

MANE 3332.03

Section 1

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Subsection 1

Lecture 3, January 30

Agenda

- Course Questions
- Start Chapter 2 lecture

Handouts

- Lecture 3 Slides
- Lecture 3 Marked Slides

Chapter 2

Our goal is to understand, quantify, and model the type of variations we encounter. When we incorporate the variation into our thinking and analyses, we can make the informed judgments from our results that are not invalidated by the variation.

Fundamental Definitions

Random experiment $\rightarrow$ stochastic

is an experiment that can result in different outcomes, even though it is repeated in the same manner

Sample Space

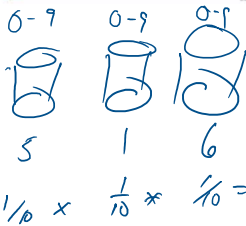
is the set of all possible outcomes of a random experiment

Event

is a subset of the sample space of a random experiment

Examples of Random Experiments, Sample Space, Events

- Consider the bead bowl
- Consider the Texas Lottery's Pick Three game (I am not encouraging gambling)



Assumption:
Independent events
Multiple probabilities

Latest Estimated Jackpots:
Mega Millions: \$42 Million for 01/20/04
Lotto Texas: \$55 Million for 01/21/04
Texas Lotto: \$400,000 for 01/20/04

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How to Play Pick 3

Play Pick 3 Twice A Day! Pick 3 is a twice daily game offering a 50-cent play, twelve drawings a week (six Day and six Night drawings) and a top prize of \$500 (on a \$1 play.)

It's easy to play. Just pick three numbers from "0" to "9", choose how you want to play them, the number of drawings you want to play and the time of day you want to play. Pick 3 drawings are held twice daily at 12:27 p.m. and 10:12 p.m. Central Time. Pick 3, 2 Times A Day, 2 Times The Fun!

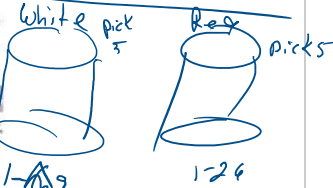
How to Play

First, pick three numbers from "0" to "9" or mark the QP (Quick Pick) box and your numbers will be randomly selected for you.

Second, select how you want to play your three numbers [Exact Order, Any Order, Exact/Any Order, Combo].

If You Play	For	And Numbers Drawn Are	You Win
Exact Order 3 like numbers 516 Odds 1:1,000	\$50 \$1.00	516	\$250 \$500
Any Order 2 like numbers 655 Odds 1:333	\$50 \$1.00	665 566 656	\$80 \$160
Any Order 3 different numbers 516 Odds 1:167	\$50 \$1.00	615 661 516 561 165 156	\$40 \$80
Exact/ Any Order 2 like numbers 787 Odds 1:333	\$50 Exact Order \$50 Any Order \$1.00	797 Exact Order 797 977 779 Any order	\$250+\$80=\$330 Pace both exact order & any order when 797 is drawn \$80
Exact/ Any Order 3 different numbers 654 Odds	\$50 Exact Order \$50 Any Order \$1.00	654 Exact Order 645 654 465 Any order	\$250+\$40=\$290 Pace both exact order & any order when 654 is drawn

