Attendance 1-A

MANE 3332.01

Lecture 26

Agenda

- Complete Chapter 9 lectures
  - Chapter 9, Case 3
- Chapter 9, Case 1 2-sided Quiz (assigned 11/25/2025, due 12/2/2025)
- Chapter 9, Case 1 Lower Quiz (assigned 11/25/2025, due 12/2/2025)
- Chapter 9, Case 1 Upper Quiz (assigned 11/25/2025, due 12/2/2025)
- Chapter 9, Case 2 2-sided Practice Problems (assigned 11/25/2025, due 12/2/2025)
- Chapter 9, Case 2 Lower Practice Problems (assigned 11/25/2025, due 12/2/2025)
- Chapter 9, Case 2 Upper Practice Problems (assigned 11/25/2025, due 12/2/2025)
- NEW: Chapter 9, Case 2 2-sided Quiz (assigned 12/2/2025, due 12/4/2025)
- NEW: Chapter 9, Case 2 Lower Quiz (assigned 12/2/2025, due 12/4/2025)
- NEW: Chapter 9, Case 2 Upper Quiz (assigned 12/2/2025, due 12/4/2025)
- NEW: Chapter 9, Case 3 2-sided Practice Problems (assigned 12/2/2025, due 12/4/2025)
- NEW: Chapter 9, Case 3 Lower Practice Problems (assigned 12/2/2025, due 12/4/2025)
- NEW: Chapter 9, Case 3 Upper Practice Problems (assigned 12/2/2025, due 12/4/2025)
- Attendance
- Questions?

Handouts

- Lecture 26 Slides
- Lecture 26 Slides - marked

Week	Tuesday Lecture	Thursday Lecture
14	**12/2** - Chapter 9, Case 3	**12/4** - Linear Regression
15	**12/9** - Review Session	**12/11** - Study Day (no class)

The final exam for MANE 3332.01 is **Thursday December 18, 2025 at 10:15 AM - 12:00 PM**.

Summary of One-Sample Hypothesis-Testing Procedures

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis	Fixed Significance Level Criteria for Rejection	*P*-value	O.C. Curve Parameter	O.C. Curve Appendix Chart VII
1.	$H_0: \mu = \mu_0$ σ^2 known	$z_0 = \dfrac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$P = 2[1 - \Phi(z_0)]$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	a, b
			$H_1: \mu > \mu_0$	$z_0 > z_\alpha$	Probability above z_0 $P = 1 - \Phi(z_0)$	$d = (\mu - \mu_0)/\sigma$	c, d
			$H_1: \mu < \mu_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	$d = (\mu_0 - \mu)/\sigma$	c, d
2.	$H_0: \mu = \mu_0$ σ^2 unknown	$t_0 = \dfrac{\bar{x} - \mu_0}{s / \sqrt{n}}$	$H_1: \mu \neq \mu_0$	$\lvert t_0 \rvert > t_{\alpha/2, n-1}$	Sum of the probability above $\lvert t_0 \rvert$ and below $-\lvert t_0 \rvert$	$d = \lvert \mu - \mu_0 \rvert / \sigma$	e, f
			$H_1: \mu > \mu_0$	$t_0 > t_{\alpha, n-1}$	Probability above t_0	$d = (\mu - \mu_0)/\sigma$	g, h
			$H_1: \mu < \mu_0$	$t_0 < -t_{\alpha, n-1}$	Probability below t_0	$d = (\mu_0 - \mu)/\sigma$	g, h
3.	$H_0: \sigma^2 = \sigma_0^2$	$x_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2, n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2, n-1}$	See text Section 9.4.	$\lambda = \sigma/\sigma_0$	i, j
			$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha, n-1}$		$\lambda = \sigma/\sigma_0$	k, l
			$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha, n-1}$		$\lambda = \sigma/\sigma_0$	m, n
4.	$H_0: p = p_0$	$z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$p = 2[1 - \Phi(z_0)]$	3–4	3–4
			$H_1: p > p_0$	$z_0 > z_\alpha$	Probability above z_0 $p = 1 - \Phi(z_0)$	3–4	3–4
			$H_1: p < p_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	3–4	3–4

