

MANE 3332.04

## Section 1

MANE 3332.04

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## Lecture 19, April 7

- Topics:
  - Chapter 5: CLT
  - Chapter 6: Multivariate Statistical Analysis
  - Chapter 7: Definitions
  - Chapter 8: Interval Estimation
- Assignments:
  - Technical Report One due today
  - Linear Combination Practice Problems due today
  - Linear Combination Quiz (assigned 4/7/25, due 4/9/25)
- Attendance
- Return and discuss Test One
- Questions?

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## Handouts

- Chapter 5
  - Chapter 5 Slides
  - Chapter 5 Slides marked
- Chapter 6
  - Chapter 6 Slides
  - Chapter 6 Slides marked
- Chapter 7
  - Chapter 7 slides
  - Chapter
- Chapter 8
  - Chapter 8 slides
  - Chapter 8 slides marked
- Final Exam Handouts

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## Class Schedule

Monday Lecture	Wednesday Lecture
4/7: Chapter 7 & 8	4/9: Chapter 8, Case 1
4/14: Chapter 8: Case 2	4/16: Chapter 8: Case 3
4/21: Chapter 9, case 1	4/23: Chapter 9, Case 2
4/28: Chpater 9, Case 3	4/30: Chapter 11
5/5: Chapter 11	5/7: Review

**10 Sessions plus final exam**

**Final Exam: Tuesday May 13, 2025 10:15 am - 12:00 pm**

## Chapter 8 Introduction

- Intervals are another method of performing estimation
- A confidence interval is an interval estimate on a population parameter (primary focus of this chapter)
- Three types of interval estimates
  - A confidence intervals bounds population or distribution parameters
  - A tolerance interval bounds a selected proportion of a distribution
  - A prediction interval bounds future observations from the population or distribution
- Interval estimates, especially confidence intervals are commonly used in science and engineering

*N or 2*  
*2 probabilities*

*Tolerance: I want a 95% tolerance interval that contains 80% of the population*

*Chapter 8* *Chapter 9*

Case	Null Hypothesis	Test Statistic	Alternative Hypothesis	Test Significance Level Criteria for Rejection	P-value	O.C. Curve Parameter	O.C. Curve Appendix Chart VII
1.	$H_0: \mu = \mu_0$ $\sigma^2$ known	$z_0 = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$	$H_1: \mu \neq \mu_0$ $H_1: \mu > \mu_0$ $H_1: \mu < \mu_0$	$ z_0  > z_{\alpha/2}$ $z_0 > z_\alpha$ $z_0 < -z_\alpha$	$p = 2(1 - \Phi( z_0 ))$ Probability above $z_0$ Probability below $z_0$ Probability below $z_0$	$d = \mu_0 - \mu_1/\sigma$ $d = (\mu_0 - \mu_1)/\sigma$ $d = (\mu_0 - \mu_1)/\sigma$	a, b c, d c, d
2.	$H_0: \mu = \mu_0$ $\sigma^2$ unknown	$t_0 = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$	$H_1: \mu \neq \mu_0$ $H_1: \mu > \mu_0$ $H_1: \mu < \mu_0$	$ t_0  > t_{\alpha/2, n-1}$ $t_0 > t_{\alpha, n-1}$ $t_0 < -t_{\alpha, n-1}$	Sum of the probability above $t_0$ and below $- t_0 $ Probability above $t_0$ Probability below $t_0$ See text Section 9.4	$d = (\mu_0 - \mu_1)/s$ $d = (\mu_0 - \mu_1)/s$ $d = (\mu_0 - \mu_1)/s$	e, f g, h i, j
3.	$H_0: \sigma^2 = \sigma_0^2$	$\chi^2_0 = \frac{(n-1)s^2}{\sigma_0^2}$	$H_1: \sigma^2 \neq \sigma_0^2$ $H_1: \sigma^2 > \sigma_0^2$ $H_1: \sigma^2 < \sigma_0^2$	$\chi^2_0 > \chi^2_{\alpha/2, n-1}$ $\chi^2_0 > \chi^2_{\alpha, n-1}$ $\chi^2_0 < \chi^2_{1-\alpha, n-1}$	Probability above $\chi^2_0$ Probability below $\chi^2_0$ Probability below $\chi^2_0$	$\lambda = \sigma_0^2/\sigma_1^2$ $\lambda = \sigma_0^2/\sigma_1^2$ $\lambda = \sigma_0^2/\sigma_1^2$	k, l m, n o, p
4.	$H_0: p = p_0$	$z_0 = \frac{\bar{p} - p_0}{\sqrt{p_0(1-p_0)}}$	$H_1: p \neq p_0$ $H_1: p > p_0$ $H_1: p < p_0$	$ z_0  > z_{\alpha/2}$ $z_0 > z_\alpha$ $z_0 < -z_\alpha$	$p = 2(1 - \Phi( z_0 ))$ Probability above $z_0$ Probability below $z_0$ Probability below $z_0$	$d = p_0 - p_1$ $d = p_0 - p_1$ $d = p_0 - p_1$	q, r s, t u, v