Power Ball



Not independent events

Simplifying without Replacement

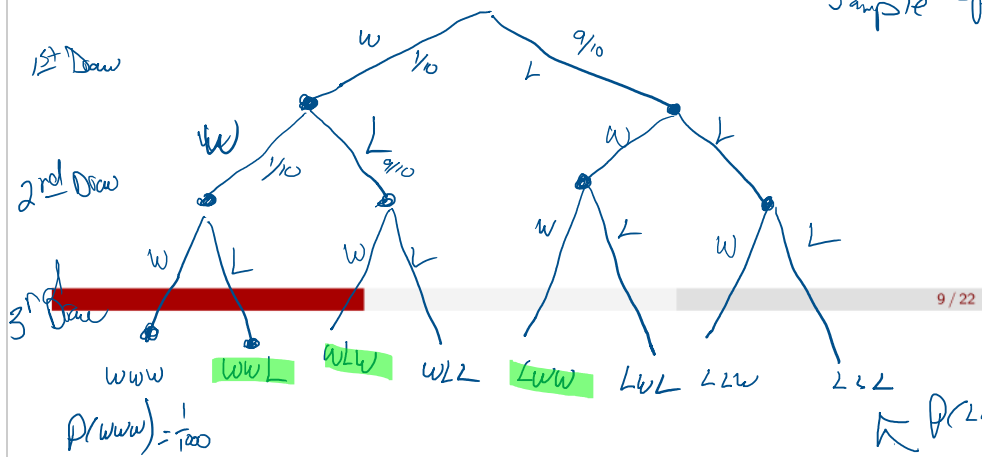
Tree Diagrams

exact order (5/16)

3 Drawings: W (winner) or L (loser)

- Tree diagrams are a useful tool for understanding sample spaces and events. Apply to Pick Three game.

Sample Space contains 8



Probability

Statisticians use decimal numbers
not percentage

- The probability of an event is the likelihood that it occurs
- Probability is expressed as a number between 0 and 1
- Probability of an event can be found by dividing the number of outcomes of the desired events divided by the total number of outcomes in the sample space (if all events are equally likely)

Counting Techniques

- Consider ordered versus unordered subsets
- Ordered subsets (Permutations)

Board of directors
Pres, VP, Treasures

uncommon
in statistics

$$P_r^n = \frac{n!}{(n-r)!}$$

- Unordered subsets (Combinations)

3 member (Jose, Leo, Miguel)

$C_{n,r}$ or C_r^n

$$C_r^n = \frac{n!}{r!(n-r)!}$$

$= \binom{n}{r}$ n choose r

- Good idea to do a calculator check

$$\binom{3}{2} = \frac{3!}{2!(3-2)!}$$

11 / 22

$3! = 3 \cdot 2!$ recursive
stopping point $0! = 1$

$$= \frac{3 \cdot 2!}{2! \cdot (1!)}$$

$$= 3$$

Calculators

Axioms (Rules) of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

① $P(S) = 1$

② $0 \leq P(E) \leq 1$

③ For two events E_1 and E_2 with $E_1 \cap E_2 = \emptyset$

E_1 intersect E_2 contains the empty (null) set

Special Case

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

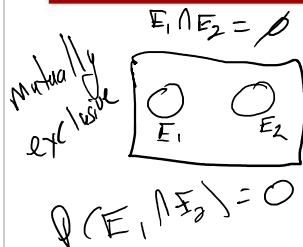
- Consider problem 2-70

union

Venn Diagrams

(always true)
General Rule

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$



Practice Problems - Single Event

Compliment of an Event



$$\begin{aligned} P(W') &= 1 - P(W) \\ &= 1 - \left(\frac{1}{10}\right) = \frac{9}{10} \end{aligned}$$

A Word of Warning

- It usually looks very easy when I work a problem
- I have been using statistics for almost 40 years
- This is something you **MUST** practice
- Rework class room examples and textbook examples

Probability of Multiple Events

Λ -cop

$P(A \cap B)$ is "the probability of A and B occurring

V - cup

$P(A \cup B)$ is "the probability of A or B (or both)"

$P(A')$ is "the probability of not A "

- Venn diagrams are a very useful tool for understanding multiple events and calculating probabilities

A - red grid
B - 1 on 4-sided die

trial 1

A - True (7-Herts)
B - False (3)

$(A \cap B) \rightarrow F$
 $(A \cup B) \rightarrow \text{True}$

trial (Block, 3)

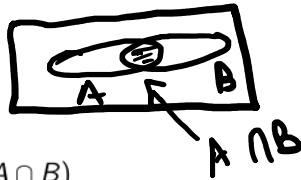
$(A \cap B) \rightarrow \text{False}$
 $(A \cup B) \rightarrow \text{False}$

