# MANE 3332.01

# LECTURE 11

# Agenda

- Continue Chapter 4 lectures
- Standard Normal Recap
- Poisson Quiz (assigned 10/2/2025, due 10/7/2025)
- Standard Normal Practice Problems (assigned 10/2/2025, due 10/7/2025)
- Standard Normal Quiz (assigned 10/7/2025, due 10/9/2025)
- Normal Practice Problems (assigned 10/7/2025, due 10/9/2025)
- Schedule

→ be delayed 1 class

# Handouts

- Lecture 11 Slides - Powerpoint
- Lecture 11 Slides - marked (pdf)

Everything will shift back

| Tuesday Date and Topic(s) | Thursday Date and Topic(s) |
|---|---|
| **10/7:** normal distribution | **10/9:** Exponential and Weibull distributions |
| **10/14:** Chapter 5 (not on midterm) | **10/16:** Midterm Review |
| **10/21:** Midterm Exam | **10/23:** Continue Part Two |

on lecture

# The Normal Distribution

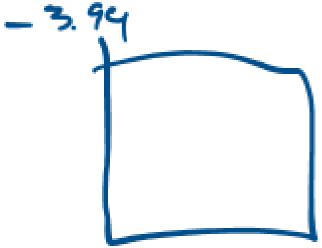
- The normal distribution is the most widely used and important distribution in statistics.
- You must master this!
- A random variable $X$ with probability density function

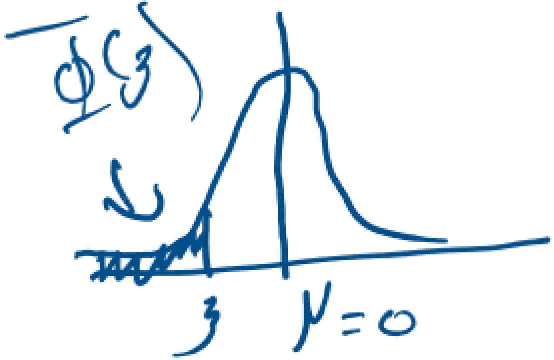
$$-\infty < x < \infty f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad \text{for}$$

has a **normal distribution** with parameters $\mu$ and $\sigma$ where $-\infty < \mu < \infty$ and $\sigma > 0$

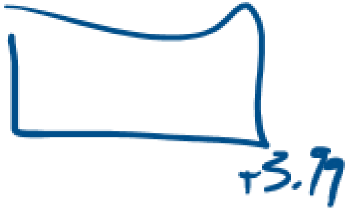
- The normal distribution with parameters $\mu$ and $\sigma$ is denoted $N(\mu, \sigma^2)$
- An interesting web-site is http://www.seeingstatistics.com/seeingTour/normal/shape3.html

$\Phi(z) = P(Z \le z) \Longrightarrow$ CDF for standard normal





$\mu = 0, \sigma = 1$

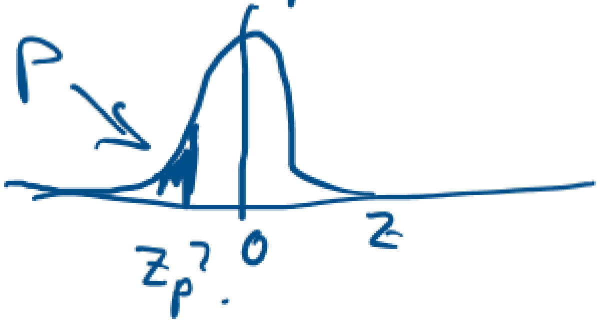


2 General Types of Problems

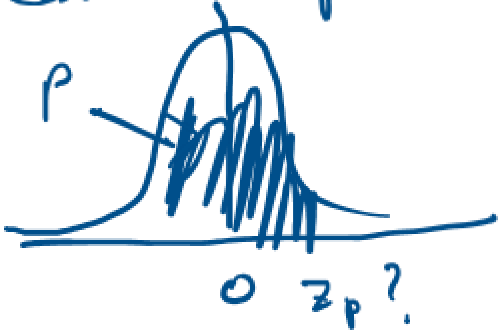
1) know $z$, find Prob
2) know Prob, find $z$

Find $x$ such that $P(Z<x) = p$

Case 1 $p < .5$



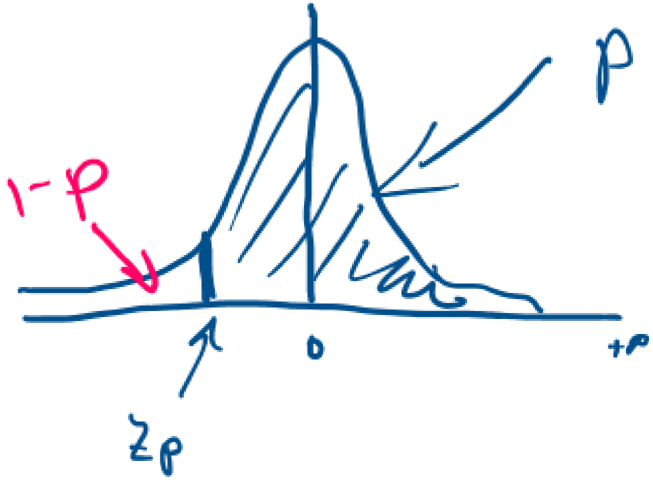
Case 2 $p > .5$



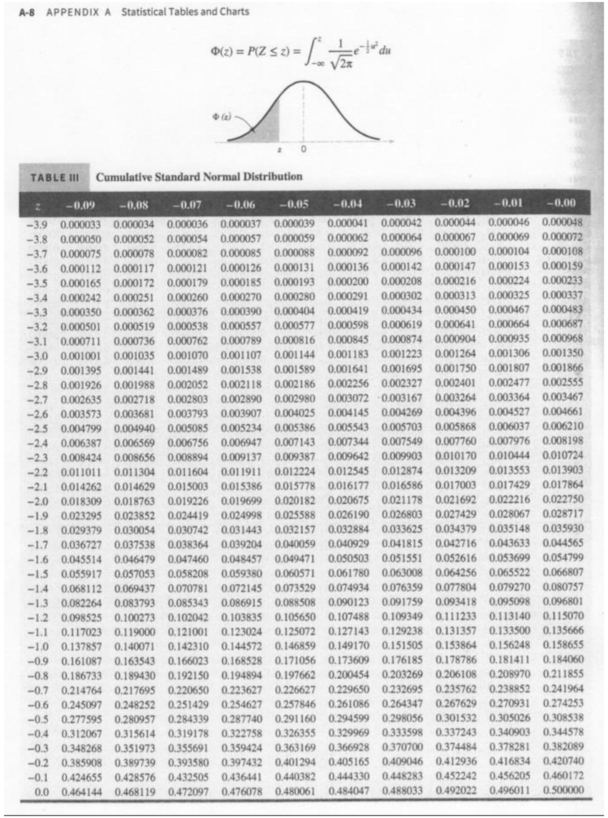
Case 2. Find x, such that $P(Z > x) = p$

