# Printout

Wednesday, April 2, 2025 1:15 PM



## Section 1

## MANE 3332.03

## Subsection 1

# Chapter Seven

# Handouts

- Chapter 7 Slides
- Chapter 7 Slides marked

## Chapter 7 Overview

- Chapter 7 contains a detailed explanation of point estimates for parameters
- Much of this chapter is of a highly statistical nature and will not be covered in this course
- Key concepts we will discuss are:
  - Statistical inference
  - Statistic
  - Sampling distribution
  - Point estimator
  - Unbiased estimate
  - MVUE estimator
  - Central limit theorem

## Statistical Inference

- Montgomery gives the following description of statistical inference. *The field of statistical inference consists of those methods used to make decisions or to draw conclusions about a population. These methods utilize the information contained in a sample from the population in drawing conclusions. This chapter begins our study of the statistical methods used for inference and decision making.*
- Statistical inference may be divided into two major areas: parameter estimation and hypothesis testing

Point estimates

Interval Estimates → Chapters

→ Chapter 9 & 10

## Point Estimate

⇒ single number

- Montgomery states that "In practice, the engineer will use sample data to compute a number that is in some sense a reasonable value (or guess) of the true mean. This number is called a **point estimate**."
- Discuss examples

  hat notation $\hat{\mu} = \bar{x}$
- A formal definition of a point estimate is
  *A **point estimate** of some population parameter $\theta$ is a single numerical value $\hat{\theta}$ of a statistic $\hat{\Theta}$. The statistic $\hat{\Theta}$ is called the point estimate.*

  → function of data
- Notice the use of the "hat" notation to denote a point estimate

## Statistic

- Point estimate requires a sample of random observations, say $X_1, X_2, \ldots, X_n$
- Any function of the sampled random variables is called a statistic
- The function of the random variables is itself a random variable
- Thus, the sample mean $\bar{x}$ and the sample variance $s^2$ are both statistics and random variables

$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

## Properties of point estimators

- We would like point estimates to be both accurate and precise
- An unbiased estimator addresses the accuracy criteria
- A minimum variance unbiased estimator addresses the precision criteria

## Unbiased Estimator

- The point estimator $\hat{\Theta}$ is an **unbiased estimator** for the parameter $\theta$ if

$$E\left(\hat{\Theta}\right) = \theta$$

- If the point estimator is not unbiased, then the difference

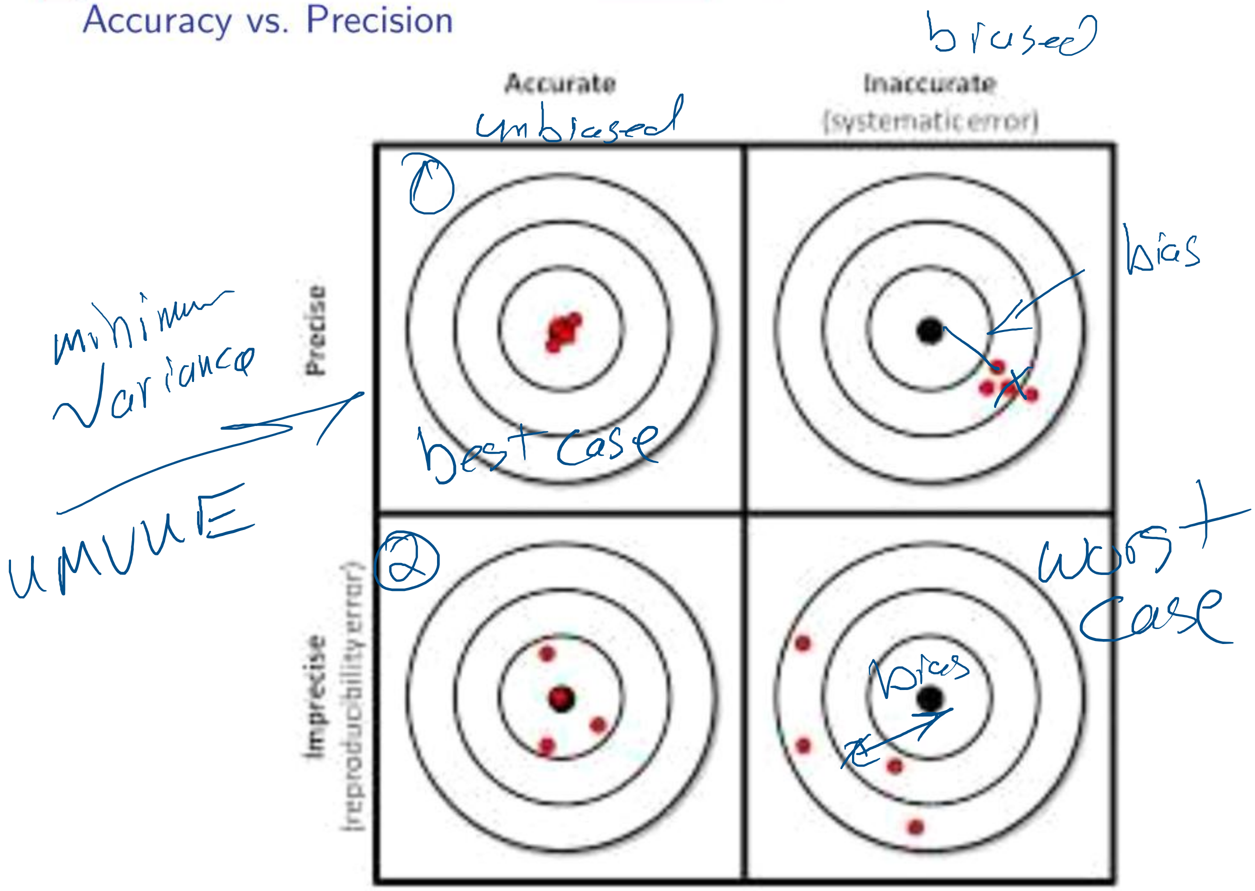
$$\text{bias} = E\left(\hat{\Theta}\right) - \theta$$

is called the **bias** of the estimator $\hat{\Theta}$

## MVUE

- Montgomery gives the following definition of a minimum variance unbiased estimator (MVUE)
  *If we consider all unbiased estimators of $\theta$, the one with the smallest variance is called the minimum variance unbiased estimator*
- An import fact is that the sample mean $\bar{x}$ is the MVUE for $\mu$ when the data comes from a normal distribution

## Accuracy vs. Precision



## Sampling Distribution

- The probability distribution of a statistic is called a **sampling distribution**

$E(x_i) = \mu$

Have data from normal distribution

$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} x_i \quad E(\bar{x}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right)$

$= \frac{1}{n}\sum_{i=1}^{n} E(x_i) = \frac{1}{n} n \mu = \mu$

$V(\bar{X}) = \sum V\left(\frac{1}{n} X_i\right) = \frac{1}{n^2}\sum_{i=1}^{n} V(x_i) = \frac{1}{n^2}(n\sigma^2) = \frac{\sigma^2}{n}$

## Central Limit Theorem

- Definition of the Central Limit Theorem is
  *If $X_1, X_2, \ldots, X_n$ is a random sample of size n taken from a population (either finite or infinite) with mean $\mu$ and finite variance $\sigma^2$, and if $\overline{X}$ is the sample mean, the limiting form of the distribution of*
  $$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$
  *as $n \to \infty$, is the standard normal distribution*
- Important result because for sufficiently large $n$, the sampling distribution of $\overline{X}$ is normally distribution
- This is a fundamental result that will be used extensively in the next four chapters of the textbook.