

MANE 3332.04

## Section 1

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## Subsection 1

Lecture 3, January 29

## Agenda

- Course Questions
- Start Chapter 2 lecture
- Single Event Practice Problems due 1/29/2024

## Handouts

- Lecture 3 Slides
- Lecture 3 Marked Slides

## Chapter 2

*Our goal is to understand, quantify, and model the type of variations we encounter. When we incorporate the variation into our thinking and analyses, we can make the informed judgments from our results that are not invalidated by the variation.*

## Fundamental Definitions

### Random experiment

*is an experiment that can result in different outcomes, even though it is repeated in the same manner*

### Sample Space

*is the set of all possible outcomes of a random experiment*

### Event

*is a subset of the sample space of a random experiment*

Rolling a 20-sided  
d.i.e  
 $\{1, 2, \dots, 20\}$

## Examples of Random Experiments, Sample Space, Events

- Consider the bead bowl
- Consider the Texas Lottery's Pick Three game (I am not encouraging gambling)

[Home](#)
[Latest Estimated Jackpots:](#)

[Mega Millions](#) **\$42 Million** for 01/20/04  
[Lotto Texas](#) **\$55 Million** for 01/20/04  
[Texas Two Step](#) **\$400,000** for 01/20/04

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### How to Play Pick 3

Play Pick 3 Twice A Day! Pick 3 is a twice daily game offering a 50-cent play, twelve drawings a week (six Day and six Night drawings) and a top prize of \$500 (on a \$1 play.)

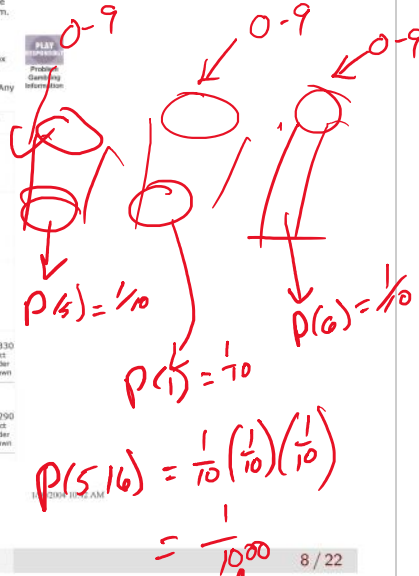
It's easy to play. Just pick three numbers from "0" to "9", choose how you want to play them, the number of drawings you want to play and the time of day you want to play. Pick 3 drawings are held twice daily at 12:27 p.m. and 10:12 p.m. Central Time. Pick 3, 2 Times A Day, 2 Times The Fun!

#### How to Play :

First, pick three numbers from "0" to "9" or mark the QP (Quick Pick) box and your numbers will be randomly selected for you.

Second, select how you want to play your three numbers (Exact Order, Any Order, Exact/Any Order, Combo).

If You Play	For	And Numbers Drawn Are	You Win
Exact Order Odds 516 \$11,000	\$ .50 \$1.00	516	\$250 \$500
Any Order 2 like numbers 665 Odds 1:333	\$ .50 \$1.00	665 566 656	\$80 \$160
Any Order 3 different numbers 516 Odds 1:167	\$ .50 \$1.00	615 651 516 561 165 156	\$40 \$80
Exact/ Any Order 2 like numbers 797 Odds 1:333	\$ .50 Exact Order \$ .50 Any Order \$1.00	797 Exact Order 797 977 779 Any order	\$250+\$80=\$330 Pays both exact order & any order when 797 is drawn \$80
Exact/ Any Order 3 different 654 numbers Odds	\$ .50 Exact Order \$ .50 Any Order \$1.00	654 Exact Order 645 654 465 456 564 546 Any order	\$250+\$40=\$290 Pays both exact order & any order when 654 is drawn



