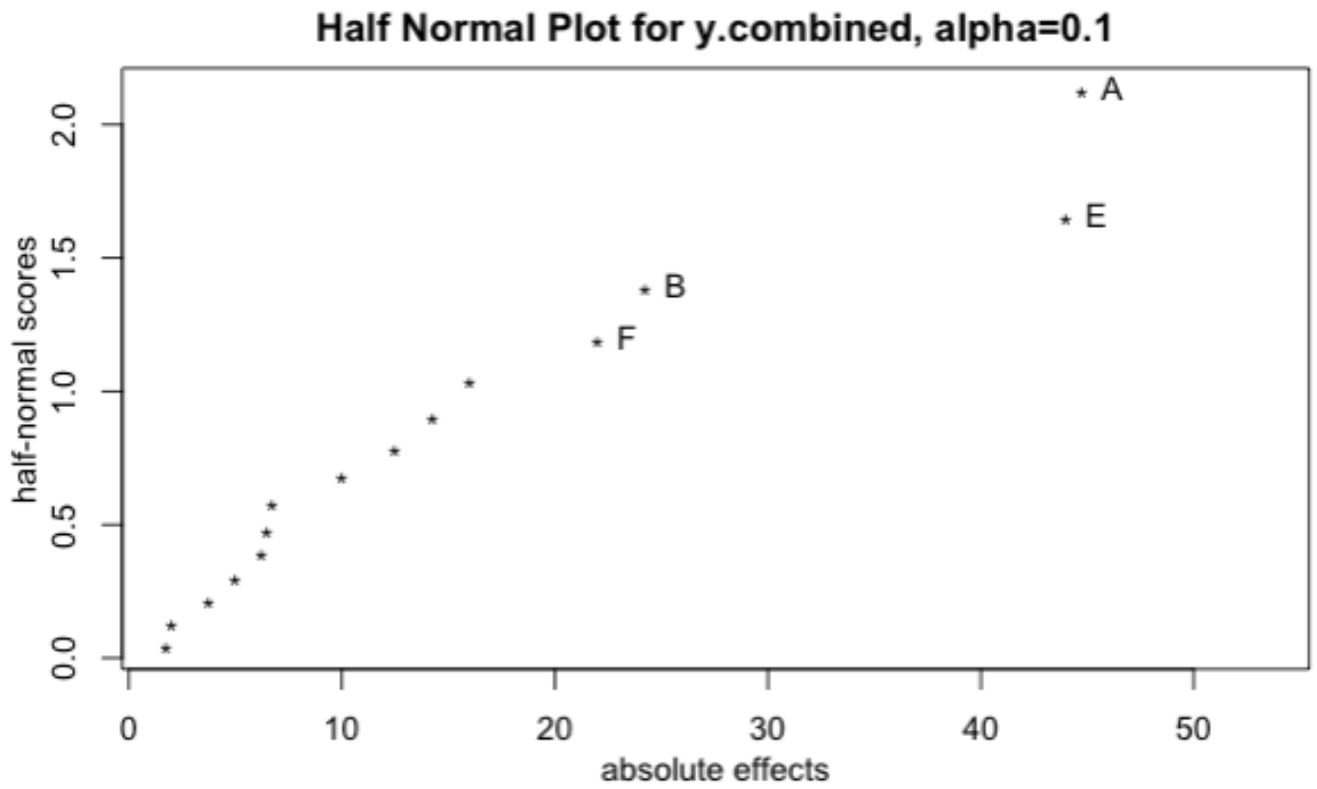

Figure 5: Half-Normal Plot for Combined Data

Initial Model

```r
combined.model1 <- aov(y.combined~A+B+E+F,data=combined.df)
summary(combined.model1)
```

```
             Df Sum Sq Mean Sq F value Pr(>F)
A             1   8010    8010   1.141  0.300
B             1   2352    2352   0.335  0.570
E             1   7744    7744   1.103  0.308
F             1   1936    1936   0.276  0.606
Residuals    18 126374    7021
```

Figure 6: Initial Model

Test for Pure Quadratic Curvature

- See Week Seven - Module G

```r
y.cp <- c(548,532,526,541,518,537,516)
n_f=nrow(base.df)
ybar_f=mean(y.base)
n_c=nrow(combined.df)-nrow(base.df)
ybar_c=mean(y.cp)
ss_pq=n_c*n_f*(ybar_f-ybar_c)^2/(n_c+n_f)
sprintf("Factorial Statistics: n_f=%i, ybar_f=%f",n_f,ybar_f)
sprintf("Center point Statistics: n_c=%i, ybar_c=%f",n_c,ybar_c)
sprintf("SS_Pure Quadratic=%f",ss_pq)
ms_residual=7021
F_0=ss_pq/ms_residual
sprintf("F0=%f",F_0)
df_resid = 18
criticalValue = qf(0.05,1,df_resid,lower.tail = FALSE)
sprintf("Reject the null hypothsis if F0>%f",criticalValue)
sprintf("Since %f > %f, the null hypothsis is rejected and quadratic curvature is present.",F_0,criticalValue)
```

```
[1] "Factorial Statistics: n_f=16, ybar_f=372.875000"
[1] "Center point Statistics: n_c=7, ybar_c=531.142857"
[1] "SS_Pure Quadratic=121976.349379"
[1] "F0=17.373074"
[1] "Reject the null hypothsis if F0>4.413873"
[1] "Since 17.373074 > 4.413873, the null hypothsis is rejected and quadratic curvature is present."
```

Figure 7: Test for Pure Quadratic Curvature

Plot of A vs. Y

```
219 ```{r}
220 plot(combined.df$A,combined.df$y.combined,xlab="Factor A",ylab="y",main="Factor A vs. y")
221 ```
```