MANE 3332.01

LECTURE 3

Agenda

- Continue Chapter 2 Lecture
- Search function in course website
- Single Event Practice Problems

Handouts

- Lecture 3 Slides - Powerpoint
- Lecture 3 Slides - marked (pdf)

Examples of Random Experiments, Sample Space, Events

- Consider the bead bowl
- Consider the Texas Lottery’s Pick Three game (I am not encouraging gambling)

Red & White Beads

paddle with 1 Hole

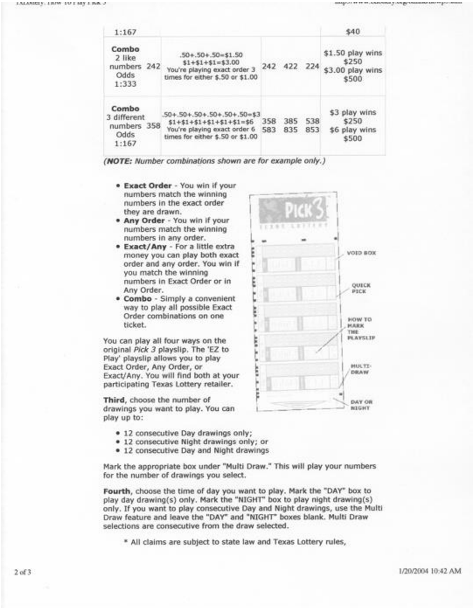
$\{R\}$ or $\{W\}$ SS: $\{\{R\}, \{W\}\}$

Paddle with 25 holes

event: $\{\underbrace{\_\ \_\ \ldots\ldots\ \_}_{25}\}$ Sample space has

$2 \times 2 \times \cdots 2 = 2^{25} =$

33,554,432



1:167			$40
Combo 2 like numbers 242 Odds 1:333	.50+.50+.50=$1.50 $1+$1+$1=$3.00 You're playing exact order 3 times for either $.50 or $1.00	242 422 224	$1.50 play wins $250 $3.00 play wins $500
Combo 3 different numbers 358 Odds 1:167	.50+.50+.50+.50+.50+.50=$3 $1+$1+$1+$1+$1+$1=$6 You're playing exact order 6 times for either $.50 or $1.00	358 385 538 583 835 853	$3 play wins $250 $6 play wins $500

*(**NOTE:** Number combinations shown are for example only.)*

- **Exact Order** - You win if your numbers match the winning numbers in the exact order they are drawn.
- **Any Order** - You win if your numbers match the winning numbers in any order.
- **Exact/Any** - For a little extra money you can play both exact order and any order. You win if you match the winning numbers in Exact Order or in Any Order.
- **Combo** - Simply a convenient way to play all possible Exact Order combinations on one ticket.

You can play all four ways on the original *Pick 3* playslip. The 'EZ to Play' playslip allows you to play Exact Order, Any Order, or Exact/Any. You will find both at your participating Texas Lottery retailer.

Third, choose the number of drawings you want to play. You can play up to:

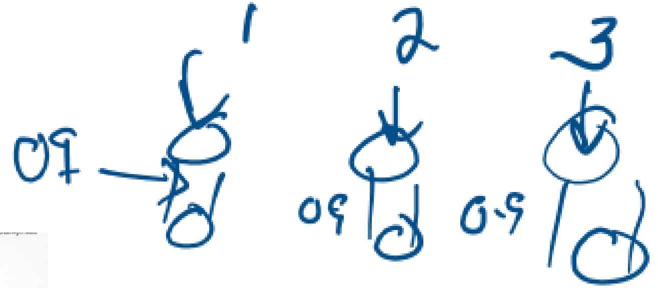
- 12 consecutive Day drawings only;
- 12 consecutive Night drawings only; or
- 12 consecutive Day and Night drawings

Mark the appropriate box under "Multi Draw." This will play your numbers for the number of drawings you select.