if $p < .5$

if $p > .5$



**Cumulative Standard Normal Distribution**


A-8 APPENDIX A Statistical Tables and Charts

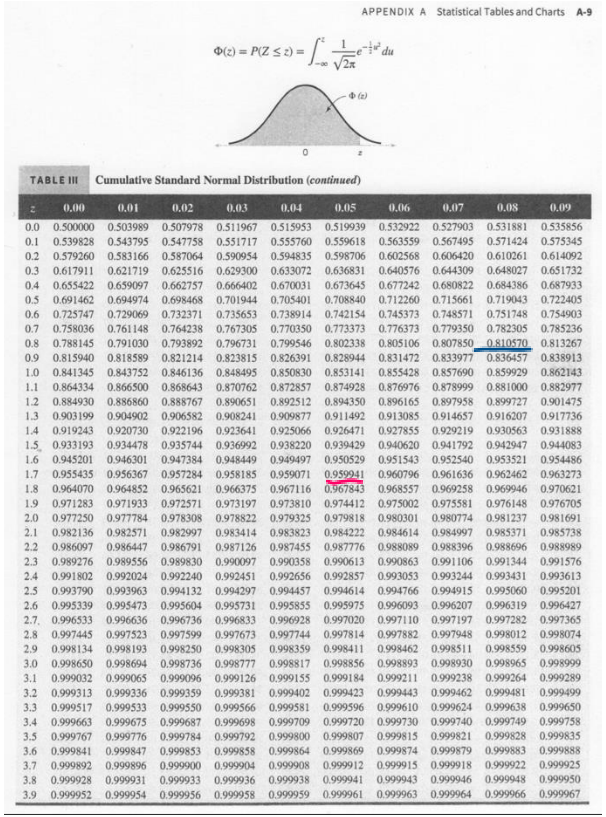
$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

**TABLE III** Cumulative Standard Normal Distribution

| z | −0.09 | −0.08 | −0.07 | −0.06 | −0.05 | −0.04 | −0.03 | −0.02 | −0.01 | −0.00 |
|---|---|---|---|---|---|---|---|---|---|---|
| −3.9 | 0.000033 | 0.000034 | 0.000036 | 0.000037 | 0.000039 | 0.000041 | 0.000042 | 0.000044 | 0.000046 | 0.000048 |
| −3.8 | 0.000050 | 0.000052 | 0.000054 | 0.000057 | 0.000059 | 0.000062 | 0.000064 | 0.000067 | 0.000069 | 0.000072 |
| −3.7 | 0.000075 | 0.000078 | 0.000082 | 0.000085 | 0.000088 | 0.000092 | 0.000096 | 0.000100 | 0.000104 | 0.000108 |
| −3.6 | 0.000112 | 0.000117 | 0.000121 | 0.000126 | 0.000131 | 0.000136 | 0.000142 | 0.000147 | 0.000153 | 0.000159 |
| −3.5 | 0.000165 | 0.000172 | 0.000179 | 0.000185 | 0.000193 | 0.000200 | 0.000208 | 0.000216 | 0.000224 | 0.000233 |
| −3.4 | 0.000242 | 0.000251 | 0.000260 | 0.000270 | 0.000280 | 0.000291 | 0.000302 | 0.000313 | 0.000325 | 0.000337 |
| −3.3 | 0.000350 | 0.000362 | 0.000376 | 0.000390 | 0.000404 | 0.000419 | 0.000434 | 0.000450 | 0.000467 | 0.000483 |
| −3.2 | 0.000501 | 0.000519 | 0.000538 | 0.000557 | 0.000577 | 0.000598 | 0.000619 | 0.000641 | 0.000664 | 0.000687 |
| −3.1 | 0.000711 | 0.000736 | 0.000762 | 0.000789 | 0.000816 | 0.000845 | 0.000874 | 0.000904 | 0.000935 | 0.000968 |
| −3.0 | 0.001001 | 0.001035 | 0.001070 | 0.001107 | 0.001144 | 0.001183 | 0.001223 | 0.001264 | 0.001306 | 0.001350 |
| −2.9 | 0.001395 | 0.001441 | 0.001489 | 0.001538 | 0.001589 | 0.001641 | 0.001695 | 0.001750 | 0.001807 | 0.001866 |
| −2.8 | 0.001926 | 0.001988 | 0.002052 | 0.002118 | 0.002186 | 0.002256 | 0.002327 | 0.002401 | 0.002477 | 0.002555 |
| −2.7 | 0.002635 | 0.002718 | 0.002803 | 0.002890 | 0.002980 | 0.003072 | 0.003167 | 0.003264 | 0.003364 | 0.003467 |
| −2.6 | 0.003573 | 0.003681 | 0.003793 | 0.003907 | 0.004025 | 0.004145 | 0.004269 | 0.004396 | 0.004527 | 0.004661 |
| −2.5 | 0.004799 | 0.004940 | 0.005085 | 0.005234 | 0.005386 | 0.005543 | 0.005703 | 0.005868 | 0.006037 | 0.006210 |
| −2.4 | 0.006387 | 0.006569 | 0.006756 | 0.006947 | 0.007143 | 0.007344 | 0.007549 | 0.007760 | 0.007976 | 0.008198 |
| −2.3 | 0.008424 | 0.008656 | 0.008894 | 0.009137 | 0.009387 | 0.009642 | 0.009903 | 0.010170 | 0.010444 | 0.010724 |
| −2.2 | 0.011011 | 0.011304 | 0.011604 | 0.011911 | 0.012224 | 0.012545 | 0.012874 | 0.013209 | 0.013553 | 0.013903 |
| −2.1 | 0.014262 | 0.014629 | 0.015003 | 0.015386 | 0.015778 | 0.016177 | 0.016586 | 0.017003 | 0.017429 | 0.017864 |
| −2.0 | 0.018309 | 0.018763 | 0.019226 | 0.019699 | 0.020182 | 0.020675 | 0.021178 | 0.021692 | 0.022216 | 0.022750 |
| −1.9 | 0.023295 | 0.023852 | 0.024419 | 0.024998 | 0.025588 | 0.026190 | 0.026803 | 0.027429 | 0.028067 | 0.028717 |
| −1.8 | 0.029379 | 0.030054 | 0.030742 | 0.031443 | 0.032157 | 0.032884 | 0.033625 | 0.034379 | 0.035148 | 0.035930 |
| −1.7 | 0.036727 | 0.037538 | 0.038364 | 0.039204 | 0.040059 | 0.040929 | 0.041815 | 0.042716 | 0.043633 | 0.044565 |
| −1.6 | 0.045514 | 0.046479 | 0.047460 | 0.048457 | 0.049471 | 0.050503 | 0.051551 | 0.052616 | 0.053699 | 0.054799 |
| −1.5 | 0.055917 | 0.057053 | 0.058208 | 0.059380 | 0.060571 | 0.061780 | 0.063008 | 0.064256 | 0.065522 | 0.066807 |
| −1.4 | 0.068112 | 0.069437 | 0.070781 | 0.072145 | 0.073529 | 0.074934 | 0.076359 | 0.077804 | 0.079270 | 0.080757 |
| −1.3 | 0.082264 | 0.083793 | 0.085343 | 0.086915 | 0.088508 | 0.090123 | 0.091759 | 0.093418 | 0.095098 | 0.096801 |
| −1.2 | 0.098525 | 0.100273 | 0.102042 | 0.103835 | 0.105650 | 0.107488 | 0.109349 | 0.111233 | 0.113140 | 0.115070 |
| −1.1 | 0.117023 | 0.119000 | 0.121001 | 0.123024 | 0.125072 | 0.127143 | 0.129238 | 0.131357 | 0.133500 | 0.135666 |
| −1.0 | 0.137857 | 0.140071 | 0.142310 | 0.144572 | 0.146859 | 0.149170 | 0.151505 | 0.153864 | 0.156248 | 0.158655 |
| −0.9 | 0.161087 | 0.163543 | 0.166023 | 0.168528 | 0.171056 | 0.173609 | 0.176185 | 0.178786 | 0.181411 | 0.184060 |
| −0.8 | 0.186733 | 0.189430 | 0.192150 | 0.194894 | 0.197662 | 0.200454 | 0.203269 | 0.206108 | 0.208970 | 0.211855 |
| −0.7 | 0.214764 | 0.217695 | 0.220650 | 0.223627 | 0.226627 | 0.229650 | 0.232695 | 0.235762 | 0.238852 | 0.241964 |
| −0.6 | 0.245097 | 0.248252 | 0.251429 | 0.254627 | 0.257846 | 0.261086 | 0.264347 | 0.267629 | 0.270931 | 0.274253 |
| −0.5 | 0.277595 | 0.280957 | 0.284339 | 0.287740 | 0.291160 | 0.294599 | 0.298056 | 0.301532 | 0.305026 | 0.308538 |
| −0.4 | 0.312067 | 0.315614 | 0.319178 | 0.322758 | 0.326355 | 0.329969 | 0.333598 | 0.337243 | 0.340903 | 0.344578 |
| −0.3 | 0.348268 | 0.351973 | 0.355691 | 0.359424 | 0.363169 | 0.366928 | 0.370700 | 0.374484 | 0.378281 | 0.382089 |
| −0.2 | 0.385908 | 0.389739 | 0.393580 | 0.397432 | 0.401294 | 0.405165 | 0.409046 | 0.412936 | 0.416834 | 0.420740 |
| −0.1 | 0.424655 | 0.428576 | 0.432505 | 0.436441 | 0.440382 | 0.444330 | 0.448283 | 0.452242 | 0.456205 | 0.460172 |
| 0.0 | 0.464144 | 0.468119 | 0.472097 | 0.476078 | 0.480061 | 0.484047 | 0.488033 | 0.492022 | 0.496011 | 0.500000 |

