Printout

Wednesday, January 29, 2025 10:50 AM

Section 1

MANE 3332.03

Subsection 1

Lecture 3, January 30

Agenda

- Course Questions
- Start Chapter 2 lecture

Handouts

- Lecture 3 Slides
- Lecture 3 Marked Slides

Chapter 2

Our goal is to understand, quantify, and model the type of variations we encounter. When we incorporate the variation into our thinking and analyses, we can make the informed judgments from our results that are not invalidated by the variation.

Fundamental Definitions

Random experiment → stochastic

is an experiment that can result in different outcomes, even though it is repeated in the same manner

Sample Space

is the set of all possible outcomes of a random experiment

Event

is a subset of the sample space of a random experiment

Examples of Random Experiments, Sample Space, Events

- Consider the bead bowl
- Consider the Texas Lottery's Pick Three game (I am not encouraging gambling)

0-9 0-9 0-9

5 1 6

$\frac{1}{10} \times \frac{1}{10} * \frac{1}{10} = \frac{1}{1000}$

Assumption: Independent events

multiple probabilities

Latest Estimated Jackpots:
$42 Million for 01/20/04
$55 Million for 01/21/04
$ 400,000 for 01/23/04

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How to Play Pick 3

Play *Pick 3* **Twice A Day!** *Pick 3* is a twice daily game offering a 50-cent play, twelve drawings a week (six Day and six Night drawings) and a **top prize of $500** (on a $1 play.)

It's easy to play. Just pick three numbers from "0" to "9", choose how you want to play them, the number of drawings you want to play and the time of day you want to play. *Pick 3* drawings are held twice daily at 12:27 p.m. and 10:12 p.m. Central Time. *Pick 3*, 2 Times A Day, 2 Times The Fun!

How to Play

First, pick three numbers from "0" to "9" or mark the QP (Quick Pick) box and your numbers will be randomly selected for you.

Second, select how you want to play your three numbers (Exact Order, Any Order, Exact/Any Order, Combo).

If You Play	For	And Numbers Drawn Are	You Win
Exact Order 516 Odds 1:1,000	$.50 $1.00	516	$250 $500
Any Order 2 like numbers 665 Odds 1:333	$.50 $1.00	665 566 656	$80 $160
Any Order 3 different numbers 516 Odds 1:167	$.50 $1.00	615 651 516 561 165 156	$40 $80
Exact/ Any Order 2 like numbers 797 Odds 1:333	$.50 Exact Order $.50 Any Order $1.00	797 Exact Order 797 977 779 Any order	$250+$80=$330 Pays both exact order & any order when 797 is drawn; $80
Exact/ Any Order 3 different numbers 654 Odds	$.50 Exact Order $.50 Any Order $1.00	654 Exact Order 645 654 465 456 564 546 Any order	$250+$40=$290 Pays both exact order & any order when 654 is drawn

1 of 3

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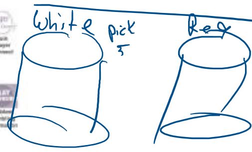
Power Ball

White pick 5 Red picks



1-69 1-26

~~not~~ Ind equaled events

Sampling without Replacement

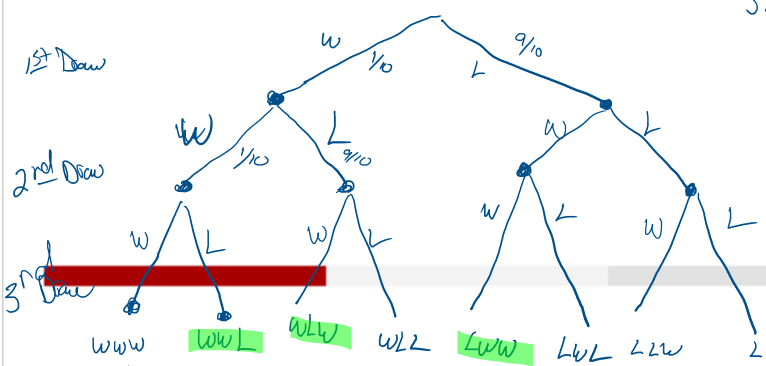
Tree Diagrams

exact order (5/6)

3 Drawings : W(inner) or L(oser)

- Tree diagrams are a useful tool for understanding sample spaces and events. Apply to Pick Three game.

