MANE 3332.01

LECTURE 5

Agenda

- Continue Chapter 2 Lecture
- Start with Two Events Practice Problems
- Single Event Quiz (assigned 9/11/2025, due 9/16/2025)
- Two Events Practice Problems (assigned 9/16/2025, due 9/18/2025)

Handouts

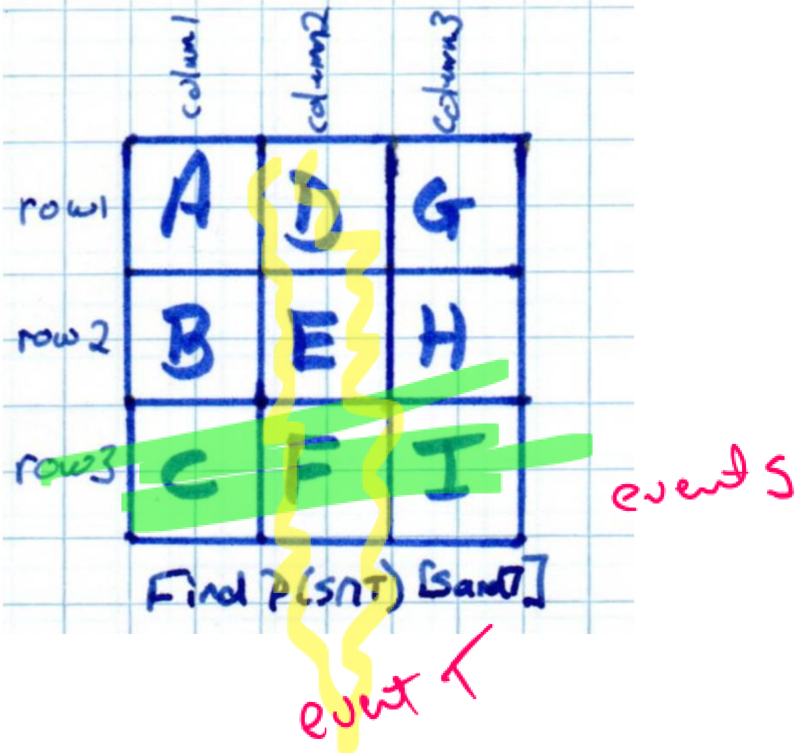
- Lecture 5 Slides - Powerpoint
- Lecture 5 Slides - marked (pdf)

Two Events Practice Problems

Question 1 (1 point)

Listen

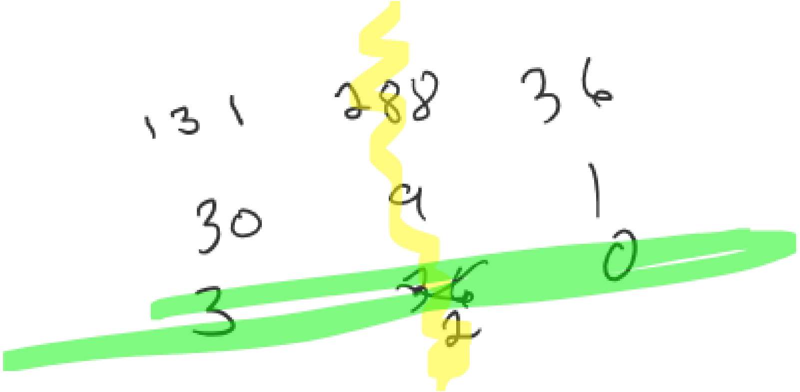
Consider a problem classified by 3 rows and 3 columns containing 500 observations. The table is described in the figure below and has the following cell counts: A=131, B=30, C=3, D=288, E=9, F=2, G=36, H=1, I=0. Let event S denote an item that occurs in row 3 and event T denote an item that occurs in column 2. Find P(S and T).