page A-8

## Cumulative Standard Normal Distribution



APPENDIX A Statistical Tables and Charts **A-9**

$$\Phi(z) = P(Z \le z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

**TABLE III** **Cumulative Standard Normal Distribution (*continued*)**

| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.0 | 0.500000 | 0.503989 | 0.507978 | 0.511967 | 0.515953 | 0.519939 | 0.532922 | 0.527903 | 0.531881 | 0.535856 |
| 0.1 | 0.539828 | 0.543795 | 0.547758 | 0.551717 | 0.555760 | 0.559618 | 0.563559 | 0.567495 | 0.571424 | 0.575345 |
| 0.2 | 0.579260 | 0.583166 | 0.587064 | 0.590954 | 0.594835 | 0.598706 | 0.602568 | 0.606420 | 0.610261 | 0.614092 |
| 0.3 | 0.617911 | 0.621719 | 0.625516 | 0.629300 | 0.633072 | 0.636831 | 0.640576 | 0.644309 | 0.648027 | 0.651732 |
| 0.4 | 0.655422 | 0.659097 | 0.662757 | 0.666402 | 0.670031 | 0.673645 | 0.677242 | 0.680822 | 0.684386 | 0.687933 |
| 0.5 | 0.691462 | 0.694974 | 0.698468 | 0.701944 | 0.705401 | 0.708840 | 0.712260 | 0.715661 | 0.719043 | 0.722405 |
| 0.6 | 0.725747 | 0.729069 | 0.732371 | 0.735653 | 0.738914 | 0.742154 | 0.745373 | 0.748571 | 0.751748 | 0.754903 |
| 0.7 | 0.758036 | 0.761148 | 0.764238 | 0.767305 | 0.770350 | 0.773373 | 0.776373 | 0.779350 | 0.782305 | 0.785236 |
| 0.8 | 0.788145 | 0.791030 | 0.793892 | 0.796731 | 0.799546 | 0.802338 | 0.805106 | 0.807850 | 0.810570 | 0.813267 |
| 0.9 | 0.815940 | 0.818589 | 0.821214 | 0.823815 | 0.826391 | 0.828944 | 0.831472 | 0.833977 | 0.836457 | 0.838913 |
| 1.0 | 0.841345 | 0.843752 | 0.846136 | 0.848495 | 0.850830 | 0.853141 | 0.855428 | 0.857690 | 0.859929 | 0.862143 |
| 1.1 | 0.864334 | 0.866500 | 0.868643 | 0.870762 | 0.872857 | 0.874928 | 0.876976 | 0.878999 | 0.881000 | 0.882977 |
| 1.2 | 0.884930 | 0.886860 | 0.888767 | 0.890651 | 0.892512 | 0.894350 | 0.896165 | 0.897958 | 0.899727 | 0.901475 |
| 1.3 | 0.903199 | 0.904902 | 0.906582 | 0.908241 | 0.909877 | 0.911492 | 0.913085 | 0.914657 | 0.916207 | 0.917736 |
| 1.4 | 0.919243 | 0.920730 | 0.922196 | 0.923641 | 0.925066 | 0.926471 | 0.927855 | 0.929219 | 0.930563 | 0.931888 |
| 1.5 | 0.933193 | 0.934478 | 0.935744 | 0.936992 | 0.938220 | 0.939429 | 0.940620 | 0.941792 | 0.942947 | 0.944083 |
| 1.6 | 0.945201 | 0.946301 | 0.947384 | 0.948449 | 0.949497 | 0.950529 | 0.951543 | 0.952540 | 0.953521 | 0.954486 |
| 1.7 | 0.955435 | 0.956367 | 0.957284 | 0.958185 | 0.959071 | 0.959941 | 0.960796 | 0.961636 | 0.962462 | 0.963273 |
| 1.8 | 0.964070 | 0.964852 | 0.965621 | 0.966375 | 0.967116 | 0.967843 | 0.968557 | 0.969258 | 0.969946 | 0.970621 |
| 1.9 | 0.971283 | 0.971933 | 0.972571 | 0.973197 | 0.973810 | 0.974412 | 0.975002 | 0.975581 | 0.976148 | 0.976705 |
| 2.0 | 0.977250 | 0.977784 | 0.978308 | 0.978822 | 0.979325 | 0.979818 | 0.980301 | 0.980774 | 0.981237 | 0.981691 |
| 2.1 | 0.982136 | 0.982571 | 0.982997 | 0.983414 | 0.983823 | 0.984222 | 0.984614 | 0.984997 | 0.985371 | 0.985738 |
| 2.2 | 0.986097 | 0.986447 | 0.986791 | 0.987126 | 0.987455 | 0.987776 | 0.988089 | 0.988396 | 0.988696 | 0.988989 |
| 2.3 | 0.989276 | 0.989556 | 0.989830 | 0.990097 | 0.990358 | 0.990613 | 0.990863 | 0.991106 | 0.991344 | 0.991576 |
| 2.4 | 0.991802 | 0.992024 | 0.992240 | 0.992451 | 0.992656 | 0.992857 | 0.993053 | 0.993244 | 0.993431 | 0.993613 |
| 2.5 | 0.993790 | 0.993963 | 0.994132 | 0.994297 | 0.994457 | 0.994614 | 0.994766 | 0.994915 | 0.995060 | 0.995201 |
| 2.6 | 0.995339 | 0.995473 | 0.995604 | 0.995731 | 0.995855 | 0.995975 | 0.996093 | 0.996207 | 0.996319 | 0.996427 |
| 2.7 | 0.996533 | 0.996636 | 0.996736 | 0.996833 | 0.996928 | 0.997020 | 0.997110 | 0.997197 | 0.997282 | 0.997365 |
| 2.8 | 0.997445 | 0.997523 | 0.997599 | 0.997673 | 0.997744 | 0.997814 | 0.997882 | 0.997948 | 0.998012 | 0.998074 |
| 2.9 | 0.998134 | 0.998193 | 0.998250 | 0.998305 | 0.998359 | 0.998411 | 0.998462 | 0.998511 | 0.998559 | 0.998605 |
| 3.0 | 0.998650 | 0.998694 | 0.998736 | 0.998777 | 0.998817 | 0.998856 | 0.998893 | 0.998930 | 0.998965 | 0.998999 |
| 3.1 | 0.999032 | 0.999065 | 0.999096 | 0.999126 | 0.999155 | 0.999184 | 0.999211 | 0.999238 | 0.999264 | 0.999289 |
| 3.2 | 0.999313 | 0.999336 | 0.999359 | 0.999381 | 0.999402 | 0.999423 | 0.999443 | 0.999462 | 0.999481 | 0.999499 |
| 3.3 | 0.999517 | 0.999533 | 0.999550 | 0.999566 | 0.999581 | 0.999596 | 0.999610 | 0.999624 | 0.999638 | 0.999650 |
| 3.4 | 0.999663 | 0.999675 | 0.999687 | 0.999698 | 0.999709 | 0.999720 | 0.999730 | 0.999740 | 0.999749 | 0.999758 |
| 3.5 | 0.999767 | 0.999776 | 0.999784 | 0.999792 | 0.999800 | 0.999807 | 0.999815 | 0.999821 | 0.999828 | 0.999835 |
| 3.6 | 0.999841 | 0.999847 | 0.999853 | 0.999858 | 0.999864 | 0.999869 | 0.999874 | 0.999879 | 0.999883 | 0.999888 |
| 3.7 | 0.999892 | 0.999896 | 0.999900 | 0.999904 | 0.999908 | 0.999912 | 0.999915 | 0.999918 | 0.999922 | 0.999925 |
| 3.8 | 0.999928 | 0.999931 | 0.999933 | 0.999936 | 0.999938 | 0.999941 | 0.999943 | 0.999946 | 0.999948 | 0.999950 |
| 3.9 | 0.999952 | 0.999954 | 0.999956 | 0.999958 | 0.999959 | 0.999961 | 0.999963 | 0.999964 | 0.999966 | 0.999967 |