Sample Space contains $\underline{8}$



$P(WWW) = \frac{1}{1000}$

$P(LLL) = \left(\frac{9}{10}\right)^3$

$= .729$

Probability

Statisticians use decimal numbers not percentage

- The probability of an event is the likelihood that it occurs
- Probability is expressed as a number between 0 and 1
- Probability of an event can be found by dividing the number of outcomes of the desired events divided by the total number of outcomes in the sample space (if all events are equally likely)

Counting Techniques

- Consider ordered versus unordered subsets
- Ordered subsets (Permutations)

Board of diretors
Pres, VP, Treasures

uncommon in statistics

$$P_r^n = \frac{n!}{(n-r)!}$$

3 member (Jose, Leo, Miguel)

- Unordered subsets (Combinations)

$C_{n,r}$ or C_r^n

$$C_r^n = \frac{n!}{r!\,(n-r)!} = \binom{n}{r}$$ n choose r

- Good idea to do a calculator check

$$\binom{3}{2} = \frac{3!}{2!\,(3-2)!}$$

$3! = 3 \cdot 2!$ recursive

stopping point $\emptyset! = 1$

$$= \frac{3 \cdot \cancel{2!}}{\cancel{2!}\,(1!)}$$

$$= 3$$

Calculators

Axioms (Rules) of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If S is the sample space and E is any event in a random experiment,

1. $P(S) = 1$
2. $0 \leq P(E) \leq 1$
3. For two events E_1 and E_2 with $E_1 \cap E_2 = \emptyset$

E1 intercept E2 contains the empty (null) set

Special Case

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

union

(always true)

General Rule

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

- Consider problem 2-70

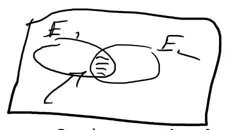
Venn Diagrams

$E_1 \cap E_2 = \emptyset$

mutually exclusive

$P(E_1 \cap E_2) = 0$

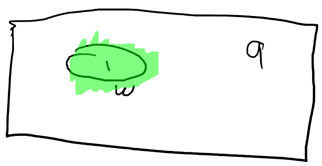
$E_1 \cap E_2 \neq \emptyset$



$E_1 \cap E_2$ (intersection)

Practice Problems - Single Event

Compliment of an Event



$$P(W') = 1 - P(W)$$
$$= 1 - \left(\frac{1}{10}\right) = \frac{9}{10}$$

A Word of Warning

- It usually looks very easy when I work a problem
- I have been using statistics for almost 40 years
- This is something you MUST practice
- Rework class room examples and textbook examples

Probability of Multiple Events → 2 or more

A - red card
B - 1 on 4-sided die

Intersection: ∩ - cap

$P(A \cap B)$ is "the probability of A and B occurring

Union: ∪ - cup

$P(A \cup B)$ is "the probability of A or B (or both)"

Complement:

$P(A')$ is "the probability of not A"

- Venn diagrams are a very useful tool for understanding multiple events and calculating probabilities

trial 1

A - True (7-Hearts)
B - False (3)

$(A \cap B) \rightarrow$ F
$(A \cup B) \rightarrow$ True

trial (Black, 3)

$(A \cap B) \rightarrow$ False
$(A \cup B) \rightarrow$ False

Size of Sample Space?
$2 \times 4 = 8$

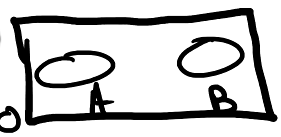
Addition Rules

Venn Diagrams

- Used to calculate the union of two events

General Rule
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If two events are mutually exclusive ($A \cap B = \emptyset$)