Case	Problem Type	Point Estimate	Two-Sided 100(1- $\alpha$ ) Percent Confidence Interval
1.	Mean $\mu$ , variance $\sigma^2$ known	$\bar{x}$	$\bar{x} \pm z_{\alpha/2} \sigma / \sqrt{n}$
2.	Mean $\mu$ of a normal distribution, variance $\sigma^2$ unknown	$\bar{x}$	$\bar{x} \pm t_{\alpha/2, n-1} s / \sqrt{n}$
3.	Variance $\sigma^2$ of a normal distribution	$s^2$	$\frac{(n-1)s^2}{K_{2, \alpha/2}} \leq \sigma^2 \leq \frac{(n-1)s^2}{K_{1, \alpha/2}}$
4.	Proportion of parameter of a binomial distribution $p$	$\bar{p}$	$\bar{p} \pm z_{\alpha/2} \sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$

## Confidence Interval on the Mean of a normal distribution, variance known (Case 1)

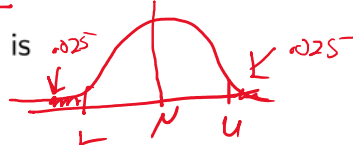
→ in practice, not common

BUT it is simplest from Statistical theory

- Suppose that  $X_1, X_2, \dots, X_n$  is a random sample from a normal population with unknown mean  $\mu$  and known variance  $\sigma^2$
- A general expression for a confidence interval is

if  $\alpha = 0.05$

$$P[L \leq \mu \leq U] = 1 - \alpha$$



- Using the sample results we calculate a  $100(1 - \alpha)\%$  confidence of the form

$$l = \bar{X} - z_{\alpha/2} \sigma / \sqrt{n}$$

$$u = \bar{X} + z_{\alpha/2} \sigma / \sqrt{n}$$

$$l \leq \mu \leq u$$

- A  $100(1 - \alpha)\%$  confidence interval for the mean of a normal distribution with variance known is

Attendance 1-A

### Problem 8-12, part a (6th edition)

- 8-12. The life in hours of a 75-watt light bulb is known to be normally distributed with  $\sigma = 25$  hours. A random sample of 20 bulbs has a mean life of  $\bar{x} = 1014$  hours.
- Construct a 95% two-sided confidence interval on the mean life.
  - Construct a 95% lower-confidence bound on the mean life.

Figure 2: image

### Interpreting Confidence Intervals

Montgomery gives the following statement regarding the correct interpretation of confidence intervals.

*The correct interpretation lies in the realization that a CI is a random interval because in the probability statement defining the end-points of the interval,  $L$  and  $U$  are random variables. Consequently, the correct interpretation of a  $100(1 - \alpha)\%$  CI depends on the relative frequency view of probability. Specifically, if an infinite number of random samples are collected and a  $100(1 - \alpha)\%$  confidence interval for  $\mu$  is computed from each sample,  $100(1 - \alpha)\%$  of these intervals will contain the true value of  $\mu$ .*