page A-9

# Standardizing (the $z$-transform)

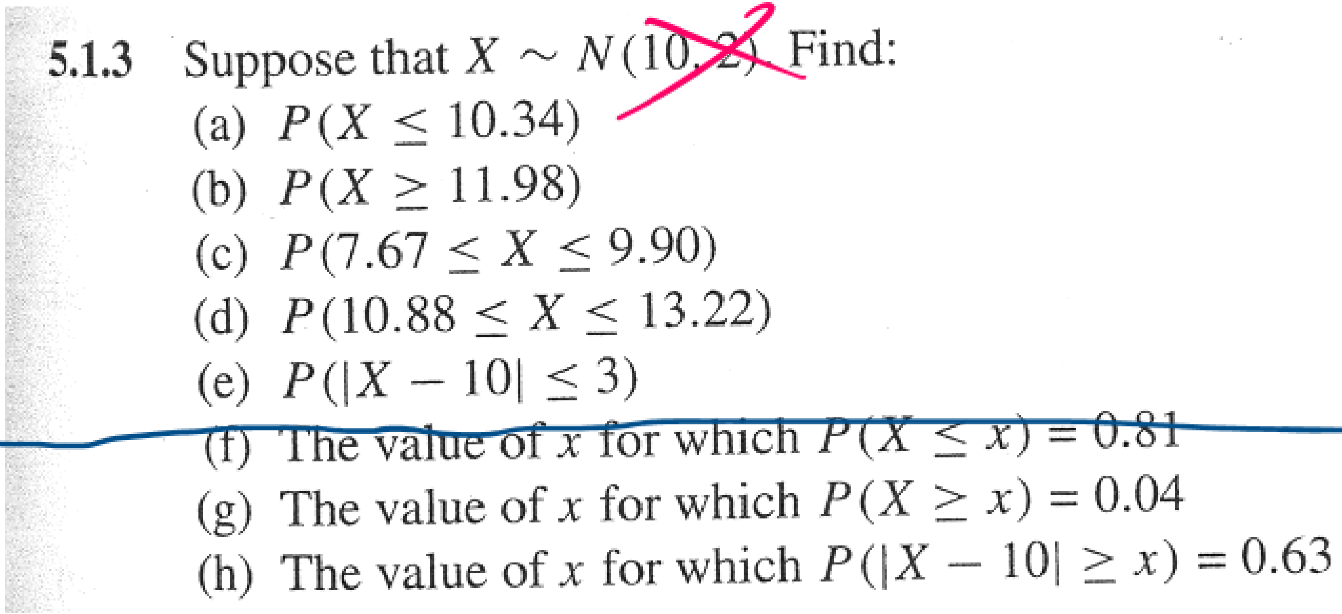
- Suppose $X$ is a normal random variable with mean $\mu$ and variance $\sigma^2$ (or standard deviation $\sigma$)

$$P(X \leq x) = P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right) = P(Z \leq z)$$

- The $z$-value is $z = (x - \mu)/\sigma$
- Result allows the standard normal tables to be used to calculate probabilities for any normal distribution

# Normal Probability Problem

$\mu = 10, \sigma = 2$



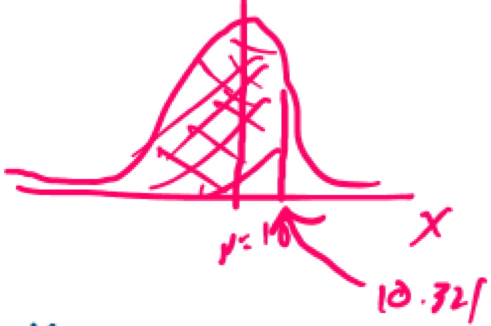
**5.1.3** Suppose that $X \sim N(10, \cancel{2})$. Find:

