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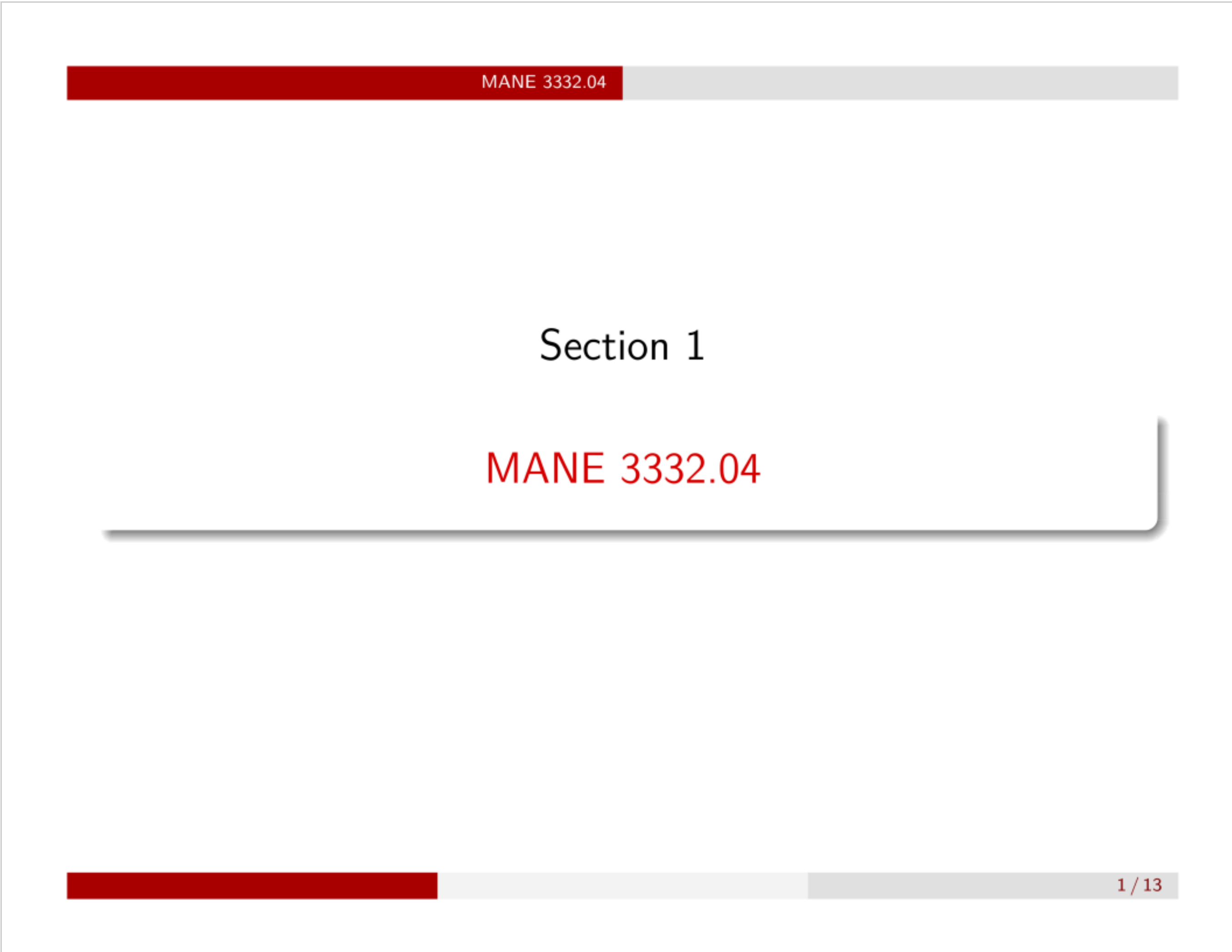


Subsection 1

Chapter Seven

Handouts

- Chapter 7 Slides
- Chapter 7 Slides marked

Chapter 7 Overview

- Chapter 7 contains a detailed explanation of point estimates for parameters
- Much of this chapter is of a highly statistical nature and will not be covered in this course
- Key concepts we will discuss are:
  - Statistical inference
  - Statistic
  - Sampling distribution
  - Point estimator
  - Unbiased estimate
  - MVUE estimator
  - Central limit theorem

Statistical Inference

- Montgomery gives the following description of statistical inference. *The field of statistical inference consists of those methods used to make decisions or to draw conclusions about a population. These methods utilize the information contained in a sample from the population in drawing conclusions. This chapter begins our study of the statistical methods used for inference and decision making.*
- Statistical inference may be divided into two major areas: parameter estimation and hypothesis testing

Chapter 9

Parameter Estimation

1) Point estimate

2) Interval Estimate - Chapter 8

Point Estimate → single number

- Montgomery states that "In practice, the engineer will use sample data to compute a number that is in some sense a reasonable value (or guess) of the true mean. This number is called a **point estimate**."
- Discuss examples
- A formal definition of a point estimate is
  *A **point estimate** of some population parameter θ is a single numerical value $\hat{\theta}$ of a statistic $\hat{\Theta}$. The statistic $\hat{\Theta}$ is called the point estimate.*
- Notice the use of the "hat" notation to denote a point estimate

hat notation

$$\hat{\mu} = \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

$$\hat{\sigma}^2 = s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

Statistic

- Point estimate requires a sample of random observations, say $X_1, X_2, \ldots, X_n$
- Any function of the sampled random variables is called a statistic
- The function of the random variables is itself a random variable
- Thus, the sample mean $\bar{x}$ and the sample variance s^2 are both statistics and random variables