### One-sided Confidence Bounds

It is possible to construct one-sided confidence bounds

- A  $100(1 - \alpha)\%$  upper-confidence bound for  $\mu$  is

$$\mu \leq u = \bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

- A  $100(1 - \alpha)\%$  lower-confidence bound for  $\mu$  is

$$\bar{x} - z_{\alpha} \frac{\sigma}{\sqrt{n}} = l \leq \mu$$

~~$$l \leq \mu \leq u$$~~

$$\mu \leq \bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

~~$$l \leq \mu \leq u$$~~

$$\bar{x} - z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

## Sample Size Considerations

Not covered in class

If  $\bar{x}$  is used as an estimate of  $\mu$ , we can be  $100(1 - \alpha)\%$  confident that the error  $|\bar{x} - \mu|$  will not exceed a specified amount  $E$  when the sample size is

$$n = \left( \frac{z_{\alpha/2} \sigma}{E} \right)^2 \rightarrow \text{round up}$$

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## Problem 8-12, part b (6th edition)

8-12. The life in hours of a 75-watt light bulb is known to be normally distributed with  $\sigma = 25$  hours. A random sample of 20 bulbs has a mean life of  $\bar{x} = 1014$  hours.

- Construct a 95% two-sided confidence interval on the mean life.
- Construct a 95% lower-confidence bound on the mean life.

$$\begin{aligned} 100(1-\alpha)\% &= 95\% \\ (1-\alpha) &= .95 \\ \alpha &= .05 \end{aligned}$$

Figure 3: image

$$\bar{x} - z_{\alpha} \frac{\sigma}{\sqrt{n}} \leq \mu$$

$$1014 - z_{.05} \frac{25}{\sqrt{20}} \leq \mu$$

$$1014 - 1.645 \frac{25}{\sqrt{20}} \leq \mu$$

$$1004.80 \leq \mu$$

$$z_{.05} = t_{\infty, .05}$$

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### A Large Sample CI for $\mu$

- When  $n$  is large (say greater than or equal to 40), the central limit theorem can be used
- It states that  $\frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$  is approximately a standard normal random variable.
- Thus, we can replace the quantity  $\sigma/\sqrt{n}$  with  $S/\sqrt{n}$  and still use the quantiles of the normal distribution to construct a confidence interval.

$$\bar{x} - z_{\alpha/2} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2} \frac{S}{\sqrt{n}}$$

- What assumption did we relax and why?

## Chapter 8, Case 1 Practice Problems

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## Confidence Interval for the mean of Normal distribution with variance unknown (Case 2)

The  $t$  distribution

- Definition.

Let  $X_1, X_2, \dots, X_n$  be a random sample from a normal distribution with unknown mean  $\mu$  and unknown variance  $\sigma^2$ . The random variable

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

has a  $t$  distribution with  $n - 1$  degrees of freedom.

- A table of percentage points (quantiles) is given in Appendix A Table 5
- Figure 8-4 on page 180 shows the relationship between the  $t$  and normal distributions.
- Figure 8-5 explains the percentage points of the  $t$  distribution

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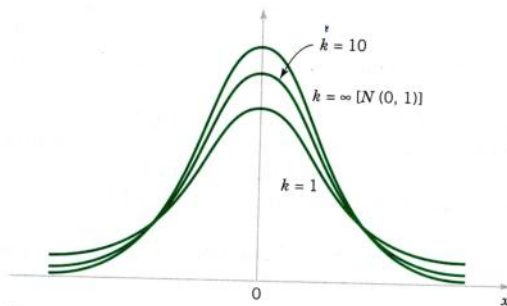


Figure 8-4 Probability density functions of several  $t$  distributions.

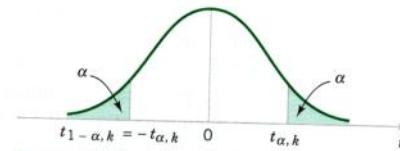


Figure 8-5 Percentage points of the  $t$  distribution.

Figure 4: image

### Confidence interval definition

- Using the  $t$  distribution it is possible to construct CIs

If  $\bar{x}$  and  $s$  are the mean and standard deviation of a random sample from a normal distribution with unknown variance  $\sigma^2$ , a  $100(1 - \alpha)\%$  confidence interval on  $\mu$  is given by

$$\bar{x} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

where  $t_{\alpha/2, n-1}$  is the upper  $100(\alpha/2)$  percentage point of the  $t$  distribution with  $n - 1$  degrees of freedom.