(a) $P(X \leq 10.34)$
(b) $P(X \geq 11.98)$
(c) $P(7.67 \leq X \leq 9.90)$
(d) $P(10.88 \leq X \leq 13.22)$
(e) $P(|X - 10| \leq 3)$
(f) The value of $x$ for which $P(X \leq x) = 0.81$
(g) The value of $x$ for which $P(X \geq x) = 0.04$
(h) The value of $x$ for which $P(|X - 10| \geq x) = 0.63$

image

$\mu = 10, \ \sigma = 2$

Find $P(X < 10.34)$



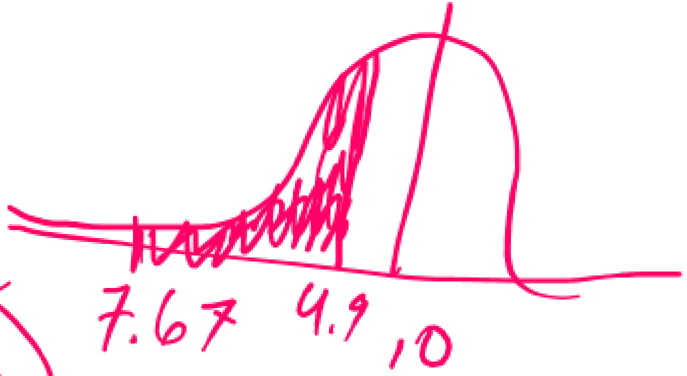
$$Z = \frac{x-\mu}{\sigma} \rightarrow P(X < 10.34) = \Phi\left(\frac{\overset{X}{\cancel{Z}} - \mu}{\sigma}\right)$$

$$= \Phi\left(\frac{10.34 - 10}{2}\right)$$

$$= \Phi(.17)$$

$$= \underline{.567495}$$

$$P(7.67 < x < 9.9)$$



$$= \Phi\left(\overset{\frac{x_2-\mu}{\sigma}}{?}\right) - \Phi\left(\overset{\frac{x_1-\mu}{\sigma}}{\phantom{?}}\right)$$

$$= \Phi\left(\frac{\overset{9.9}{\cancel{7.67}}-10}{2}\right) - \Phi\left(\frac{7.67-10}{2}\right)$$

$$= \Phi(-.05) - \Phi(-1.17)$$

$\mu = 10, \sigma = 2$

$P(X > \overset{x}{11.98})$

$= 1 - \Phi(?)$

$= 1 - \Phi\left(\frac{11.98 - 10}{2}\right)$

$= 1 - \Phi(.49)$

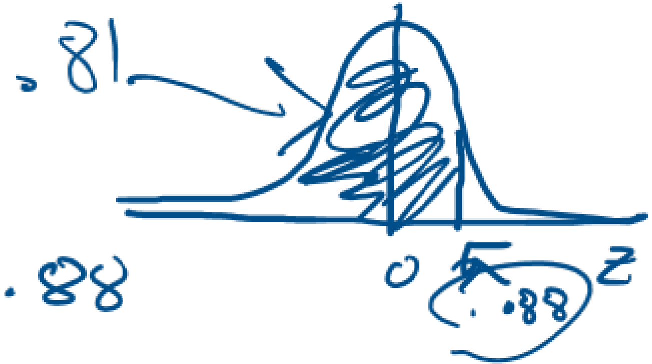
$= 1 - .687933$

$= \cancel{1}\ .312067$

$Z = \frac{x - \mu}{\sigma}$

$\mu = 10, \sigma = 2$

Find x, such that $P(X<x) = .81$



Probability

Transform from $z = .88$
to $x = ?$

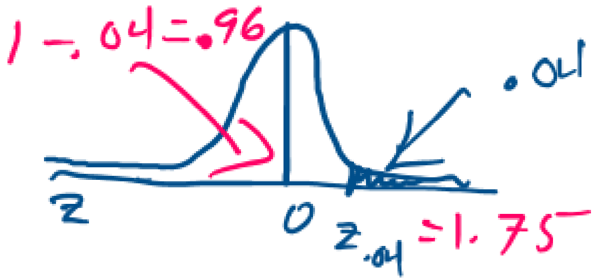
$$z = \frac{x-\mu}{\sigma}$$

$$.88 = \frac{x-10}{2} \rightarrow x - 10 = 1.76$$

$$x = \underline{11.76}$$

$\mu = 10$ $\sigma = 2$

Find $x$ s.t. $P(X > x) = .04$



Know $z = 1.75$, $\mu = 10$, $\sigma = 2$

$$z = \frac{x - \mu}{\sigma} \Rightarrow 1.75 = \frac{X - 10}{2}$$

$$x - 10 = 3.5$$

$$x = 13.5$$

10/17 : Attendance → <u>1 — B</u>

# Normal Practice Problems

## Normal Approximation to the Binomial Distribution

- If $X$ is a binomial random variable,

$$Z = \frac{X - np}{\sqrt{np(1-p)}}$$

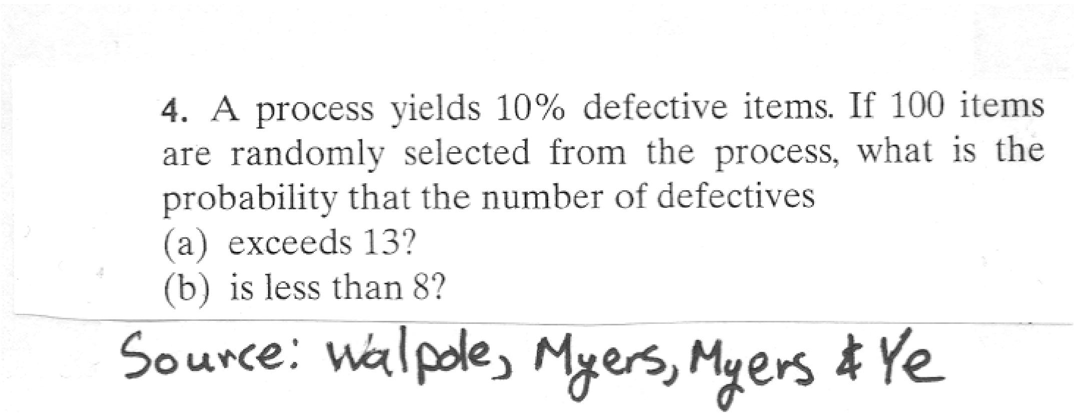
is approximately a standard normal random variable. Consequently, probabilities computed from $Z$ can be used to approximate probabilities for $X$

- Usually holds when

$$np > 5 \quad \text{and } n(1-p) > 5$$

- How good are the approximations?

**Problem**



**4.** A process yields 10% defective items. If 100 items are randomly selected from the process, what is the probability that the number of defectives

(a) exceeds 13?

(b) is less than 8?

Source: Walpole, Myers, Myers & Ye

image

**Normal Approximation - Figure**

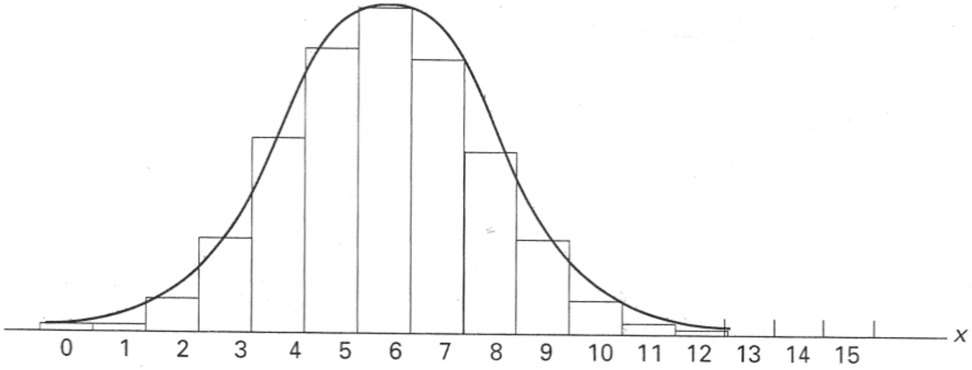


**Figure 6.22** Normal approximation of $b(x; 15, 0.4)$.

Source: Walpole, Myers, Myers & Ye

image

## Rework Problem using Continuity Correction Factor

- Are the approximations improved?