Summary of One-Sample Confidence Interval Procedures

Case	Problem Type	Point Estimate	Two-sided $100(1-\alpha)$ Percent Confidence Interval
1.	Mean μ, variance σ^2 known	$\bar{x}$	$\bar{x} - z_{\alpha/2}\sigma/\sqrt{n} \leq \mu \leq \bar{x} + z_{\alpha/2}\sigma/\sqrt{n}$
2.	Mean μ of a normal distribution, variance σ^2 unknown	$\bar{x}$	$\bar{x} - t_{\alpha/2, n-1}\, s/\sqrt{n} \leq \mu \leq \bar{x} + t_{\alpha/2, n-1} s/\sqrt{n}$
3.	Variance σ^2 of a normal distribution	s^2	$\dfrac{(n-1)s^2}{\chi^2_{\alpha/2, n-1}} \leq \sigma^2 \leq \dfrac{(n-1)s^2}{\chi^2_{1-\alpha/2, n-1}}$
4.	Proportion or parameter of a binomial distribution p	$\hat{p}$	$\hat{p} - z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\dfrac{\hat{p}(1-\hat{p})}{n}}$

Case 3. Hypothesis Test on Variance of Normal Population

The test statistics is a χ^2 random variable

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$$

Test on standard deviation

$H_0: \sigma^2 = \sigma_0^2$ or $H_0: \sigma = \sigma_0$

Tests on the Variance of a Normal Distribution

Null hypothesis: H_0: $\sigma^2 = \sigma^2{}_0$

Test statistic: $\chi_0^2 = \dfrac{(n-1)S^2}{\sigma_0^2}$

Alternative Hypothesis	Rejection Criteria
$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$
$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$
$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$

- The table below summarizes the three possible hypothesis tests. The rejection regions are clearly shown in Figure 9-17 on page 222

Figure 9-17

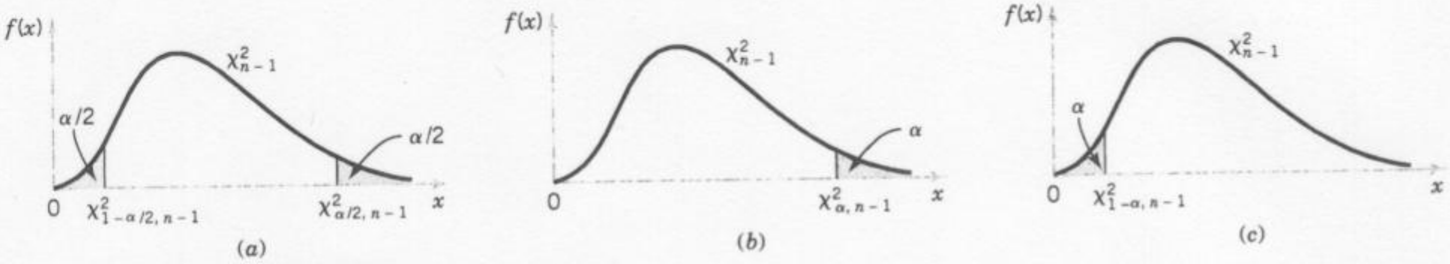


FIGURE 9.17

Reference distribution for the test of H_0: $\sigma^2 = \sigma_0^2$ with critical region values for (a) H_1: $\sigma^2 \neq \sigma_0^2$, (b) H_1: $\sigma^2 > \sigma_0^2$, and (c) H_1: $\sigma^2 < \sigma_0^2$.

Test Summary

See summary in your textbook

.	$H_0: \sigma^2 = \sigma_0^2$	$x_0^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha/2,n-1}$ or $\chi_0^2 < \chi^2_{1-\alpha/2,n-1}$	See text Section 9.4.	$\lambda = \sigma/\sigma_0$	*i, j*
			$H_1: \sigma^2 > \sigma_0^2$	$\chi_0^2 > \chi^2_{\alpha,n-1}$		$\lambda = \sigma/\sigma_0$	*k, l*
			$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 < \chi^2_{1-\alpha,n-1}$		$\lambda = \sigma/\sigma_0$	*m, n*

Problem 7.108

Problem taken from Ostle, Turner, Hicks and McElrath (1996). *Engineering Statistics: The Industrial Experience.* Duxbury Press.