Fourth, choose the time of day you want to play. Mark the "DAY" box to play day drawing(s) only. Mark the "NIGHT" box to play night drawing(s) only. If you want to play consecutive Day and Night drawings, use the Multi Draw feature and leave the "DAY" and "NIGHT" boxes blank. Multi Draw selections are consecutive from the draw selected.

* All claims are subject to state law and Texas Lottery rules,

Pick 3 Back Page



event : {W}, {L}

Random event

{0 99}

Size of Sample space

$10 \times 10 \times 10 = 1000$

Tree Diagrams

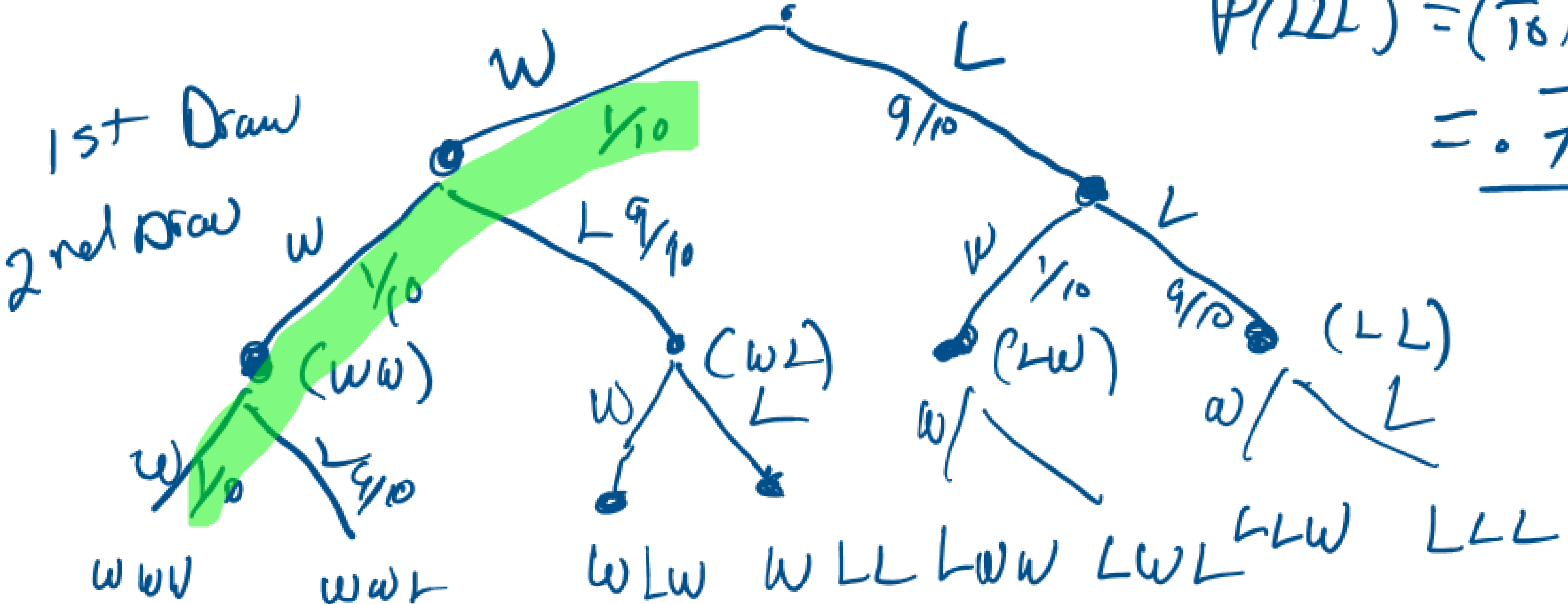
$P(WWW) = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10}$

$= \underline{1/1000}$

- Tree diagrams are a useful tool for understanding sample spaces and events. Apply to Pick Three game.