**Normal Approximation to the Poisson Distribution**

- If $X$ is a Poisson random variable with $E(X) = \lambda$ and $V(X) = \lambda$,

$$Z = \frac{X - \lambda}{\sqrt{\lambda}}$$

is approximately a standard normal random variable.

# Exponential Distribution

- The exponential distribution is widely used in the area of reliability and life-test data.
- Ostle, et. al. (1996) list the following applications of the exponential distribution
  - the number of feet between two consecutive erroneous records on a computer tape,
  - the lifetime of a component of a particular device,
  - the length of a life of a radioactive material and
  - the time to the next customer service call at a service desk

## Exponential Distribution

- The PDF for an exponential distribution with parameter $\lambda > 0$ is

$$f(x) = \lambda e^{-\lambda x}, \quad \text{for } 0 \le x < \infty$$

- The mean of $X$ is

$$\mu = E(X) = \frac{1}{\lambda}$$

- The variance of $X$ is

$$\sigma^2 = V(X) = \frac{1}{\lambda^2}$$

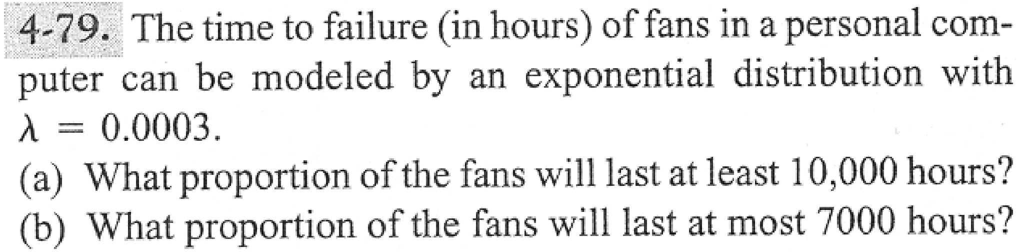
Note that other authors define $f(x) = \frac{1}{\theta} e^{-x/\theta}$. Either definition is acceptable. However one must be aware of which definition is being used.

## The Exponential CDF

The CDF for the exponential distribution is easy to derive

$$\begin{aligned}
F(x) &= P(X \leq x) = \int_{-\infty}^{x} \lambda\, e^{-\lambda y}\, dy \\
&= \int_{0}^{x} \lambda\, e^{-\lambda y}\, dy \\
&= \left(-e^{-\lambda y}\right)\Big|_{y=0}^{x} \\
&= -e^{-\lambda x} - (-e^{0}) \\
&= -e^{-\lambda x} + 1 \\
&= 1 - e^{-\lambda x}
\end{aligned}$$

**Problem 4-79**



**4-79.** The time to failure (in hours) of fans in a personal computer can be modeled by an exponential distribution with $\lambda = 0.0003$.

(a) What proportion of the fans will last at least 10,000 hours?

(b) What proportion of the fans will last at most 7000 hours?

image

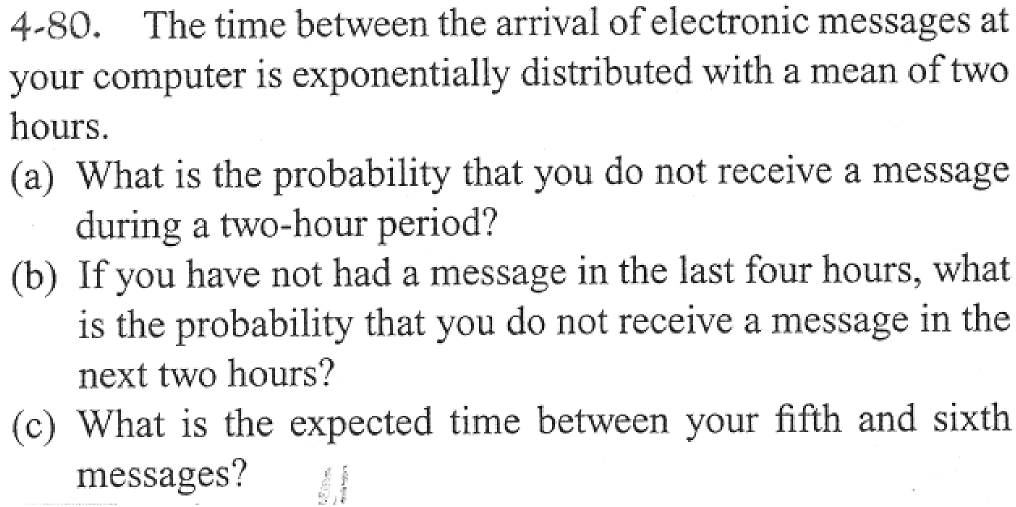
**Lack of Memory Property**

- The mathematical definition is

$$P(X < t_1 + t_2 | X > t_1) = P(X < t_2)$$

- That is “the probability of a failure time that is less than $t_1 + t_2$ given the failure time is greater than $t_1$ is the probability that the item’s failure time is less than $t_2$
- This property is unique to the exponential distribution
- Often used to model the reliability of electronic components.

**Problem 4–80**



4-80. The time between the arrival of electronic messages at your computer is exponentially distributed with a mean of two hours.

(a) What is the probability that you do not receive a message during a two-hour period?

(b) If you have not had a message in the last four hours, what is the probability that you do not receive a message in the next two hours?

(c) What is the expected time between your fifth and sixth messages?

image

## Relationship to the Poisson Distribution

- Let $Y$ be a Poisson random variable with parameter $\lambda$. Note: $Y$ represents the number of occurrences per unit
- Let $X$ be a random variable that records the time between occurrences for the same process as $Y$
- $X$ has an exponential distribution with parameter $\lambda$

# Lognormal Distribution

- Let $W$ have a normal distribution with mean $\theta$ and variance $\omega^2$; then $X = \exp(W)$ is a **lognormal random variable** with pdf

$$f(x) = \frac{1}{x\omega\sqrt{2\pi}} \exp\left[-\frac{(\ln(x)-\theta)^2}{2\omega^2}\right] \quad 0 < x < \infty$$

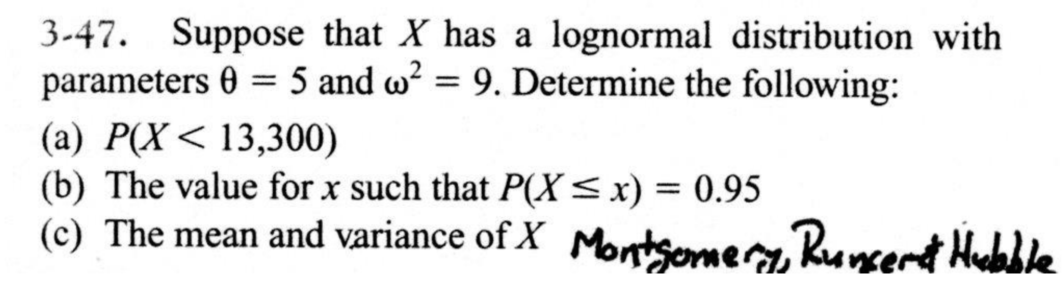
- The mean of $X$ is

$$E(X) = e^{\theta+\omega^2/2}$$

- The variance of $X$ is

$$V(X) = e^{2\theta+\omega^2}\left(e^{\omega^2} - 1\right)$$

**Example Problem**



3-47. Suppose that $X$ has a lognormal distribution with parameters $\theta = 5$ and $\omega^2 = 9$. Determine the following:

(a) $P(X < 13{,}300)$

(b) The value for $x$ such that $P(X \leq x) = 0.95$

(c) The mean and variance of $X$

Montgomery, Runger & Hubble

image

# Gamma Distribution

- The random variable $X$ with pdf

$$f(x) = \frac{\lambda^r x^{r-1} e^{-\lambda x}}{\Gamma(r)}, \quad \text{for } x > 0$$

is a **gamma random variable** with parameters $\lambda > 0$ and $r > 0$.