independent events

independent



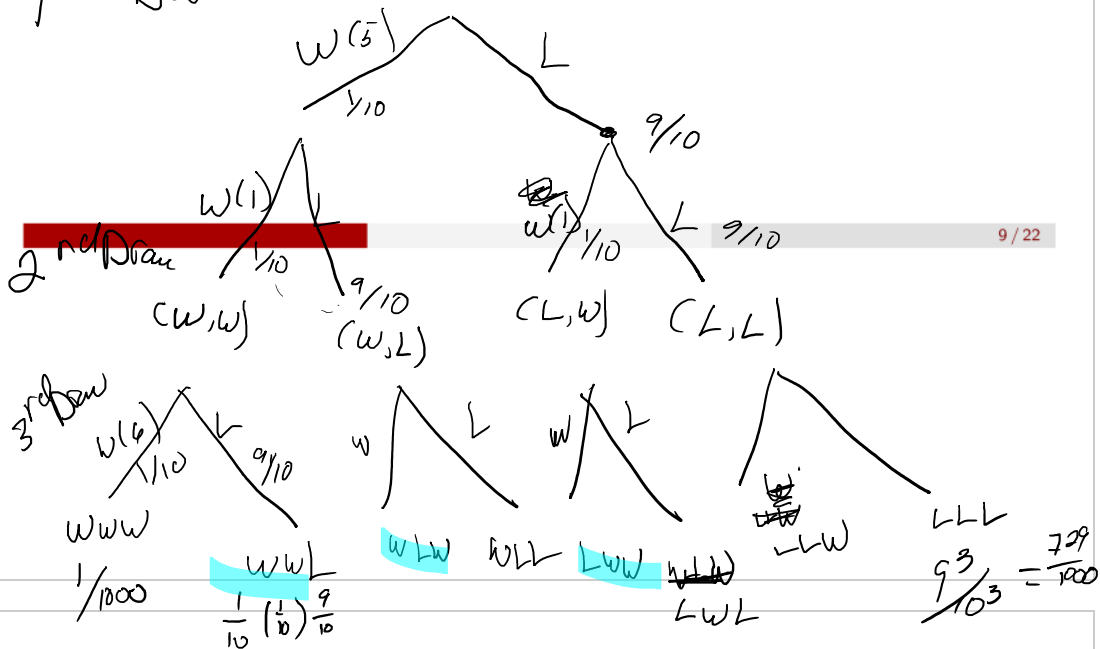
## Tree Diagrams

exact order (5/16)

- Tree diagrams are a useful tool for understanding sample spaces and events. Apply to Pick Three game.

w w w

1st Draw



## Probability

## Probability

Decimal Number, not a Percentage

- The probability of an event is the likelihood that it occurs
- Probability is expressed as a number between 0 and 1
- Probability of an event can be found by dividing the number of outcomes of the desired events divided by the total number of outcomes in the sample space (if all events are equally likely)

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$$

## Counting Techniques

- Consider ordered versus unordered subsets
- Ordered subsets (Permutations)

order is important

$$P_r^n = \frac{n!}{(n-r)!}$$

- Unordered subsets (Combinations)

★  $C_{n,r}$   
 $C_r^n$

order is not important

$$C_r^n = \frac{n!}{r!(n-r)!}$$

$= \binom{n}{r} \rightarrow n \text{ choose } r$

- Good idea to do a calculator check

$$\binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{3 \cdot 2 \cdot 1}{2! \cdot 1!} = 3$$

$$\binom{3}{3} = \frac{3!}{3!(3-3)!} = 1$$

## Axioms (Rules) of Probability

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If  $S$  is the sample space and  $E$  is any event in a random experiment,

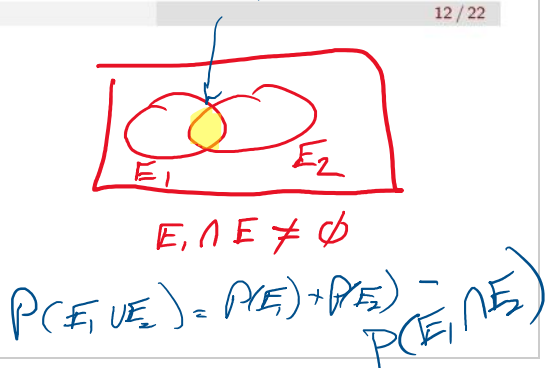
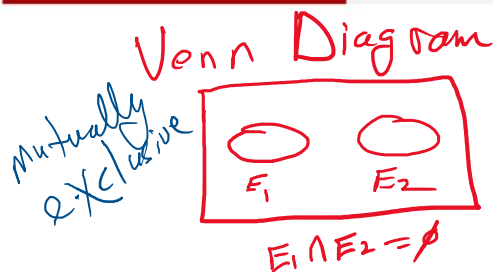
- 1  $P(S) = 1$
- 2  $0 \leq P(E) \leq 1$
- 3 For two events  $E_1$  and  $E_2$  with  $E_1 \cap E_2 = \emptyset$ 