Special Case
$$P(A \cup B) = P(A) + P(B)$$

- Consider problems 2-82 and 2-85

Addition Rule for 3 or More Events

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \cdot 3 \cdot \cancel{2!}}{2! \cancel{2!}} = 6$$

- For three events

$$\begin{aligned} P(A \cup B \cup C) = & \quad P(A) + P(B) + P(C) \\ & -P(A \cap B) - P(A \cap C) - P(B \cap C) \\ & +P(A \cap B \cap C) \end{aligned}$$

- For a set of events to mutually exclusive all pairs of variables must satisfy $E_1 \cap E_2 = \emptyset$
- For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \ldots \cup E_k) = P(E_1) + P(E_2) + \cdots + P(E_k)$$

$\cancel{P(A \cap B}$

$$\begin{aligned} P(A \cup B \cup C \cup D) = & \; P(A) + P(B) + P(C) + P(D) \\ & - P(A \cap B) - P(A \cap C) - P(A \cap D) \\ & - P(B \cap C) - P(B \cap D) - P(C \cap D) \\ & + P(A \cap B \cap C) + P(A \cap B \cap D) + P(A \cap C \cap D) \\ & + P(B \cap C \cap D) \\ & - P(A \cap B \cap C \cap D) \end{aligned}$$

Conditional Probability

- Hayter (2002) states that "For experiments with two or more events of interest, attention is often directed not only at the probabilities of individual events but also at the probability of an event occurring **conditional** on the knowledge that another event has occurred."
- The **conditional probability** of an event B given an event A, denoted $P(B|A)$ is

$B/A \rightarrow B$ given A

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

for $P(A) > 0$

- Consider problems 2-99

Multiplication Rules

- This rule provides another method for calculating $P(A \cap B)$

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

- This leads to the total probability rule

$$\begin{aligned} P(B) &= P(B \cap A) + P(B \cap A') \\ &= P(B|A)P(A) + P(B|A')P(A') \end{aligned}$$

- Consider problems from 3rd edition (next slide) and 2-129

Bayesian Statistics

Conditional Probability

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

A - 1st Card Ace
B - 2nd Card Ace

$P(B|A) = \dfrac{3}{51}$

$P(A) = \dfrac{4}{52}$

$P(B|A') = \dfrac{4}{51}$

$P(A') = \dfrac{48}{52}$

$P(B) = \dfrac{3}{51}\left(\dfrac{4}{52}\right) + \dfrac{4}{51}\left(\dfrac{48}{52}\right) = .07692$

Example Problem 2-76

E/ 2-76. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2 and 1% of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

Figure 3: problem 2-76

$P(\text{break} \mid \text{small}) = .02$

$P(\text{break} \mid \text{large}) = .01$

$P(\text{large}) = .6$

$P(\text{Small}) = .4$

$P(\text{break}) = P(\text{break} \mid \text{small})P(\text{small}) + P(\text{break} \mid \text{Large})P(\text{large})$

$= .02(.4) + .01(.6)$

$= .014$

Independent Events

if one is true, all are true

- Two events are independent if any one of the following is true:
  1. $P(A|B) = P(A)$
  2. $P(B|A) = P(B)$
  3. $P(A \cap B) = P(A)P(B)$
- Consider problem 2-146

Reliability Analysis

- Reliability is the application of statistics and probability to determine the probability that a system will be working properly
- Components can be arranged in series. All components must work for the system to work.

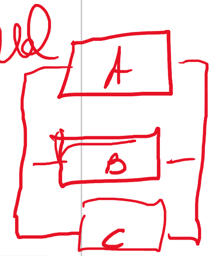
Series

$$P(\text{system works}) = P(A \text{ works})P(B \text{ works})$$

← if independent

- Components can be arranged in parallel. As long as one component works, the system works.