### Problem 8-30 (6th edition)

8-30. An article in *Nuclear Engineering International* (February 1988, p. 33) describes several characteristics of fuel rods used in a reactor owned by an electric utility in Norway. Measurements on the percentage of enrichment of 12 rods were reported as follows:

2.94	3.00	2.90	2.75	3.00	2.95
2.90	2.75	2.95	2.82	2.81	3.05

- Use a normal probability plot to check the normality assumption.
- Find a 99% two-sided confidence interval on the mean percentage of enrichment. Are you comfortable with the statement that the mean percentage of enrichment is 2.95 percent? Why?

Figure 5: image

Table IV Percentage Points  $t_{\alpha, \nu}$  of the  $t$ -Distribution

$\alpha \backslash \nu$	40	25	10	.05	.025	.01	.005	.0025	.001	.0005
1	.325	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	.289	.816	1.886	2.920	4.303	6.965	9.925	14.089	23.326	31.598
3	.277	.765	1.638	2.353	3.182	4.541	5.841	7.453	10.213	12.924
4	.271	.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	.267	.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	.265	.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	.263	.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	.262	.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	.261	.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	.260	.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	.260	.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	.259	.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	.259	.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	.258	.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	.258	.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	.258	.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	.257	.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	.257	.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	.257	.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	.257	.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	.257	.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	.256	.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	.256	.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.767
24	.256	.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	.256	.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	.256	.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	.256	.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	.256	.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	.256	.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	.256	.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	.255	.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	.254	.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
120	.254	.677	1.289	1.658	1.980	2.358	2.617	2.860	3.160	3.373
$\infty$	.253	.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291

 $\nu$  = degrees of freedom.

### One-sided confidence bounds

- Are easy to construct
- Use only the appropriate upper or lower bound
- Change  $t_{\alpha/2, n-1}$  to  $t_{\alpha, n-1}$

### Chapter 8, Case 2 Practice Problems

## Confidence Interval for $\sigma^2$ and $\sigma$ (Case 3)

- Section 8-3 presents a CI for  $\sigma^2$  or  $\sigma$
- Requires the  $\chi^2$  (chi-squared) distribution

Let  $X_1, X_2, \dots, X_n$  be a random sample from a normal distribution with mean  $\mu$  and variance  $\sigma^2$  and let  $S^2$  be the sample variance. Then the random variable

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-square ( $\chi^2$ ) distribution with  $n - 1$  degrees of freedom

- A table of the upper percentage points of the  $\chi^2$  distribution are given in Table 4 in the appendix
- Figure 8-9 on page 183 explains the percentage points of the  $\chi^2$  distribution