- The gamma function is

$$\Gamma(r) = \int_0^\infty x^{r-1}\, e^{-x}\, dx \quad \text{for } r > 0$$

with special properties:

- $\Gamma(r)$ is finite
- $\Gamma(r) = (r-1)\Gamma(r-1)$
- For any positive integer $r$, $\Gamma(r) = (r-1)!$
  - $\Gamma(1/2) = \pi^{1/2}$

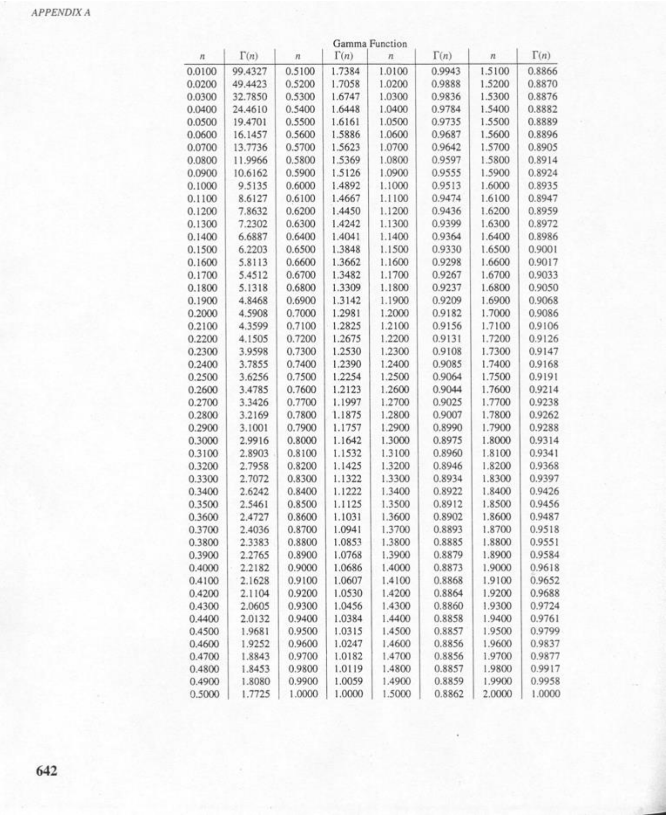
## Gamma Distribution

- The mean and variance are

$$\mu = E(X) = r/\lambda \text{ and } \sigma^2 = V(X) = r/\lambda^2$$

- We will not work any probability problems using the gamma distribution

## Gamma Tables



APPENDIX A

Gamma Function

| $n$ | $\Gamma(n)$ | $n$ | $\Gamma(n)$ | $n$ | $\Gamma(n)$ | $n$ | $\Gamma(n)$ |
|---|---|---|---|---|---|---|---|
| 0.0100 | 99.4327 | 0.5100 | 1.7384 | 1.0100 | 0.9943 | 1.5100 | 0.8866 |
| 0.0200 | 49.4423 | 0.5200 | 1.7058 | 1.0200 | 0.9888 | 1.5200 | 0.8870 |
| 0.0300 | 32.7850 | 0.5300 | 1.6747 | 1.0300 | 0.9836 | 1.5300 | 0.8876 |
| 0.0400 | 24.4610 | 0.5400 | 1.6448 | 1.0400 | 0.9784 | 1.5400 | 0.8882 |
| 0.0500 | 19.4701 | 0.5500 | 1.6161 | 1.0500 | 0.9735 | 1.5500 | 0.8889 |
| 0.0600 | 16.1457 | 0.5600 | 1.5886 | 1.0600 | 0.9687 | 1.5600 | 0.8896 |
| 0.0700 | 13.7736 | 0.5700 | 1.5623 | 1.0700 | 0.9642 | 1.5700 | 0.8905 |
| 0.0800 | 11.9966 | 0.5800 | 1.5369 | 1.0800 | 0.9597 | 1.5800 | 0.8914 |
| 0.0900 | 10.6162 | 0.5900 | 1.5126 | 1.0900 | 0.9555 | 1.5900 | 0.8924 |
| 0.1000 | 9.5135 | 0.6000 | 1.4892 | 1.1000 | 0.9513 | 1.6000 | 0.8935 |
| 0.1100 | 8.6127 | 0.6100 | 1.4667 | 1.1100 | 0.9474 | 1.6100 | 0.8947 |
| 0.1200 | 7.8632 | 0.6200 | 1.4450 | 1.1200 | 0.9436 | 1.6200 | 0.8959 |
| 0.1300 | 7.2302 | 0.6300 | 1.4242 | 1.1300 | 0.9399 | 1.6300 | 0.8972 |
| 0.1400 | 6.6887 | 0.6400 | 1.4041 | 1.1400 | 0.9364 | 1.6400 | 0.8986 |
| 0.1500 | 6.2203 | 0.6500 | 1.3848 | 1.1500 | 0.9330 | 1.6500 | 0.9001 |
| 0.1600 | 5.8113 | 0.6600 | 1.3662 | 1.1600 | 0.9298 | 1.6600 | 0.9017 |
| 0.1700 | 5.4512 | 0.6700 | 1.3482 | 1.1700 | 0.9267 | 1.6700 | 0.9033 |
| 0.1800 | 5.1318 | 0.6800 | 1.3309 | 1.1800 | 0.9237 | 1.6800 | 0.9050 |
| 0.1900 | 4.8468 | 0.6900 | 1.3142 | 1.1900 | 0.9209 | 1.6900 | 0.9068 |
| 0.2000 | 4.5908 | 0.7000 | 1.2981 | 1.2000 | 0.9182 | 1.7000 | 0.9086 |
| 0.2100 | 4.3599 | 0.7100 | 1.2825 | 1.2100 | 0.9156 | 1.7100 | 0.9106 |
| 0.2200 | 4.1505 | 0.7200 | 1.2675 | 1.2200 | 0.9131 | 1.7200 | 0.9126 |
| 0.2300 | 3.9598 | 0.7300 | 1.2530 | 1.2300 | 0.9108 | 1.7300 | 0.9147 |
| 0.2400 | 3.7855 | 0.7400 | 1.2390 | 1.2400 | 0.9085 | 1.7400 | 0.9168 |
| 0.2500 | 3.6256 | 0.7500 | 1.2254 | 1.2500 | 0.9064 | 1.7500 | 0.9191 |
| 0.2600 | 3.4785 | 0.7600 | 1.2123 | 1.2600 | 0.9044 | 1.7600 | 0.9214 |
| 0.2700 | 3.3426 | 0.7700 | 1.1997 | 1.2700 | 0.9025 | 1.7700 | 0.9238 |
| 0.2800 | 3.2169 | 0.7800 | 1.1875 | 1.2800 | 0.9007 | 1.7800 | 0.9262 |
| 0.2900 | 3.1001 | 0.7900 | 1.1757 | 1.2900 | 0.8990 | 1.7900 | 0.9288 |
| 0.3000 | 2.9916 | 0.8000 | 1.1642 | 1.3000 | 0.8975 | 1.8000 | 0.9314 |
| 0.3100 | 2.8903 | 0.8100 | 1.1532 | 1.3100 | 0.8960 | 1.8100 | 0.9341 |
| 0.3200 | 2.7958 | 0.8200 | 1.1425 | 1.3200 | 0.8946 | 1.8200 | 0.9368 |
| 0.3300 | 2.7072 | 0.8300 | 1.1322 | 1.3300 | 0.8934 | 1.8300 | 0.9397 |
| 0.3400 | 2.6242 | 0.8400 | 1.1222 | 1.3400 | 0.8922 | 1.8400 | 0.9426 |
| 0.3500 | 2.5461 | 0.8500 | 1.1125 | 1.3500 | 0.8912 | 1.8500 | 0.9456 |
| 0.3600 | 2.4727 | 0.8600 | 1.1031 | 1.3600 | 0.8902 | 1.8600 | 0.9487 |
| 0.3700 | 2.4036 | 0.8700 | 1.0941 | 1.3700 | 0.8893 | 1.8700 | 0.9518 |
| 0.3800 | 2.3383 | 0.8800 | 1.0853 | 1.3800 | 0.8885 | 1.8800 | 0.9551 |
| 0.3900 | 2.2765 | 0.8900 | 1.0768 | 1.3900 | 0.8879 | 1.8900 | 0.9584 |
| 0.4000 | 2.2182 | 0.9000 | 1.0686 | 1.4000 | 0.8873 | 1.9000 | 0.9618 |
| 0.4100 | 2.1628 | 0.9100 | 1.0607 | 1.4100 | 0.8868 | 1.9100 | 0.9652 |
| 0.4200 | 2.1104 | 0.9200 | 1.0530 | 1.4200 | 0.8864 | 1.9200 | 0.9688 |
| 0.4300 | 2.0605 | 0.9300 | 1.0456 | 1.4300 | 0.8860 | 1.9300 | 0.9724 |
| 0.4400 | 2.0132 | 0.9400 | 1.0384 | 1.4400 | 0.8858 | 1.9400 | 0.9761 |
| 0.4500 | 1.9681 | 0.9500 | 1.0315 | 1.4500 | 0.8857 | 1.9500 | 0.9799 |
| 0.4600 | 1.9252 | 0.9600 | 1.0247 | 1.4600 | 0.8856 | 1.9600 | 0.9837 |
| 0.4700 | 1.8843 | 0.9700 | 1.0182 | 1.4700 | 0.8856 | 1.9700 | 0.9877 |
| 0.4800 | 1.8453 | 0.9800 | 1.0119 | 1.4800 | 0.8857 | 1.9800 | 0.9917 |
| 0.4900 | 1.8080 | 0.9900 | 1.0059 | 1.4900 | 0.8859 | 1.9900 | 0.9958 |
| 0.5000 | 1.7725 | 1.0000 | 1.0000 | 1.5000 | 0.8862 | 2.0000 | 1.0000 |