7.108 Incoming coal at a coking plant is routinely analyzed for sulfur content (in percent). In the past, samples taken from barges loaded with coal from a particular mine have had a variance of 0.000196. When a new analyst was hired, the results of an assay of coal from the mine produced percentages of 0.83, 0.79, 0.77, 0.81, and 0.80.

(a) Using $\alpha = 0.05$, does the sample variance provide sufficient evidence to conclude that the results from the new analyst indicate more variability than in the past? State all assumptions.

(b) Based on these data, is an assumption of normality reasonable? Justify by using a normal quantile plot *and* a formal test such as the Shapiro-Wilk W test.

$H_0: \sigma^2 = .000196$

$H_1: \sigma^2 > .000196$

TS:

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(5-1)s^2}{.000196} = \frac{(4)(.02)^2}{.000196}$$

$$= \underline{8.1637}$$

Statistics for Problem 7.108

```
x<-c(0.83,0.79,0.77,0.81,0.80)
library(psych)
describe(x)

##    vars n mean   sd median trimmed  mad  min  max range skew kurtosis   se
## X1    1 5  0.8 0.02    0.8     0.8 0.01 0.77 0.83  0.06    0    -1.69 0.01

print(var(x))

## [1] 5e-04
```

s

s = .02

Classical Approach

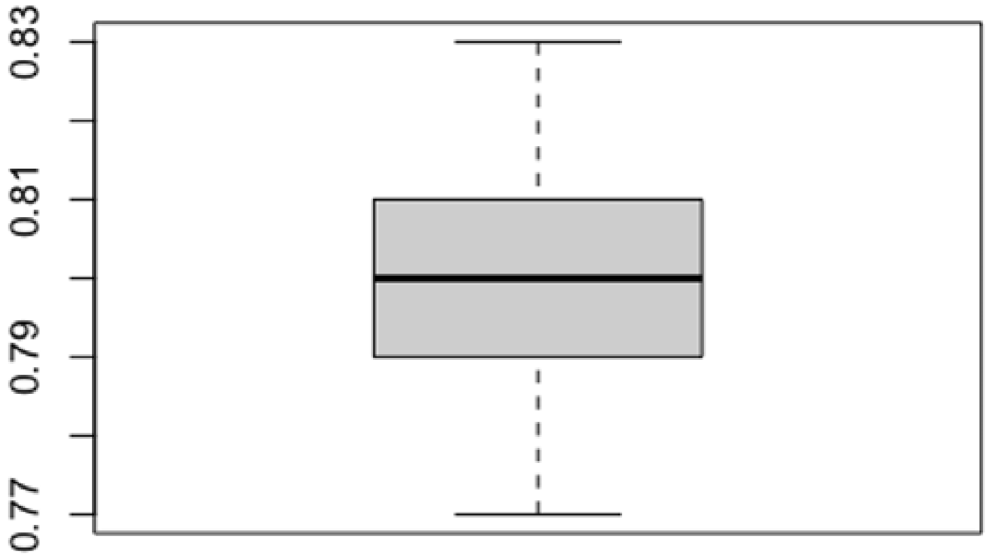
Rejection Region

Reject H_0 if $\chi^2_0 > \chi^2_{\alpha, n-1}$

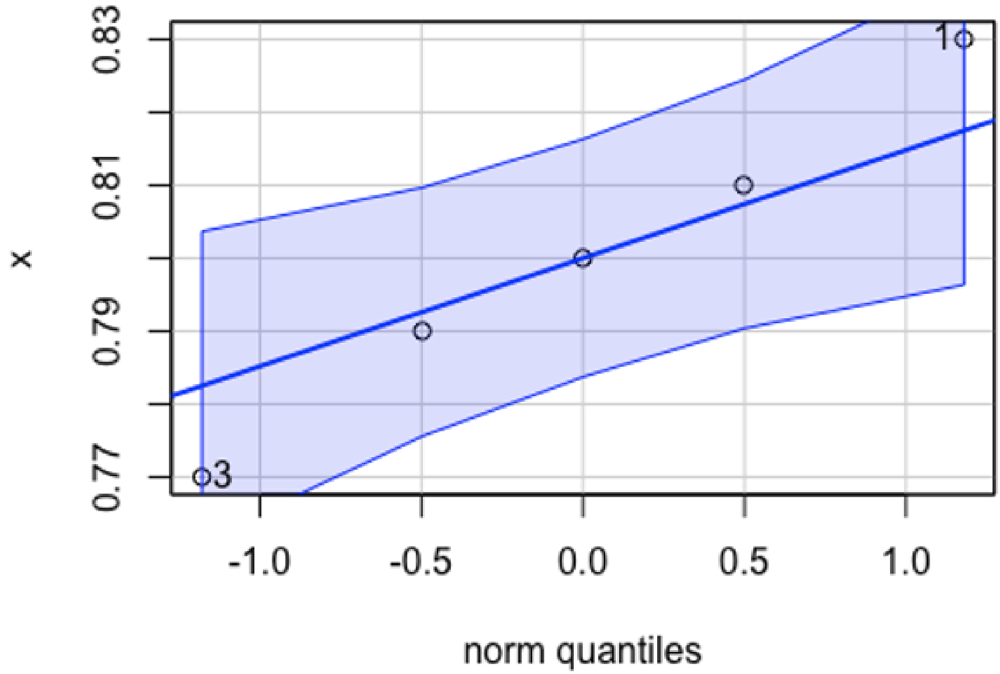
need $\chi^2_{.05, 5-1} = 9.49$

Conclusion: Since $\chi^2_0 = 8.16$ which is not greater than 9.49, we fail to reject H_0

Problem 7.108 Plots



Problem 7.108 Plots



Non-Parametric Test

Problem 7.108 Shapiro-Wilks Test

H_0: data is normal

H_1: data is not normal

```
## [1] 3 1

shapiro.test(x)

##
##  Shapiro-Wilk normality test
##
## data:  x
## W = 0.99929, p-value = 0.9998
```

***p*-values**

- Very similar to the case for the mean of a normal population with variance unknown
- Difficult to calculate since the χ^2-tables only contain a few quantiles