find $P(S \cap T)$

$P(S \cap T) = \frac{F}{n} = \frac{F}{500}$

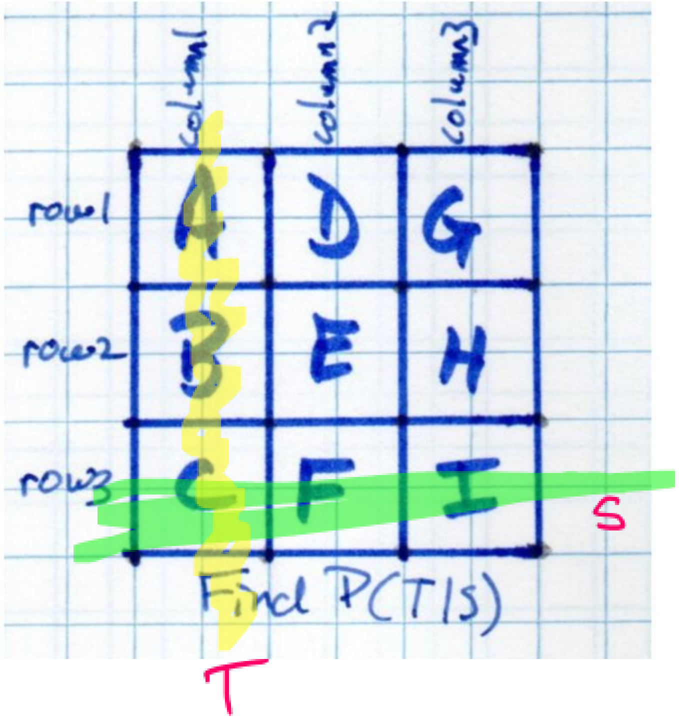
$= 2/500$



Question 3 (1 point)

Listen

Consider a problem classified by 3 rows and 3 columns containing 200 observations. The table is described in the figure below and has the following cell counts: A=161, B=3, C=0, D=27, E=2, F=0, G=3, H=4, and I=0. Let event S denote an item that occurs in row 3 and event T denote an item that occurs in column 1. Find P(T|S).



$$P(T|S) = \frac{P(T \cap S)}{P(T)}$$

$n = 200$

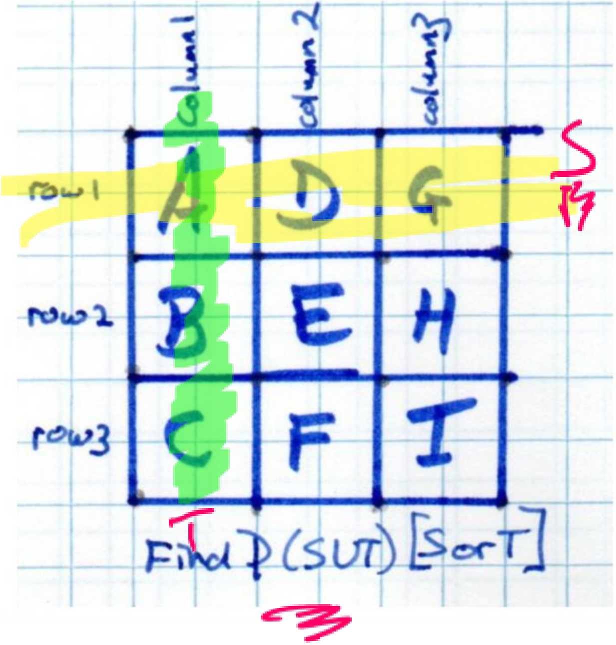
$$P(T \cap S) = \frac{C}{n} = \frac{0}{200} = 0$$

$$P(T) = \frac{A+B+C}{n} = \frac{161+3+0}{200} = \frac{164}{200} = .82$$

Question 5 (1 point)

Listen

Consider a problem classified by 3 rows and 3 columns containing 300 observations. The table is described in the figure below and has the following cell counts: A=111, B=2, C=16, D=49, E=77, F=2, G=28, H=12, and I=3. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in column 1. Find P(S or T).