Size of Sample space?
 $2 \times 4 = 8$

Addition Rules

Venn Diagrams

- Used to calculate the union of two events



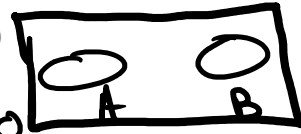
General Rule

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If two events are mutually exclusive ($A \cap B = \emptyset$)

Special Case

$$P(A \cup B) = P(A) + P(B)$$



- Consider problems 2-82 and 2-85

Addition Rule for 3 or More Events

- For three events

$$P(A \cup B \cup C) =$$

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \cdot 3 \cdot 2!}{2! \cdot 2!} = 6$$

$$P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- For a set of events to mutually exclusive all pairs of variables must satisfy $E_1 \cap E_2 = \emptyset$
- For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \dots \cup E_k) = P(E_1) + P(E_2) + \dots + P(E_k)$$

~~P(A \cap B)~~

$$P(A \cup B \cup C \cup D)$$

$$= P(A) + P(B) + P(C) + P(D) - P(A \cap B) - P(A \cap C) - P(A \cap D) - P(B \cap C) - P(B \cap D) - P(C \cap D) + P(A \cap B \cap C) + P(A \cap B \cap D) + P(A \cap C \cap D) + P(B \cap C \cap D) - P(A \cap B \cap C \cap D)$$

Conditional Probability

- Hayter (2002) states that “For experiments with two or more events of interest, attention is often directed not only at the probabilities of individual events but also at the probability of an event occurring **conditional** on the knowledge that another event has occurred.”
- The **conditional probability** of an event B given an event A , denoted $P(B|A)$ is

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

for $P(A) > 0$

- Consider problems 2-99

Multiplication Rules

- This rule provides another method for calculating $P(A \cap B)$

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

- This leads to the total probability rule

$$\begin{aligned} P(B) &= P(B \cap A) + P(B \cap A') \\ &= P(B|A)P(A) + P(B|A')P(A') \end{aligned}$$

- Consider problems from 3rd edition (next slide) and 2-129

Example Problem 2-76

✓ 2-76. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2 and 1% of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

Figure 3: problem 2-76

Independent Events

- Two events are independent if any one of the following is true:
 - 1 $P(A|B) = P(A)$
 - 2 $P(B|A) = P(B)$
 - 3 $P(A \cap B) = P(A)P(B)$
- Consider problem 2-146

Reliability Analysis

- Reliability is the application of statistics and probability to determine the probability that a system will be working properly
- Components can be arranged in series. All components must work for the system to work.

$$P(\text{system works}) = P(A \text{ works})P(B \text{ works})$$

- Components can be arranged in parallel. As long as one component works, the system works.

$$P(\text{system works}) = 1 - (1 - P(A \text{ works})) \times (1 - P(B \text{ works}))$$

- Consider problem 2-157

Economics of Pick 3

Thursday, January 30, 2025 11:23 AM

Only exact match

Pay \$1 if right win \$500
 if wrong ~~lose~~ lose dollar

$$\frac{\text{winning}}{(500-1) \left(\frac{1}{1000}\right)} + -1 \left(\frac{999}{1000}\right) = \text{expected winning} \\ = -.50$$

Single event pp

Tuesday, February 4, 2025 11:07 AM

QUESTION 1

Consider a problem classified by 2 rows and 3 columns containing 200 observations. The table is described in the figure below and has the following cell counts: A=109, B=52, C=14,

D=12, E=8, F=5. Let event S denote an item that occurs in row 1. Find P(S).