Parallel



$$P(\text{system works}) = 1 - (1 - P(A \text{ works})) \times 1 - P(B \text{ works}))$$

$\times\ 1 - P(\text{works}))$

- Consider problem 2-157

Economics of Pick 3

Thursday, January 30, 2025 11:23 AM

Only exact match

Pay $1 if right win $500
if wrong ~~loose~~ loss dollar

$$\frac{\text{winning}}{(500-1)\left(\frac{1}{1000}\right)} + -1\left(\frac{999}{1000}\right) = \text{expected winning}$$

$$= -.50$$

Single event pp

Tuesday, February 4, 2025 11:07 AM

QUESTION 1

Consider a problem classified by 2 rows and 3 columns containing 200 observations. The table is described in the figure below and has the following cell counts: A=109, B=52, C=14, D=12, E=8, F=5. Let event S denote an item that occurs in row 1. Find P(S).

	Column 1	column 2	Column 3
row 1	A	B	C
row 2	D	E	F

$n = 200$

109	52	14
12	8	5

- The correct answer is not provided
- 0.605
- 0.3
- 0.6904
- 0.4001
- 0.095
- 0.125

$$P(S) = \frac{A+B+C}{n} = \frac{109+52+14}{200} = 0.875$$

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Tuesday, February 4, 2025 11:10 AM

QUESTION 3

Consider a problem classified by 2 rows and 3 columns containing 500 observations. The table is described in the figure below and has the following cell counts: A=306, B=47, C=92, D=17, E=26, F=12. Let event T denote an item that occurs in column 3. Find P(T').

	Column 1	column 2	Column 3
row 1	A	B	C
row 2	D	E	F

$n = 500$

306	47	92
17	26	12

- The correct answer is not provided
- 0.646
- 0.354
- 0.146
- 0.854
- 0.792

$$P(T') = 1 - P(T)$$
$$= 1 - \left(\frac{C+F}{n}\right) = 1 - \frac{92+12}{\cancel{200}\ 500} = .792$$

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Single event pp

Tuesday, February 4, 2025 11:14 AM

QUESTION 5

Consider a problem classified by 2 rows and 3 columns containing 1000 observations. The table is described in the figure below and has the following cell counts: A=317, B=237, C=270, D=11, E=46, F=119. Let event T denote an item that occurs in column 1. Find P(T).

	Column 1	column 2	Column 3
row 1	A	B	C
row2	D	E	F

n = 1000

317	237	270
11	46	119

- 0.283
- 0.389
- 0.328
- The correct answer is not provided
- 0.672

$$P(T) = \frac{A+D}{n} = \frac{317+11}{1000} = .328$$

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Tuesday, February 4, 2025 11:16 AM

QUESTION 7

Consider a problem classified by 2 rows and 3 columns containing 300 observations. The table is described in the figure below and has the following cell counts: A=115, B=55, C=5, D=16, E=12, F=97. Let event S denote an item that occurs in row 1. Find P(S').

	Column 1	column 2	column 3
row 1	A	B	C
row2	D	E	F

115 55 5

16 12 97

$n = 300$

- 0.5833
- 0.6883
- X 0.4167
- 0.34
- The correct answer is not provided
- 0.4367
- 0.2233

$$P(S') = 1 - P(S) = 1 - \frac{A+B+C}{n}$$

$$= 1 - \frac{115+55+5}{300} = 0.41\overline{66}$$

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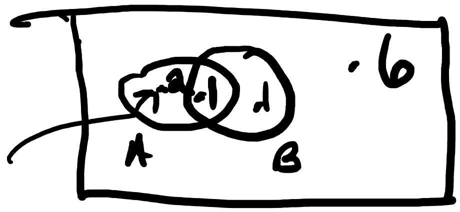
$$\frac{16+12+97}{300} = .41\overline{66}$$