$P(S \cap T) = \frac{A}{n} = \frac{111}{300} = .37$

$P(S \cup T) = P(S) + P(T) - P(S \cap T)$

$P(S) = \frac{A+D+G}{300} = \frac{111+49+28}{300} = .62667$

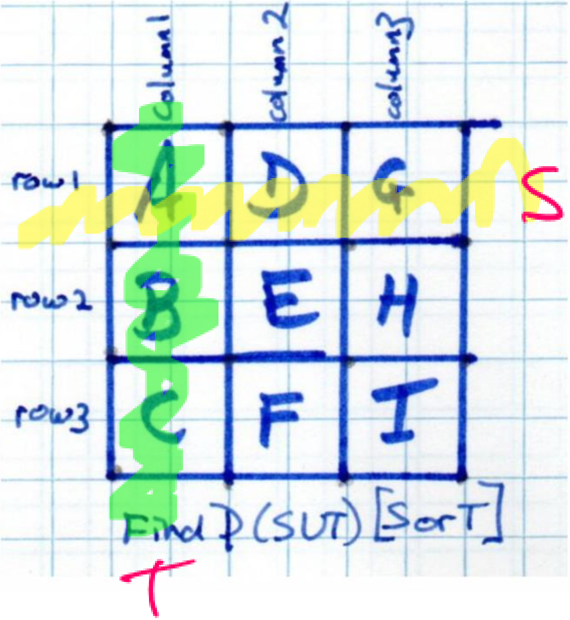
$P(T) = \frac{A+B+C}{n} = \frac{111+2+16}{300} = .43$

$P(S \cup T) = .62667 + .43 - .37 = .68667$

Question 5 (1 point)

Listen

Consider a problem classified by 3 rows and 3 columns containing 300 observations. The table is described in the figure below and has the following cell counts: A=111, B=2, C=16, D=49, E=77, F=2, G=28, H=12, and I=3. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in column 1. Find P(S or T).



$$P(S \cup T) = \frac{A+B+C+D+G}{n}$$

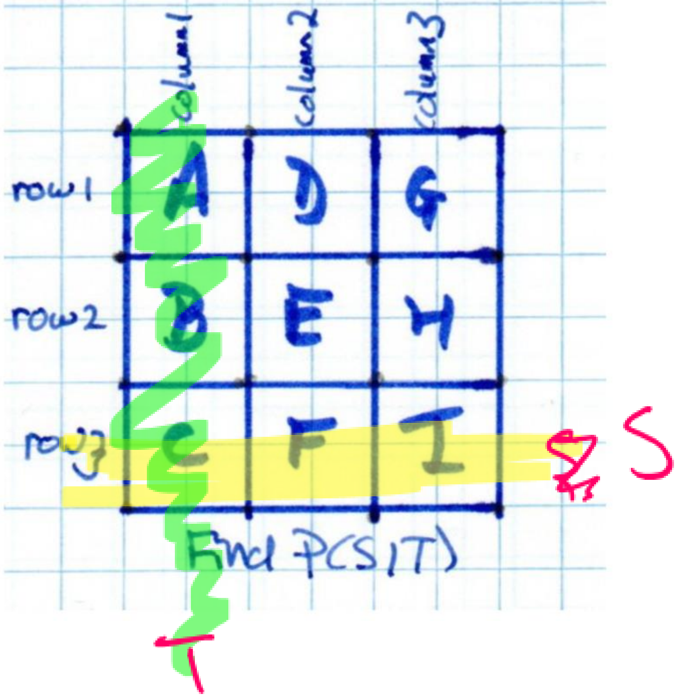
$$= \frac{111+2+16+49+28}{300}$$

$$= .68667$$

Question 7 (1 point)

Listen

Consider a problem classified by 3 rows and 3 columns containing 500 observations. The table is described in the figure below and has the following cell counts: A=250, B=0, C=0, D=244, E=0, F=0, G=6, H=0, and I=0. Let event S denote an item that occurs in row 3 and event T denote an item that occurs in column 1. Find P(S|T).