	column 1	column 2	column 3
row 1	A	B	C
row 2	D	E	F

$n = 200$

109	52	14
12	8	5

☒ The correct answer is not provided

- ☐ 0.605
- ☐ 0.3
- ☐ 0.6904
- ☐ 0.4001
- ☐ 0.095
- ☐ 0.125

$$P(S) = \frac{A+B+C}{n} = \frac{109+52+14}{200} = 0.875$$

Screen clipping taken: 2/4/2025 11:08 AM

QUESTION 3

Consider a problem classified by 2 rows and 3 columns containing 500 observations. The table is described in the figure below and has the following cell counts: A=306, B=47, C=92,

D=17, E=26, F=12. Let event T denote an item that occurs in column 3. Find $P(T')$.

	column 1	column 2	column 3	
row 1	A	B	C	306 47 92
row 2	D	E	F	17 26 12

☐ The correct answer is not provided

☐ 0.646

☐ 0.354

☐ 0.146

☐ 0.854

☒ 0.792

$$\begin{aligned}
 P(T') &= 1 - P(T) \\
 &= 1 - \left(\frac{C+F}{n} \right) = 1 - \frac{92+12}{500} = .792
 \end{aligned}$$

$n = 500$

Single event pp

Tuesday, February 4, 2025 11:14 AM

QUESTION 5

Consider a problem classified by 2 rows and 3 columns containing 1000 observations. The table is described in the figure below and has the following cell counts: A=317, B=237, C=270,

D=11, E=46, F=119. Let event T denote an item that occurs in column 1. Find P(T).

	column 1	column 2	column 3	
row 1	A	B	C	317 237 270
row 2	D	E	F	11 46 119

$n = 1000$

- ☐ 0.283
- ☐ 0.389
- ☒ 0.328
- ☐ The correct answer is not provided
- ☐ 0.672

$$P(T) = \frac{A+D}{n} = \frac{317+11}{1000} = .328$$

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QUESTION 7

Consider a problem classified by 2 rows and 3 columns containing 300 observations. The table is described in the figure below and has the following cell counts: A=115, B=55, C=5,

D=16, E=12, F=97. Let event S denote an item that occurs in row 1. Find $P(S)$.

	column 1	column 2	column 3
row 1	A	B	C
row 2	D	E	F

115 55 5
16 12 97

$$n = 300$$

- ☐ 0.5833
- ☐ 0.6883
- ☒ 0.4167
- ☐ 0.34
- ☐ The correct answer is not provided
- ☐ 0.4367
- ☐ 0.2233

$$P(S') = 1 - P(S) = 1 - \frac{A+B+C}{n}$$

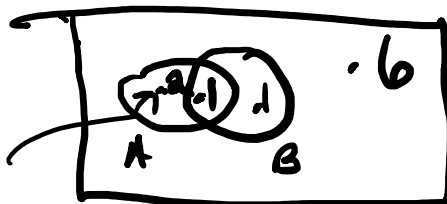
$$= 1 - \frac{115 + 55 + 5}{300} = 0.21\overline{66}$$

$$\frac{16 + 12 + 97}{300} = .41\overline{66}$$

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$$P(A) = .3, P(B) = .2, P(A \cap B) = .1$$

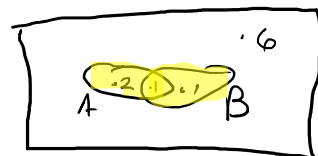
$$\begin{aligned} a) P(A') &= 1 - P(A) \\ &= 1 - .3 = .7 \end{aligned}$$



not to scale



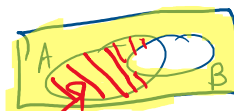
$$\begin{aligned} b) P(A \cup B) &= P(A) + P(B) \\ &\quad - P(A \cap B) \\ &= .3 + .2 - .1 = .4 \end{aligned}$$



$$c) P(A' \cap B) = .1$$



$$d) P(A \cap B')$$



.2

$$\begin{aligned} f) P(A' \cup B) &= P(A') + P(B) - P(A' \cap B) \\ &= (1 - .3) + .2 - .1 = .8 \end{aligned}$$



$$\rightarrow .6 + .1 + .1 = .8$$