642

image

# Weibull Distribution

- The random variable $X$ with pdf

$$f(x) = \frac{\beta}{\delta}\left(\frac{x}{\delta}\right)^{\beta-1} \exp\left[-\left(\frac{x}{\delta}\right)^{\beta}\right], \quad \text{for } x > 0$$

is a **Weibull random variable** with scale parameter $\delta > 0$ and shape parameter $\beta > 0$

- The CDF for the Weibull distribution is

$$F(x) = 1 - \exp\left[-\left(\frac{x}{\delta}\right)^{\beta}\right]$$

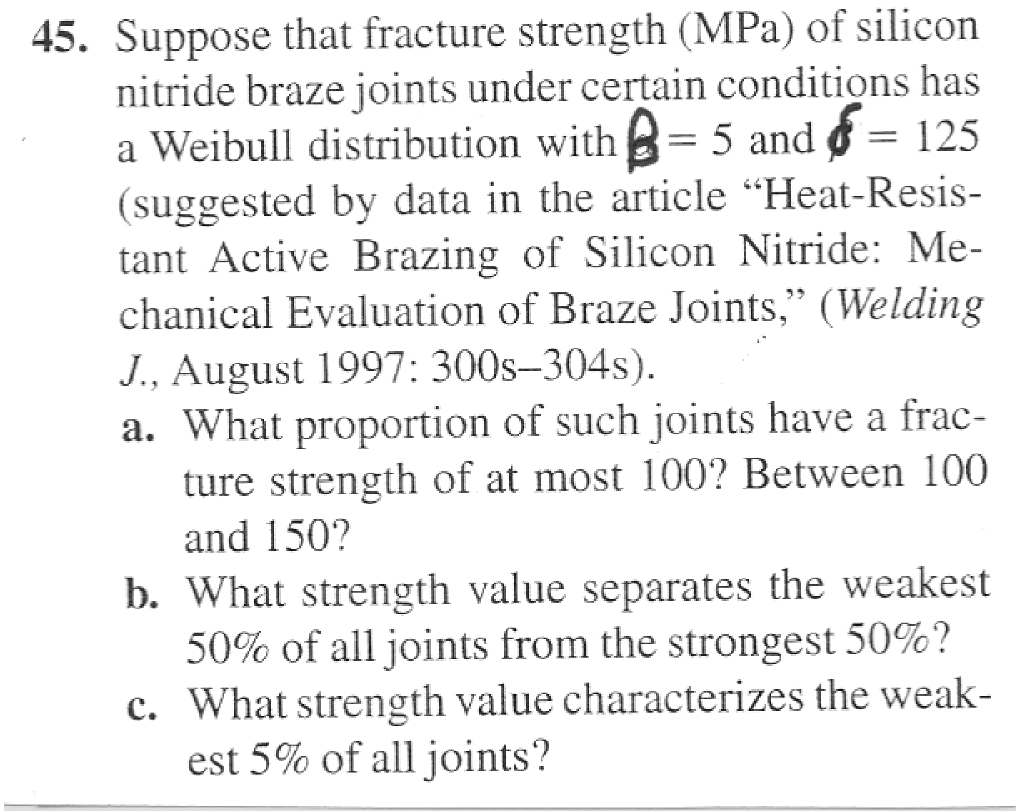
- The mean of the Weibull distribution is

$$\mu = E(X) = \delta\Gamma\left(1 + \frac{1}{\beta}\right)$$

- The variance of the Weibull distribution is

$$\sigma^2 = V(X) = \delta^2\Gamma\left(1 + \frac{2}{\beta}\right) - \delta^2\left[\Gamma\left(1 + \frac{1}{\beta}\right)\right]^2$$

**Weibull Problem**



**45.** Suppose that fracture strength (MPa) of silicon nitride braze joints under certain conditions has a Weibull distribution with $\beta = 5$ and $\delta = 125$ (suggested by data in the article "Heat-Resistant Active Brazing of Silicon Nitride: Mechanical Evaluation of Braze Joints," (*Welding J.*, August 1997: 300s–304s).

**a.** What proportion of such joints have a fracture strength of at most 100? Between 100 and 150?

**b.** What strength value separates the weakest 50% of all joints from the strongest 50%?

**c.** What strength value characterizes the weakest 5% of all joints?

image

# Weibull Practice Problems