$P(LLL) = \left(\frac{9}{10}\right)^3$

$= \underline{.729}$



$P(W) = \frac{1}{1000} \rightarrow \$500$

$P(L) = .729 \rightarrow -1$

---

expected income $= -1.00 + 500\left(\frac{1}{1000}\right)$

$= -1 + .5$

$= -.50$

Power ball

big bowl

1 - 69

pick 5

little bowl (red)

1-26

pick 1

Sample space for big bowl

$\underline{69} \times \underline{68} \times \underline{67} \times \underline{66} \times \underline{65} =$

1,348,621,560

Probability

- The probability of an event is the likelihood that it occurs
- Probability is expressed as a number between 0 and 1
- Probability of an event can be found by dividing the number of outcomes of the desired events divided by the total number of outcomes in the sample space (if all events are equally likely)

Counting Techniques

Fact: $0! = 1$

- Consider ordered versus unordered subsets
- Ordered subsets (Permutations)

$$P_r^n = \frac{n!}{(n-r)!}$$

- Unordered subsets (Combinations)

C_n^n or $C_{n,r}$

$$C_r^n = \frac{n!}{r!\,(n-r)!}$$

$\binom{n}{r}$

- Good idea to do a calculator check

Flip 3 coins & record results

Events

H H H

T H H

H T H

T T H

H H T

T H T

H T T

T T T

X is a head

$$\binom{3}{3} = \frac{3!}{3!\,(3-3)!} = \frac{\cancel{3!}}{\cancel{3!}\,0!} = 1$$

$$\binom{3}{2} = \frac{3!}{2!\,(3-2)!} = \frac{3!}{2!\,1!} = 3$$

Bridge

Bridge Hand is 13 Cards

$$\binom{52}{4} = \frac{52!}{4!\,(52-4)!}$$

$$= \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot \cancel{48!}}{4!\ (\cancel{48!})}$$

$$= 635{,}013{,}559{,}600$$

Axioms (Rules) of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

1. $P(S) = 1$
2. $0 \leq P(E) \leq 1$
3. For two events E_1 and E_2 with $E_1 \cap E_2 = \emptyset$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

- Consider problem 2-70

Compliment of an Event

T' is the complement of T

$$P(T') = 1 - P(T)$$

Simple case

Head vs Tail

Bead Bowl

W, R, BL

W' — R or BL

409	126	311	846
128	8	18	154
537	134	319	

$n = 1000$

Question 1 (1 point)

Listen

Consider a problem classified by 2 rows and 3 columns containing 1000 observations. The table is described in the figure below and has the following cell counts: A=409, B=126, C=311, D=128, E=8, F=18. Let event S denote an item that

	Column 1	column 2	Column 3
row 1	A	B	C
row 2	D	E	F

occurs in row 2. Find P(S').

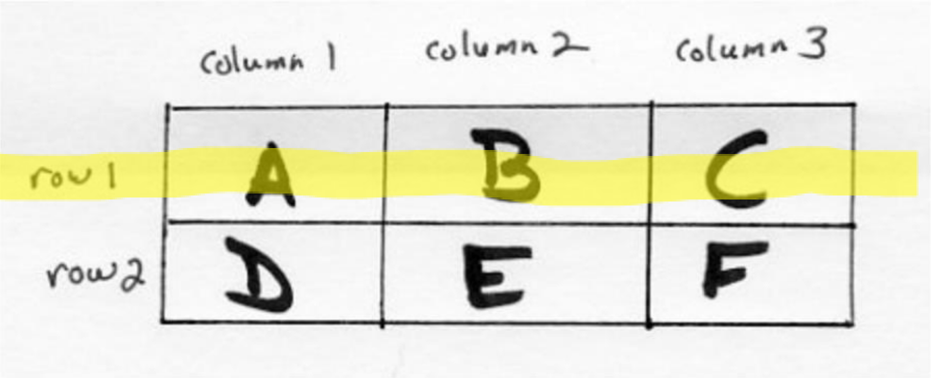
$$P(S') = 1 - P(S) = 1 - \frac{154}{1000} = .\cancel{154} \quad .846$$

$$P(S) = \frac{A + B + C}{1000} = \frac{385 + 574 + 38}{1000} = .997$$

Question 3 (1 point)

Listen

Consider a problem classified by 2 rows and 3 columns containing 1000 observations. The table is described in the figure below and has the following cell counts: A=385, B=574, C=38, D=0, E=1, F=2. Let event S denote an item that occurs



in row 1. Find P(S).