- Can use tables to generate bounds on the p-value
- Software will provide p-values

Test on Variance using R (EnvStats)

```
varTest(x,alternative="greater",conf.level=0.95,sigma.squared=0.000196)

##
## Results of Hypothesis Test
## --------------------------
##
## Null Hypothesis:                 variance = 0.000196
##
## Alternative Hypothesis:          True variance is greater than 0.000196
##
## Test Name:                       Chi-Squared Test on Variance
##
## Estimated Parameter(s):          variance = 5e-04
##
## Data:                            x
##
## Test Statistic:                  Chi-Squared = 10.20408
##
## Test Statistic Parameter:        df = 4
##
## P-value:                         0.03712675
##
## 95% Confidence Interval:         LCL = 0.0002107986
##                                  UCL =          Inf
```

if $\alpha = .05$, conclusion is to Reject Ho

Difference in outcome is due to round-off ($s = .02$)

Power Calculations

- Can be done with OC curves found in Table VII*i*–*n*
- Can be done in software such as R

Test on Standard Deviation

- What about test on standard deviation?

Chapter 9, Case 3 2-sided Practice Problems

Question 1 (2 points)

Listen

$H_1: \sigma^2 \neq \sigma_0^2 = 11.56$

Calculate the test statistic for a test of hypothesis for the variance of normal distribution. The null hypothesis is sigma^2=11.56 versus sigma^2 not equal to 11.56 using alpha=0.01. The sample statistics are n=2, xbar=33.93, S=1.799.

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(2-1)(1.799)^2}{11.56}$$

$$= \underline{0.27997}$$

- ChiSquared0=3.5719
- ChiSquared0=0.5291
- ChiSquared0=1.8899
- ✓ ChiSquared0=0.28

Question 2 (2 points)

Listen

Construct the rejection region for a test of hypothesis for the variance of normal distribution. The null hypothesis is sigma^2=324.0 versus sigma^2 not equal to 324.0 using alpha=0.05. The sample statistics are n=5, xbar=124.47, S=8.957.