$$P(S|T) = \frac{P(S \cap T)}{P(T)} = 0$$

$$P(S \cap T) = \frac{C}{n} = \frac{0}{500} = 0$$

$$P(T) = \frac{A+B+C}{n} = \frac{250+0+0}{500} = .5$$

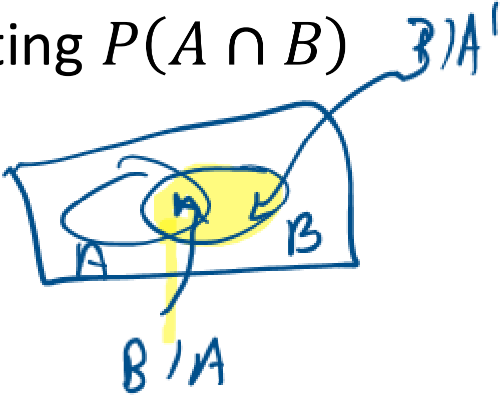
Multiplication Rules

$P(S|T) = \frac{P(S \cap T)}{P(T)}$

- This rule provides another method for calculating $P(A \cap B)$
- $P(A \cap B) \quad = \quad P(A|B)P(B) = P(B|A)P(A)$
- This leads to the total probability rule

$$P(B) \quad = \quad P(B \cap A) + P(B \cap A')$$

- $$= \quad P(B|A)P(A) + P(B|A')P(A')$$



B - 2nd card Ace, A - 1st card

- Consider problems from 3rd edition (next slide) and 2-129

P6

Example Problem 2-76

P(small) = .4
P(large) = .6

what is overall rate of breakage

2-76. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2 and 1% of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

P(break | small) = .02
P(break | large) = .01

problem 2-76

P(break) = .4(.02) + .6(.01) = .014

Independent Events

- Two events are independent if any one of the following is true:
  1. $P(A|B) = P(A)$
  2. $P(B|A) = P(B)$
  3. $P(A \cap B) = P(A)P(B)$
- Consider problem 2-146

note: only have to prove one

	A	A'
B	70	9
B'	16	5

is A & B independent events

is $P(A|B) = P(A)$? No

$P(A) = .86$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{.70}{.79}$

$.88608$

Reliability Analysis

all components have independent failures

- Reliability is the application of statistics and probability to determine the probability that a system will be working properly
- Components can be arranged in series. All components must work for the system to work.

$$P(\text{system works}) = P(A\text{ works})P(B\text{ works})$$

- Components can be arranged in parallel. As long as one component works, the system works.

$$P(\text{system works}) = 1 - \big(1 - P(A\text{ works})\big) \times 1 - P(B\text{ works}))$$

- Consider problem 2-157

Series - Flashlight

Case - battery - switch - bulb

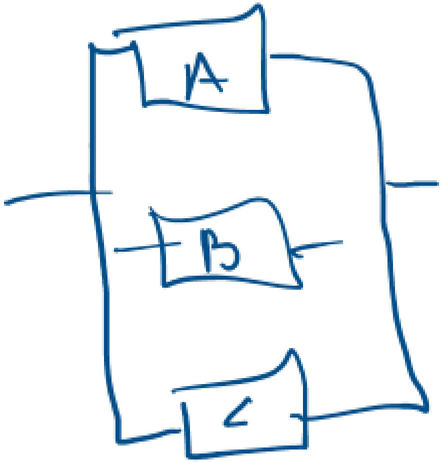
$P(\text{system works}) = P(\text{case} \cap \text{battery} \cap \text{switch} \cap \text{bulb})$