Question 5 (1 point)

Listen

Consider a problem classified by 2 rows and 3 columns containing 1000 observations. The table is described in the figure below and has the following cell counts: A=298, B=191, C=160, D=108, E=44, F=199. Let event T denote an item that occurs in column 3. Find P(T).

	Column 1	column 2	column 3
row 1	A	B	C
row 2	D	E	F

$$P(T) = \frac{C+F}{n}$$

$$= \frac{160+199}{1000}$$

$$= .359$$

$$P(T') = 1 - P(T)$$
$$= 1 - \left(\frac{C + F}{n}\right)$$

Question 7 (1 point)

Listen

Consider a problem classified by 2 rows and 3 columns containing 2000 observations. The table is described in the figure below and has the following cell counts: A=394, B=1353, C=1, D=51, E=160, F=41. Let event T denote an item that occurs in column 3. Find P(T').

	Column 1	column 2	Column 3
row 1	A	B	C
row 2	D	E	F

$$= 1 - \left(\frac{1 + 41}{2000}\right)$$
$$= .979$$

Practice Problems - Single Event

A Word of Warning

- It usually looks very easy when I work a problem
- I have been using statistics for ~~almost~~ over 40 years
- This is something you MUST practice
- Rework class room examples and textbook examples

Probability of Multiple Events

Intersection:

$P(A \cap B)$ is "the probability of A and B occurring

Union:

$P(A \cup B)$ is "the probability of A or B (or both)"

Complement:

$P(A')$ is "the probability of not A"

- Venn diagrams are a very useful tool for understanding multiple events and calculating probabilities

Addition Rules

- Used to calculate the union of two events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If two events are mutually exclusive ($A \cap B = \emptyset$)

$$P(A \cup B) = P(A) + P(B)$$

- Consider problems 2-82 and 2-85

Addition Rule for 3 or More Events

- For three events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- For a set of events to mutually exclusive all pairs of variables must satisfy $E_1 \cap E_2 = \emptyset$
- For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \cdots \cup E_k) = P(E_1) + P(E_2) + \cdots + P(E_k)$$

Conditional Probability

- Hayter (2002) states that "For experiments with two or more events of interest, attention is often directed not only at the probabilities of individual events but also at the probability of an event occurring **conditional** on the knowledge that another event has occurred."
- The **conditional probability** of an event B given an event A, denoted $P(B|A)$ is

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

for $P(A) > 0$

- Consider problems 2-99

Multiplication Rules

- This rule provides another method for calculating $P(A \cap B)$
- $P(A \cap B) \quad = \quad P(A|B)P(B) = P(B|A)P(A)$
- This leads to the total probability rule
- $$\begin{aligned} P(B) \quad &= \quad P(B \cap A) + P(B \cap A') \\ &= \quad P(B|A)P(A) + P(B|A')P(A') \end{aligned}$$
- Consider problems from 3rd edition (next slide) and 2-129

Example Problem 2-76

E 2-76. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2 and 1% of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

problem 2-76

Independent Events

- Two events are independent if any one of the following is true:
  1. $P(A|B) = P(A)$
  2. $P(B|A) = P(B)$
  3. $P(A \cap B) = P(A)P(B)$
- Consider problem 2-146

Reliability Analysis

- Reliability is the application of statistics and probability to determine the probability that a system will be working properly
- Components can be arranged in series. All components must work for the system to work.

$$P(\text{system works}) = P(A\text{ works})P(B\text{ works})$$

- Components can be arranged in parallel. As long as one component works, the system works.

$$P(\text{system works}) = 1 - \big(1 - P(A\text{ works})\big) \times 1 - P(B\text{ works}))$$

- Consider problem 2-157