Problem 2.4.2

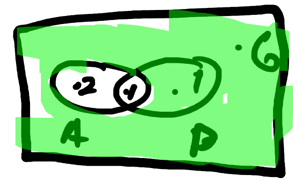
Tuesday, February 4, 2025 11:47 AM

$P(A) = .3$, $P(B) = .2$ $P(A \cap B) = .1$

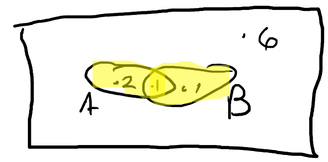
a) $P(A') = 1 - P(A)$
$= 1 - .3 = .7$



not to scale

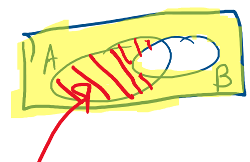


b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= .3 + .2 - .1 = .4$



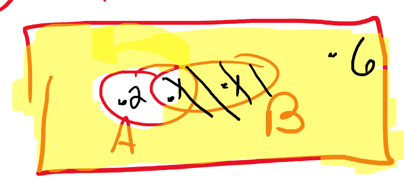
c) $P(A' \cap B) = .1$

d) $P(A \cap B')$



.2

~~e~~ f) $P(A' \cup B) = P(A') + P(B) - P(A' \cap B)$
$(1 - .3) + .2 - .1 = \underline{.8}$



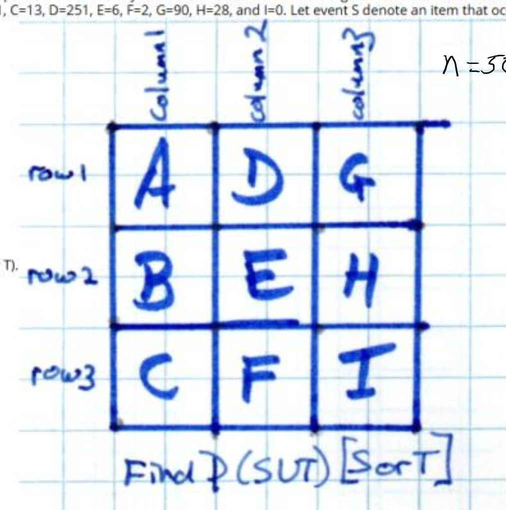
$\rightarrow .6 + .1 + .1 = .8$

Two event pp

Thursday, February 6, 2025 11:13 AM

Consider a problem classified by 3 rows and 3 columns containing 500 observations. The table is described in the figure below and has the following cell counts: A=69, B=41, C=13, D=251, E=6, F=2, G=90, H=28, and I=0. Let event S denote an item that occurs in row 2 and event T denote an item that occurs in column 1.

Find P(S or T).



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	69	251	90	410
S	41	6	28	75
	13	2	0	15
T	123	259	118	500

~~322~~

$$P(S \cup T) = P(S) + P(T) - P(S \cap T)$$

$$= \frac{75}{500} + \frac{123}{500} - \frac{41}{500}$$

$$= .314$$

Highlighted Cells

$$\frac{69+41+13+6+28}{500} = \frac{157}{500}$$

$$= .314$$

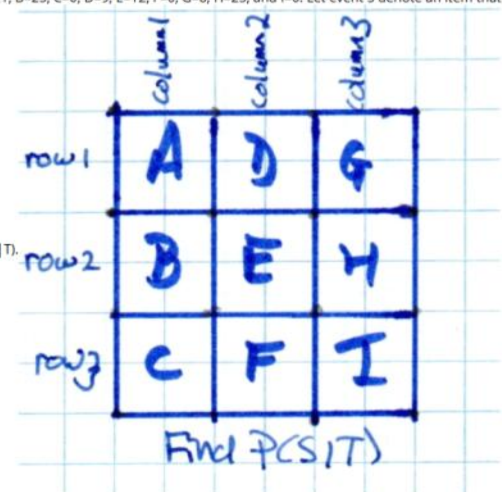
Two events pp

Thursday, February 6, 2025 11:25 AM

QUESTION 3

$n = 100$

Consider a problem classified by 3 rows and 3 columns containing 100 observations. The table is described in the figure below and has the following cell counts: A=21, B=25, C=0, D=9, E=12, F=0, G=8, H=25, and I=0. Let event S denote an item that occurs in row 2 and event T denote an item that occurs in column 3. Find P(S|T).