$= P(\text{case})P(\text{battery})P(\text{switch})P(\text{bulb})$

$= .9(.9).9(.9)$

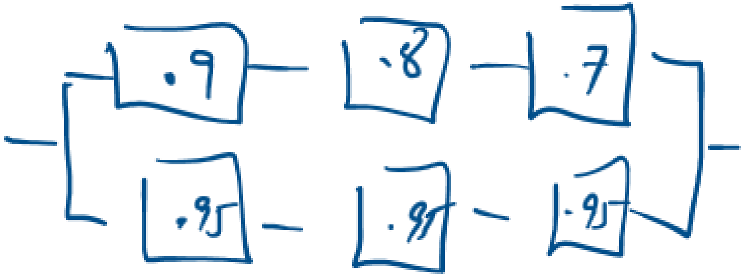
$= .6510$

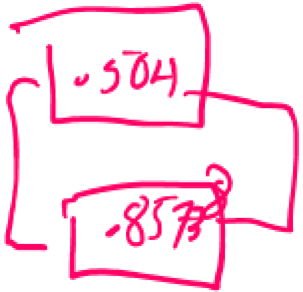
Parallel Systems



$$P(\text{system works}) = 1 - \left[(1-P(A))(1-P(B))(1-P(C))\right]$$

P2.7.12



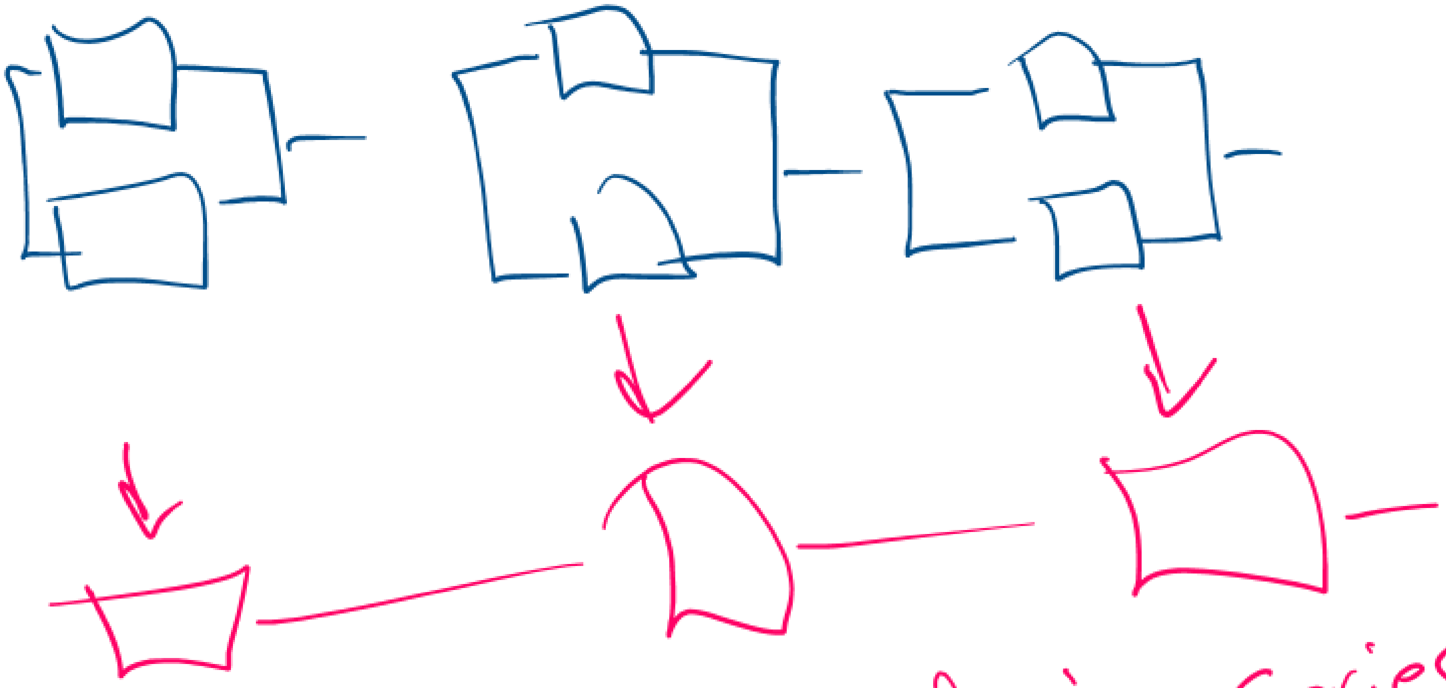


Top Row $P(\text{Top}) = .9(.8)(.7) = .504$

Bottom Row $P(\text{Bottom}) = (.95)^3 = .857375$

$P(\text{system}) = 1 - [(1 - .504)(1 - .857375)]$

$= .92926$



Simple 3 elements in series