$\rightarrow E_1 \text{ intersect (and) } E_2 \text{ is the empty set}$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

- Consider problem 2-70  $\uparrow$   
 Union of  $E_1$  &  $E_2 \rightarrow E_1 \text{ or } E_2 \text{ or both}$   
 $E_1 \cap E_2$

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## Practice Problems - Single Event

Compliment of an event

$A' \rightarrow$  compliment of  $A$

$$P(A') = 1 - P(A)$$

## A Word of Warning

- It usually looks very easy when I work a problem
- I have been using statistics for almost 40 years
- This is something you MUST practice
- Rework class room examples and textbook examples

→ Practice Problem

Reschedule Single Event  
Practice Problems

## Probability of Multiple Events

2 events

Intersection:

$P(A \cap B)$  is "the probability of A (and) B occurring"

$\cap$  - cap

Union:

$P(A \cup B)$  is "the probability of A or B (or both)"

$\cup$  - cup

Complement:

$P(A')$  is "the probability of not A"

- Venn diagrams are a very useful tool for understanding multiple events and calculating probabilities

A - Red Card  
B - 1 on 6-sided die

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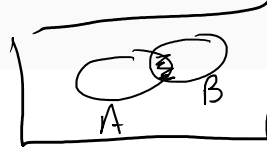
Trial 1  $\rightarrow$  black, 4  
Trial 2  $\rightarrow$  red, 3

How big is Sample Space

is  $A \cap B$  true?  
is  $A \cup B$  true?  
is  $A \cap B$  true?  
is  $A \cup B$  true?

false  
false  
false  
True

## Addition Rules



- Used to calculate the union of two events

general Rule  $\rightarrow$  always true

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If two events are mutually exclusive ( $A \cap B = \emptyset$ )  $\rightarrow$  mutually exclusive

special case

$$P(A \cup B) = P(A) + P(B)$$



- Consider problems 2-82 and 2-85



## Addition Rule for 3 or More Events

$$\binom{3}{2} = 3$$

- For three events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

- For a set of events to mutually exclusive all pairs of variables must satisfy  $E_1 \cap E_2 = \emptyset$
- For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \dots \cup E_k) = P(E_1) + P(E_2) + \dots + P(E_k)$$

$$\checkmark \binom{4}{1} = 4 \quad \binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4 \cdot 3 \cdot 2!}{2! \cdot 2!} = 6$$

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$$\begin{aligned} P(A \cup B \cup C \cup D) &= P(A) + P(B) + P(C) + P(D) \\ &\quad - P(A \cap B) - P(A \cap C) - P(A \cap D) - P(B \cap C) - P(B \cap D) - P(C \cap D) \\ &\quad + P(A \cap B \cap C) + P(A \cap B \cap D) + P(A \cap C \cap D) + P(B \cap C \cap D) \\ &\quad - P(A \cap B \cap C \cap D) \end{aligned}$$

$$\begin{aligned} \binom{4}{3} &= 4 \\ \binom{4}{4} &= 1 \end{aligned} \quad \rightarrow$$

## Conditional Probability

- Hayter (2002) states that “For experiments with two or more events of interest, attention is often directed not only at the probabilities of individual events but also at the probability of an event occurring **conditional** on the knowledge that another event has occurred.”
- The **conditional probability** of an event  $B$  given an event  $A$ , denoted  $P(B|A)$  is

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

for  $P(A) > 0$

- Consider problems 2-99

$B | A$   
↑  
given

## Multiplication Rules

- This rule provides another method for calculating  $P(A \cap B)$

$$P(A \cap B) = P(A|B)P(B) = P(B|A)P(A)$$

- This leads to the total probability rule

$$\begin{aligned} P(B) &= P(B \cap A) + P(B \cap A') \\ &= P(B|A)P(A) + P(B|A')P(A') \end{aligned}$$

- Consider problems from 3rd edition (next slide) and 2-129

## Example Problem 2-76

2-76. Samples of laboratory glass are in small, light packaging or heavy, large packaging. Suppose that 2 and 1% of the sample shipped in small and large packages, respectively, break during transit. If 60% of the samples are shipped in large packages and 40% are shipped in small packages, what proportion of samples break during shipment?