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	21	9	8	
S →	25	12	25	62
	0	0	0	
			T ↑ 33	

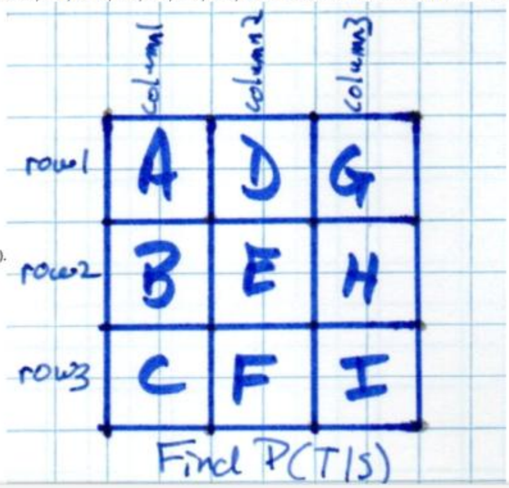
$$P(S|T) = \frac{P(S \cap T)}{P(T)} = \frac{25/100}{33/100} = 0.75758$$

Two events

Thursday, February 6, 2025 11:31 AM

QUESTION 5

Consider a problem classified by 3 rows and 3 columns containing 100 observations. The table is described in the figure below and has the following cell counts: A=36, B=40, C=0, D=11, E=0, F=0, G=6, H=7, and I=0. Let event S denote an item that occurs in row 3 and event T denote an item that occurs in column 1. Find P(T|S).



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$n = 100$

	36	11	6
	40	0	7
S	0	0	0
	T		

Problem: $S = \emptyset$

$$P(T|S) = \frac{P(S \cap T)}{P(S)}$$

for $P(S) > 0$

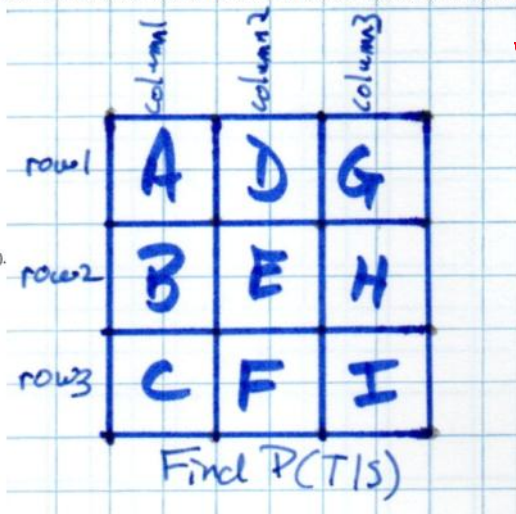
Does not exist! Because $P(S) = 0$

$$\frac{\text{any}}{0} \rightarrow \infty$$

Two events pp

Thursday, February 6, 2025 11:40 AM

Consider a problem classified by 3 rows and 3 columns containing 400 observations. The table is described in the figure below and has the following cell counts: A=393, B=1, C=0, D=4, E=0, F=0, G=0, H=1, and I=1. Let event S denote an item that occurs in row 3 and event T denote an item that occurs in column 1. Find P(T|S).