Population

Parameter: μ, σ^2

Sample

Statistic: $\hat{\mu} = \bar{x} = f(x_i) = \frac{1}{n}\sum x_i$

Properties of point estimators

- We would like point estimates to be both accurate and precise
- An unbiased estimator addresses the accuracy criteria
- A minimum variance unbiased estimator addresses the precision criteria

Unbiased Estimator

- The point estimator $\hat{\Theta}$ is an **unbiased estimator** for the parameter θ if

$$E\left(\hat{\Theta}\right) = \theta$$

- If the point estimator is not unbiased, then the difference

$$E\left(\hat{\Theta}\right) - \theta$$

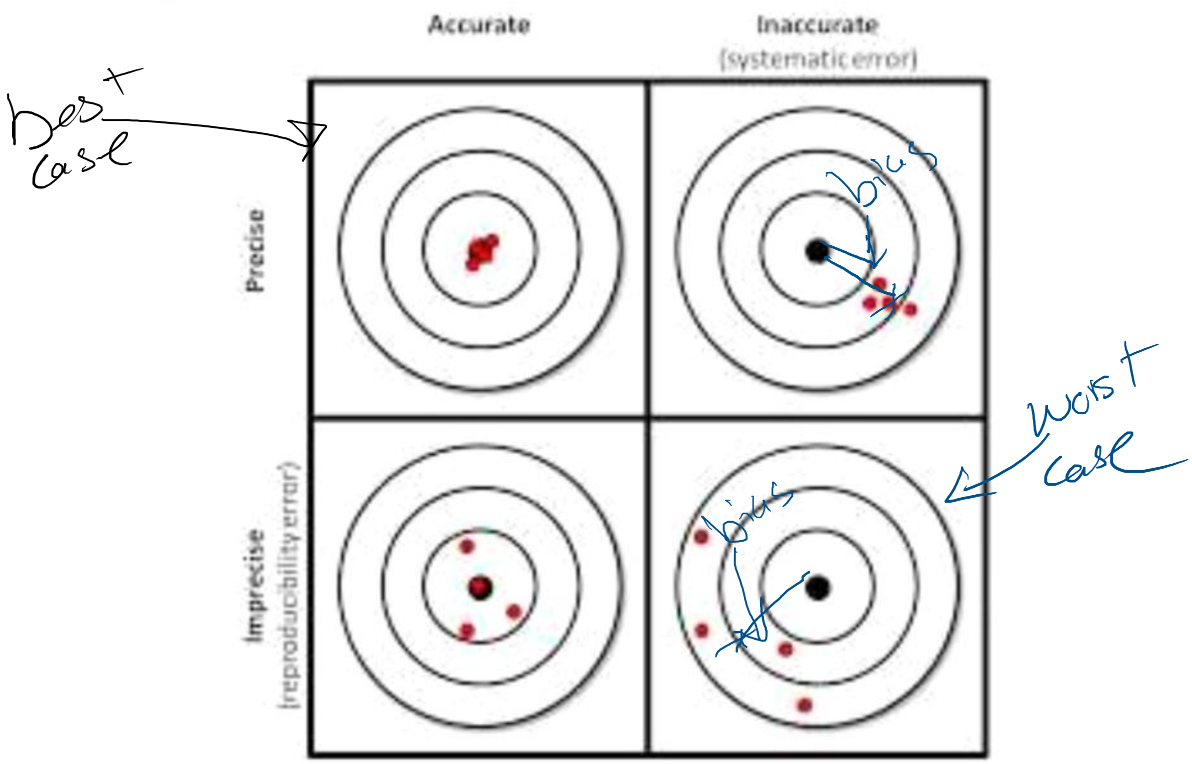
is called the **bias** of the estimator $\hat{\Theta}$

1) everything in this class will use unbiased estimator
2) Mean of Weibull Distribution is biased

MVUE

- Montgomery gives the following definition of a minimum variance unbiased estimator (MVUE)
  If we consider all unbiased estimators of θ, the one with the smallest variance is called the minimum variance unbiased estimator
- An import fact is that the sample mean $\bar{x}$ is the MVUE for μ when the data comes from a normal distribution

Accuracy vs. Precision



$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ (linear combination $c_i = \frac{1}{n}$)

Sampling Distribution

- The probability distribution of a statistic is called a **sampling distribution**

$E(\bar{X}) = E\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum E(X_i)$
$= \frac{1}{n}(\mu \cdot n)$
$= \mu$

$V(\bar{X}) = V\left(\frac{1}{n}\sum X_i\right) = \frac{1}{n^2}\sum(V(X_i))$
$= \frac{1}{n^2}(n \cdot \sigma^2) = \frac{\sigma^2}{n}$

Central Limit Theorem

- Definition of the Central Limit Theorem is
  If $X_1, X_2, \ldots, X_n$ is a random sample of size n taken from a population (either finite or infinite) with mean μ and finite variance σ^2, and if $\overline{X}$ is the sample mean, the limiting form of the distribution of
  $$Z = \frac{\overline{X} - \mu}{\sigma/\sqrt{n}}$$
  as $n \to \infty$, is the standard normal distribution
- Important result because for sufficiently large n, the sampling distribution of $\overline{X}$ is normally distribution
- This is a fundamental result that will be used extensively in the next four chapters of the textbook.

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