Figure 3: problem 2-76

## Independent Events

- Two events are independent if any one of the following is true:
  - ①  $P(A|B) = P(A)$
  - ②  $P(B|A) = P(B)$
  - ③  $P(A \cap B) = P(A)P(B)$
- Consider problem 2-146

## Reliability Analysis

- Reliability is the application of statistics and probability to determine the probability that a system will be working properly
- Components can be arranged in series. All components must work for the system to work.

$$P(\text{system works}) = P(A \text{ works})P(B \text{ works})$$

- Components can be arranged in parallel. As long as one component works, the system works.

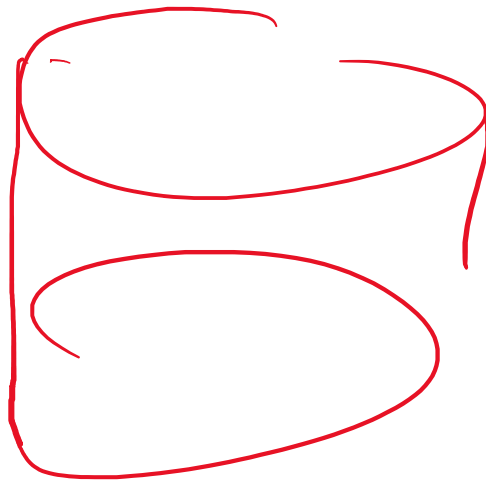
$$P(\text{system works}) = 1 - (1 - P(A \text{ works})) \times 1 - P(B \text{ works}))$$

- Consider problem 2-157

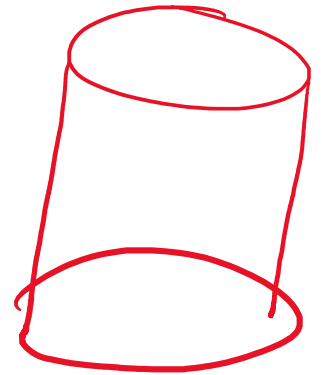
# Non-independent event

Wednesday, January 29, 2025 8:23 AM

white 1-69



Red 1-26



Draw 5  
drawing w/o replacement

exact pick

\$ 1.00  $\rightarrow$  \$500

$$P(\text{win}) = \frac{1}{1000}$$

$$P(\text{lose}) = \frac{999}{1000}$$

Player's

$$\frac{500}{499} \left( \frac{1}{1000} \right) - 1 \left( \frac{999}{1000} \right) \rightarrow -.499$$

Slightly wrong  
even if you win you still  
spend \$100



# Single Event PP

Monday, February 3, 2025 8:18 AM

## QUESTION 1

$$N = 200$$

Consider a problem classified by 2 rows and 3 columns containing 200 observations. The table is described in the figure below and has the following cell counts: A=192, B=3, C=1, D=3,

E=1, F=0. Let event S denote an item that occurs in row 2. Find P(S).

	column 1	column 2	column 3
row 1	A	B	C
row 2	D	E	F

192	3	1
3	1	0

$$\begin{aligned} P(S) &= \frac{3+1+0}{200} \\ &= \frac{4}{200} \\ &= \frac{1}{50} \\ &= .02 \end{aligned}$$

- ☐ 0.975
- ☐ 0.6665
- ☐ 0.98
- ☐ 0.6143
- ☒ The correct answer is not provided
- ☐ 0.463
- ☐ 0.005

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## Single Event PP

Monday, February 3, 2025 8:20 AM

### QUESTION 3

$$n = 100$$

Consider a problem classified by 2 rows and 3 columns containing 100 observations. The table is described in the figure below and has the following cell counts: A=49, B=9, C=24, D=2,