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$n = 400$

T

393	4	$\emptyset$
1	$\emptyset$	1
$\emptyset$	$\emptyset$	1

S

$$P(T/S) = \frac{P(S \cap T)}{P(S)}$$

$$= \frac{0/400}{1/400} \to \emptyset$$

Two events pp

Thursday, February 6, 2025 11:46 AM

Consider a problem classified by 3 rows and 3 columns containing 400 observations. The table is described in the figure below and has the following cell counts: A=231, B=2, C=0, D=165, E=0, F=0, G=2, H=0, I=0. Let event S denote an item that occurs in row 2 and event T denote an item that occurs in column 1. Find P(S and T).

	column 1	column 2	column 3
row 1	A	D	G
row 2	B	E	H
row 3	C	F	I

Find P(S∩T) [S and T]

	231	165	2
S	2	0	0
	0	0	0
	T		

$$P(S \cap T) = 2/400 = \frac{1}{200}$$

$$= .005$$

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Independent events

Tuesday, February 11, 2025 11:20 AM

Shock R.

		High	Low	
Scratch Resistance	High	70	9	79 ← B
	Low	16	5	21
		86	14	100
		A		

are A&B independent

$$P(A \mid B) = P(A)$$

① $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{70/100}{79/100} = \dfrac{70}{79}$

② $P(B) = \cancel{77}\ \dfrac{79}{100}$

$= ?$

not equal, so the events are not independent

Series

Tuesday, February 11, 2025 11:30 AM

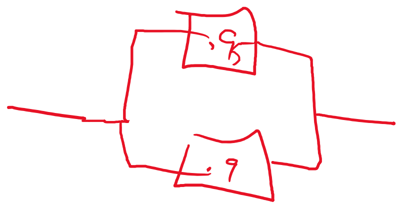
$-\boxed{.8}-\boxed{.1}-$ {independent}

P(works) = .8(.1) = .08

$-\boxed{.8}-\boxed{.9}-$ → .8(.9) = .72

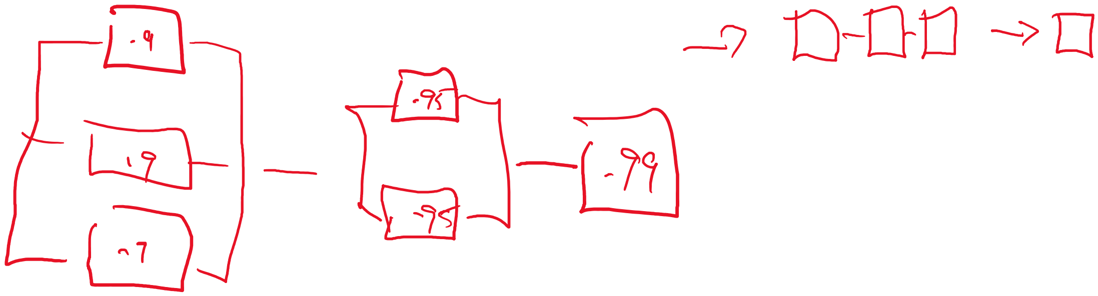
Parallel

Tuesday, February 11, 2025 11:33 AM

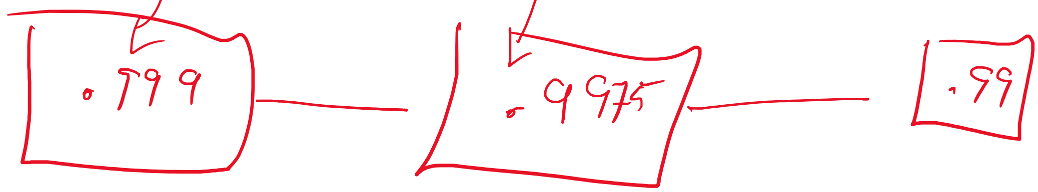


$$P(\text{system works}) = 1 - [(1-.95)(1-.9)]$$

$$= .995$$



$1 - [(1-.9)^3]$ $\quad$ $1 - [(1-.95)^2]$ $\quad$ $.99$



$P(\text{system works}) = .98651 = .999(.9975)(.99)$