- ~~Reject Ho if chiSquare0>9.49~~
- Reject H0 if ChiSquare0<0.71 or ChiSquare0>9.49
- ~~Reject Ho if ChiSquare0>11.14~~
- Reject H0 if ChiSquare0<0.48 or ChiSquare0>11.14
- ~~Reject Ho if chiSquare0<0.71~~
- ~~Reject Ho if chiSquare0<0.48~~

Reject Ho if

$$\chi_0^2 > \chi^2_{\alpha/2,\, n-1} = \chi^2_{.025,\, 5-1} = 11.14$$

or

$$\chi_0^2 < \chi^2_{1-\alpha/2,\, n-1} = \chi^2_{.975,\, 5-1} = 0.48$$

Question 3 (2 points)

Listen

What is the correct conclusion for a test of hypothesis for the variance of a normal distribution. The null hypothesis is Sigma^2=11.56 versus sigma^2 not equal to 11.56 using alpha=0.01. The sample statistics are n=24, xbar=36.45, S=11.606. The value of ChiSquare0 is 268.0002 and the rejection region is reject HO if ChiSquare0<9.26 or ChiSquare0>44.18

- ( ) Reject H0
- ( ) Fail to reject H0

is 268.0002 either

> 44.18 or < 9.26

? yes → Reject H_0

Question 4 (2 points)

Listen

Using the p-value from a test of hypothesis for the variance of a normal distribution, determine the correct conclusion for the hypothesis test. The null hypothesis is Sigma^2=0.0036 versus sigma^2 not equal to 0.0036 using alpha=0.1. The sample statistics are n=5, xbar=0.5, S=0.105. The results of hypothesis test include ChiSquare0= 12.25 and p-value=0.031.

- Fail to reject H0
- Reject H0

p-value = .031 & α = .1

is .031 < .1 ? yes → reject H0

Practice Problem Chapter 9, Case 3 Lower

Question 1 (2 points)

Listen

Calculate the test statistic for a test of hypothesis for the variance of normal distribution. The null hypothesis is sigma^2=0.0036 versus sigma^2 less than 0.0036 using alpha=0.05. The sample statistics are n=6, xbar=0.49, S=0.034.

- ChiSquared0=2.8333
- ~~ChiSquared0=15.5709~~
- $\boxed{\text{ChiSquared0=1.6056}}$ ✓
- ChiSquared0=8.8235

$$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{(6-1)(.034)^2}{.0036} = 1.60556$$

Question 2 (2 points)

Listen

$H_1: \sigma^2 < 17.64 \rightarrow$

Construct the rejection region for a test of hypothesis for the variance of normal distribution. The null hypothesis is sigma^2=17.64 versus sigma^2 less than 17.64 using alpha=0.005. The sample statistics are n=5, xbar=19.79, S=2.396.

Reject Ho if

$\chi_0^2 < \chi^2_{1-\alpha, n-1}$

$= \chi^2_{.995, 4} = .21$

- ○ Reject Ho if chiSquare0>14.86
- ○ Reject Ho if chiSquare0<0.14
- ○ Reject Ho if ChiSquare0>16.42
- ○ Reject H0 if ChiSquare0>0.21 or ChiSquare0>14.86
- ✓ Reject Ho if chiSquare0<0.21
- ○ Reject H0 if ChiSquare0<0.14 or ChiSquare0>16.42

Practice Problem Chapter 9, Case 3 Upper

Question 2 (2 points)

Listen

$H_1: \sigma_0^2 > 44.89$

Construct the rejection region for a test of hypothesis for the variance of normal distribution. The null hypothesis is sigma^2=44.89 versus sigma^2 greater than 44.89 using alpha=0.1. The sample statistics are n=26, xbar=27.64, S=6.601.