E=14, F=2. Let event T denote an item that occurs in column 1. Find  $P(T)$ .

	column 1	column 2	column 3
row 1	A	B	C
row 2	D	E	F

49  
2

9  
14

24  
2

☐ The correct answer is not provided

☐ 0.26

☐ 0.49

☐ 0.23

☒ 0.51

$$P(T) = \frac{A+D}{n} = \frac{49+2}{100} = \frac{51}{100} = 0.51$$

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QUESTION 5

Consider a problem classified by 2 rows and 3 columns containing 1000 observations. The table is described in the figure below and has the following cell counts: A=63, B=868, C=39,

D=27, E=1, F=2. Let event S denote an item that occurs in row 2. Find  $P(S)$ .

column 1    column 2    column 3

row 1	A	B	C	63	868	39
row 2	D	E	F	27	1	2

- $S \rightarrow \text{row 2}$
- ☐ 0.041
  - ☐ 0.9304
  - ☐ 0.869
  - ☒ 0.97
  - ☐ 0.09
  - ☐ The correct answer is not provided
  - ☐ 0.03

$$\begin{aligned}
 P(S') &= 1 - P(S) \\
 &= 1 - \frac{D+E+F}{1000} = 1 - \frac{27+1+2}{1000} \\
 &= 1 - \frac{30}{1000} = 1 - 0.03 = 0.97
 \end{aligned}$$

## Single event pp

Monday, February 3, 2025 8:28 AM

### QUESTION 7

$n = 200$

Consider a problem classified by 2 rows and 3 columns containing 200 observations. The table is described in the figure below and has the following cell counts: A=5, B=69, C=44, D=1,

E=24, F=57. Let event T denote an item that occurs in column 2. Find  $P(T)$ .

	column 1	column 2	column 3
row 1	A	B	C
row 2	D	E	F

5 69 44  
1 24 57

☒ The correct answer is not provided

☐ 0.495

☐ 0.97

☐ 0.505

☐ 0.4804

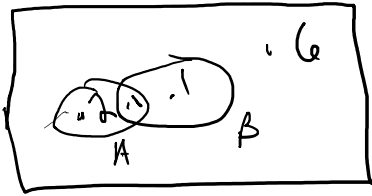
☐ 0.03

$$\begin{aligned} P(T^c) &= 1 - P(T) = 1 - \frac{B+E}{n} \\ &= 1 - \frac{69+24}{200} = 1 - \frac{93}{200} \\ &= \underline{.565 + .535} \end{aligned}$$

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# Problem 2.4.2

Monday, February 3, 2025 8:57 AM



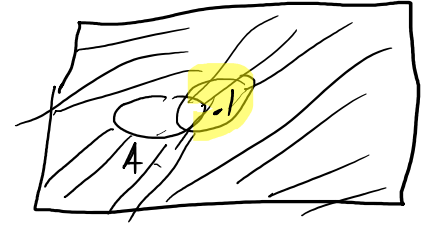
Not to scale

$$\text{If } P(A) = .3, P(B) = .2, P(A \cap B) = .1$$

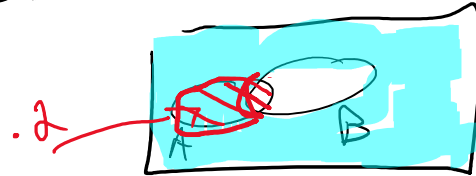
$$a) P(A') = 1 - P(A) = 1 - .3 = .7$$

$$b) P(A \cup B) = P(A) + P(B) - P(A \cap B) = .3 + .2 - .1 = .4$$

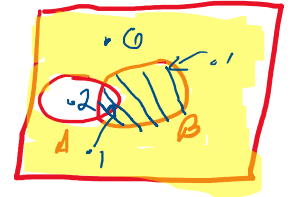
$$c) P(A' \cap B) = .1$$



$$d) P(A \cap B')$$



$$f) P(A' \cup B) = .8$$



$$\begin{aligned} &= P(A') + P(B) - P(A' \cap B) \\ &= (1 - .3) + (.2) - .1 \\ &= .8 \end{aligned}$$

## 2 event pp

Monday, February 10, 2025 8:11 AM

Consider a problem classified by 3 rows and 3 columns containing 300 observations. The table is described in the figure below and has the following cell counts: A=147, B=11, C=0, D=14, E=0, F=0, G=128, H=0, and I=0. Let event S denote an item that occurs in row 2 and event T denote an item that occurs in column 3. Find

	column 1	column 2	column 3
row 1	A	D	G
row 2	B	E	H
row 3	C	F	I

Find  $P(T|S)$

$n = 300$

			$\downarrow T$
	147	14	128
$S \rightarrow$	11	0	0
	0	0	0

$$P(T|S) = \frac{P(T \cap S)}{P(S)} = \frac{0/300}{1/300} = 0.0$$

for  $P(S) > 0$

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## 2 events pp

Monday, February 10, 2025 8:14 AM

Consider a problem classified by 3 rows and 3 columns containing 1000 observations. The table is described in the figure below and has the following cell counts: A=46, B=36, C=0, D=847, E=12, F=2, G=54, H=0, I=3. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in column 2.

$N = 1000$

	column 1	column 2	column 3
row 1	A	D	G
row 2	B	E	H
row 3	C	F	I

Find  $P(S \cap T) | S \text{ and } T$

Handwritten counts and calculations:

- Row 1 counts: 46 (A), 847 (D), 54 (G) ← event S
- Column 2 counts: 36 (B), 12 (E), 2 (F) ← event T
- Row 2 counts: 0 (B), 12 (E), 0 (H)
- Row 3 counts: 0 (C), 2 (F), 3 (I)
- Calculation:  $P(S \cap T) = \frac{847}{1000}$

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## 2 events pp

Monday, February 10, 2025 8:19 AM

### QUESTION 5

Consider a problem classified by 3 rows and 3 columns containing 2000 observations. The table is described in the figure below and has the following cell counts: A=1319, B=7, C=0, D=516, E=0, F=1, G=157, H=0, and I=0. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in

	column 1	column 2	column 3
row 1	A	D	G
row 2	B	E	H
row 3	C	F	I

Find  $P(S \cup T) [S \text{ or } T]$

column 1. Find  $P(S \text{ or } T)$ .

$n = 2000$

1319	516	157	$\leftarrow S$ 1992
7	0	0	
0	1	0	
<u>1326</u>			

$$P(S \cup T) = P(S) + P(T) - P(S \cap T)$$

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$$\frac{1319 + 516 + 157 + 7 + 0}{2000}$$

$$= \frac{1992}{2000} + \frac{1326}{2000} - \frac{1319}{2000}$$

$$= .9995$$



## 2 events pp

Monday, February 10, 2025 8:27 AM

Consider a problem classified by 3 rows and 3 columns containing 500 observations. The table is described in the figure below and has the following cell counts: A=141, B=157, C=0, D=33, E=3, F=0, G=164, H=1, and I=1. Let event S denote an item that occurs in row 3 and event T denote an item that occurs in column 1.

	column 1	column 2	column 3
row 1	A	D	G
row 2	B	E	H
row 3	C	F	I

Find  $P(S|T)$ .

Find  $P(S|T)$ .

$$P(S|T) = \frac{P(S \cap T)}{P(T)} = \frac{0/500}{298/500} = 0$$

for  $P(T) > 0$

$A = 500$

	T		
	141	33	164
	157	3	1
S →	0	0	1
	<u>298</u>		

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## 2 events pp

Monday, February 10, 2025 8:30 AM

### QUESTION 8

Consider a problem classified by 3 rows and 3 columns containing 300 observations. The table is described in the figure below and has the following cell counts: A=172, B=16, C=1, D=60, E=0, F=0, G=47, H=3, and I=1. Let event S denote an item that occurs in row 1 and event T denote an item that occurs in column 1. Find

	column1	column2	column3
row1	A	D	G
row2	B	E	H
row3	C	F	I

Find  $P(S|T)$

$n = 300$

S	T	
172	60	47
16	0	3
1	0	1

$$P(S|T) = \frac{P(S \cap T)}{P(T)} = \frac{172/300}{189/300} = .91005$$

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