- Reject H0 if ChiSquare0<14.61 or ChiSquare0>37.65
- Reject Ho if ChiSquare0>37.65
- Reject H0 if ChiSquare0<16.47 or ChiSquare0>34.38
- Reject Ho if chiSquare0<16.47
- ✓ Reject Ho if chiSquare0>34.38
- Reject Ho if chiSquare0<14.61

Reject Ho if

$\chi_0^2 > \chi^2_{\alpha, n-1}$

$= \chi_{.1, 26-1}$

$= 34.28$

Case 4. Hypothesis Test on a Population Proportion

- The test statistics for the hypothesis test is

$$Z_0 = \frac{x - np_0}{\sqrt{np_0(1 - p_0)}}$$

.	$H_0: p = p_0$	$z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$	$H_1: p \neq p_0$	$\lvert z_0 \rvert > z_{\alpha/2}$	$p = 2[1 - \Phi(z_0)]$	3–4	3–4
			$H_1: p > p_0$	$z_0 > z_\alpha$	Probability above z_0 $p = 1 - \Phi(z_0)$	3–4	3–4
			$H_1: p < p_0$	$z_0 < -z_\alpha$	Probability below z_0 $P = \Phi(z_0)$	3–4	3–4

Problem 9.5.2

9.5.2 WP Suppose that of 1000 customers surveyed, 850 are satisfied or very satisfied with a corporation's products and services.

a. Test the hypothesis H_0: $p = 0.9$ against H_1: $p \neq 0.9$ at $\alpha = 0.05$. Find the P-value.

b. Explain how the question in part (a) could be answered by constructing a 95% two-sided confidence interval for p.

Problem 9.5.2 Classic Approach

Power Calculations

- For the two-sided alternative hypothesis

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{p_0 - p - z_{\alpha/2}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

- If the alternative is $H_1\!: p < p_0$

$$\beta = 1 - \Phi\left(\frac{p_0 - p - z_{\alpha}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

- and finally if the alternative hypothesis is $H_1\!: p > p_0$

$$\beta = \Phi\left(\frac{p_0 - p + z_{\alpha}\sqrt{p_0(1-p_0)/n}}{\sqrt{p(1-p)/n}}\right)$$

Sample Size

- Sample size requirements to satisfy type II(β) error constraints for a two-tailed hypothesis test is given by

$$n = \left[\frac{z_{\alpha/2}\sqrt{p_0(1-p_0)} + z_\beta\sqrt{p(1-p)}}{p-p_0}\right]^2.$$

- For a sample size for a one-sided test substitute z_α for $z_{\alpha/2}$.
- Problem 9.95

Testing for Goodness of Fit

- Material is presented in section 9-7 of your textbook
- Procedure determines if the sample data is from a specified underlying distribution
- Procedure uses a χ^2 distribution (GOF)
- Example 9-12 presents a χ^2 goodness of fit test for a Poisson example
- Example 9-13 presents a χ^2 goodness of fit test for a normal example

Procedure

1. Collect a random sample of size n from a population with an unknown distribution,
2. Arrange the n observations in a frequency distribution containing k classes
3. Calculate the observed frequency in each class O_i,
4. From the hypothesized distribution, calculate the expected frequency in class i, denoted E_i (if E_i is small combine classes)
5. Calculate the test statistic

$$\chi_0^2 = \frac{\sum_{i=1}^{k}(O_i - E_i)^2}{E_i}$$

6. Reject the null hypothesis if the calculated value of the test statistic $\chi_0^2 > \chi_{\alpha,k-p-1}^2$ where p is the number of parameters in the hypothesized distribution

Example 9.12, part 1

EXAMPLE 9.12 | Printed Circuit Board Defects—Poisson Distribution

The number of defects in printed circuit boards is hypothesized to follow a Poisson distribution. A random sample of $n = 60$ printed circuit boards has been collected, and the following number of defects observed.

Number of Defects	Observed Frequency
0	32
1	15
2	9
3 or more	4

The mean of the assumed Poisson distribution in this example is unknown and must be estimated from the sample data. The estimate of the mean number of defects per board is the sample average, that is, $(32 \cdot 0 + 15 \cdot 1 + 9 \cdot 2 + 4 \cdot 3)/60 = 0.75$. From the Poisson distribution with parameter 0.75, we may compute p_i, the theoretical, hypothesized probability associated with the ith class interval. Because each class interval corresponds to a particular number of defects, we may find the p_i as follows:

$$p_1 = P(X = 0) = \frac{e^{-0.75}(0.75)^0}{0!} = 0.472$$

$$p_2 = P(X = 1) = \frac{e^{-0.75}(0.75)^1}{1!} = 0.354$$

$$p_3 = P(X = 2) = \frac{e^{-0.75}(0.75)^2}{2!} = 0.133$$

$$p_4 = P(X \geq 3) = 1 - (p_1 + p_2 + p_3) = 0.041$$

Example 9.12, part 2

The expected frequencies are computed by multiplying the sample size $n = 60$ times the probabilities p_i. That is, $E_i = np_i$. The expected frequencies follow:

Number of Defects	Probability	Expected Frequency
0	0.472	28.32
1	0.354	21.24
2	0.133	7.98
3 (or more)	0.041	2.46

Because the expected frequency in the last cell is less than 3, we combine the last two cells:

Number of Defects	Observed Frequency	Expected Frequency
0	32	28.32
1	15	21.24
2 (or more)	13	10.44

The seven-step hypothesis-testing procedure may now be applied, using $\alpha = 0.05$, as follows:

1. **Parameter of interest:** The variable of interest is the form of the distribution of defects in printed circuit boards.
2. **Null hypothesis:** H_0: The form of the distribution of defects is Poisson.
3. **Alternative hypothesis:** H_1: The form of the distribution of defects is not Poisson.
4. **Test statistic:** The test statistic is $\chi_0^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$
5. **Reject H_0 if:** Because the mean of the Poisson distribution was estimated, the preceding chi-square statistic will have $k - p - 1 = 3 - 1 - 1 = 1$ degree of freedom. Consider whether the P-value is less than 0.05.
6. **Computations:**

$$\chi_0^2 = \frac{(32 - 28.32)^2}{28.32} + \frac{(15 - 21.24)^2}{21.24} + \frac{(13 - 10.44)^2}{10.44} = 2.94$$

7. **Conclusions:** We find from Appendix Table III that $\chi^2_{0.10,1} = 2.71$ and $\chi^2_{0.05,1} = 3.84$. Because $\chi_0^2 = 2.94$ lies between these values, we conclude that the P-value is between 0.05 and 0.10. Therefore, because the P-value exceeds 0.05, we are unable to reject the null hypothesis that the distribution of defects in printed circuit boards is Poisson. The exact P-value computed from software is 0.0864.

Chapter 9 Summary

- You should be prepared to work any practice problems assigned: Cases 1-3 with three different alternatives
- All other information is conceptual knowledge that can be questioned with multiple choice
  - Name 3 ways to test if data is from a normal distribution

1) Normal Probability Plot
2) Wilks - Shapiro (non-parametric test)
3) χ^2 Goodness of fit

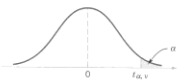


Table IV Percentage Points $t_{\alpha,\nu}$ of the t-Distribution

ν \ α	.40	.25	.10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
∞	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

ν = degrees of freedom.

$\chi^2_{\alpha,\nu}$

Table III Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Squared Distribution

ν \ α	.995	.990	.975	.950	.900	.500	.100	.050	.025	.010	.005
1	.00+	.00+	.00+	.00+	.02	.45	2.71	3.84	5.02	6.63	7.88
2	.01	.02	.05	.10	.21	1.39	4.61	5.99	7.38	9.21	10.60
3	.07	.11	.22	.35	.58	2.37	6.25	7.81	9.35	11.34	12.84
4	.21	.30	.48	.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	.41	.55	.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	.68	.87	1.24	1.64	2.20	5.35	10.65	12.59	14.45	16.81	18.55
7	.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	8.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.72	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.27	7.26	8.55	14.34	22.31	25.00	27.49	30.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.09	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.87	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	24.34	34.28	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.89	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.65
28	12.46	13.57	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
50	27.99	29.71	32.36	34.76	37.69	49.33	63.17	67.50	71.42	76.15	79.49
60	35.53	37.48	40.48	43.19	46.46	59.33	74.40	79.08	83.30	88.38	91.95
70	43.28	45.44	48.76	51.74	55.33	69.33	85.53	90.53	95.02	100.42	104.22
80	51.17	53.54	57.15	60.39	64.28	79.33	96.58	101.88	106.63	112.33	116.32
90	59.20	61.75	65.65	69.13	73.29	89.33	107.57	113.14	118.14	124.12	128.30
100	67.33	70.06	74.22	77.93	82.36	99.33	118.50	124.34	129.56	135.81	140.17

ν = degrees of freedom.