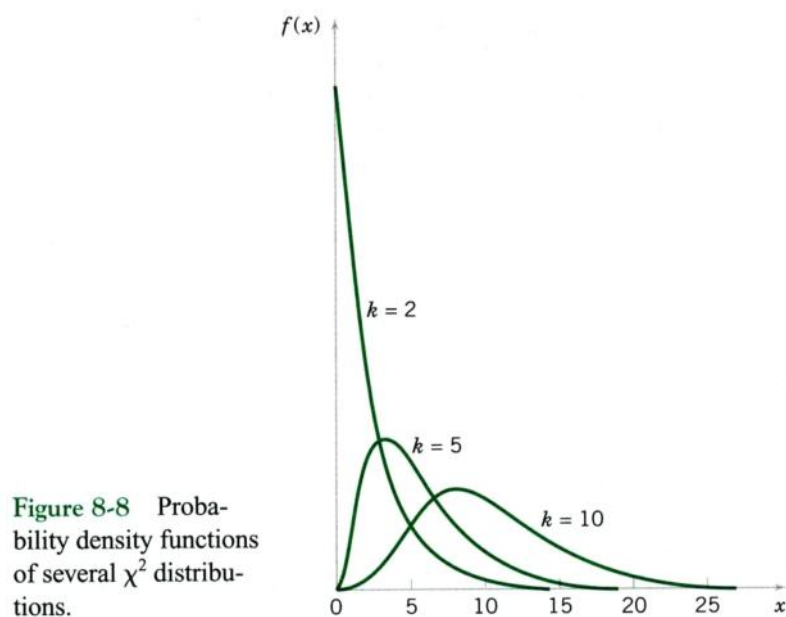


Figure 7: image

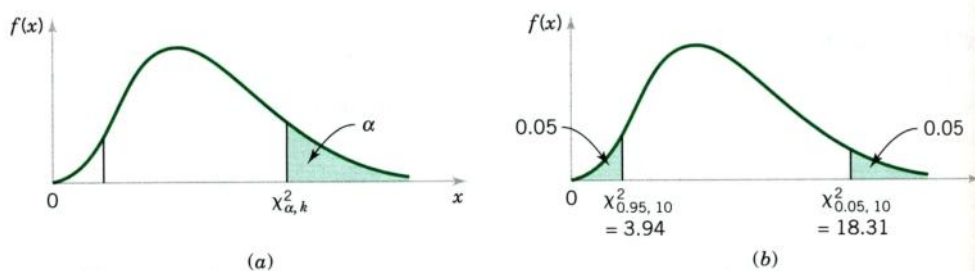


Figure 8: image

### Confidence Intervals for $\sigma^2$ and $\sigma$

If  $s^2$  is the sample variance from a random sample of  $n$  observations from a normal distribution with unknown variance  $\sigma^2$ , then a  $100(1 - \alpha)\%$  confidence interval on  $\sigma^2$  is

$$\frac{(n-1)s^2}{\chi_{\alpha/2, n-1}^2} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi_{1-\alpha/2, n-1}^2}$$

where  $\chi_{\alpha/2, n-1}^2$  and  $\chi_{1-\alpha/2, n-1}^2$  are the upper and lower  $100\alpha/2$  percentage points of the  $\chi^2$ -distribution with  $n - 1$  degrees of freedom

## Problem 8-36 (6th edition)

8-36. The sugar content of the syrup in canned peaches is normally distributed. A random sample of  $n = 10$  cans yields a sample standard deviation of  $s = 4.8$  milligrams. Find a 95% two-sided confidence interval for  $\sigma$ .

Figure 9: image



Table III Percentage Points  $\chi^2_{\alpha}$  of the Chi-Squared Distribution

$\alpha$	.995	.990	.975	.950	.900	.800	.700	.600	.500	.400	.300	.200	.100	.050	.025	.010	.005
1	.004	.005	.008	.012	.020	.032	.045	.063	.088	.121	.159	.203	.254	.314	.386	.473	.578
2	.010	.012	.016	.020	.032	.050	.075	.109	.151	.203	.267	.344	.436	.545	.675	.831	1.023
3	.017	.021	.028	.035	.058	.092	.136	.196	.274	.374	.500	.657	.859	1.119	1.461	1.914	2.526
4	.021	.026	.035	.044	.072	.116	.176	.256	.364	.506	.687	.919	1.217	1.603	2.104	2.776	3.688
5	.024	.030	.040	.050	.080	.132	.204	.296	.416	.572	.774	1.044	1.399	1.879	2.526	3.326	4.362
6	.028	.034	.046	.058	.092	.152	.232	.336	.472	.644	.864	1.164	1.564	2.084	2.776	3.688	4.862
7	.032	.038	.052	.064	.104	.172	.264	.384	.532	.724	.984	1.324	1.764	2.324	3.044	3.964	5.162
8	.036	.042	.058	.072	.116	.192	.284	.404	.564	.772	1.064	1.424	1.884	2.484	3.244	4.204	5.442
9	.040	.048	.064	.080	.128	.212	.312	.444	.616	.844	1.164	1.564	2.044	2.684	3.484	4.524	5.802
10	.044	.052	.070	.088	.140	.232	.344	.484	.672	.924	1.264	1.684	2.184	2.844	3.684	4.764	6.082
11	.048	.058	.076	.096	.152	.252	.372	.524	.724	.992	1.364	1.804	2.324	3.004	3.884	5.004	6.362
12	.052	.062	.082	.104	.164	.272	.404	.564	.784	1.084	1.484	1.944	2.484	3.184	4.104	5.244	6.642
13	.056	.066	.088	.112	.176	.292	.432	.604	.832	1.164	1.604	2.084	2.644	3.364	4.324	5.504	6.922
14	.060	.070	.094	.120	.188	.312	.464	.644	.892	1.244	1.724	2.224	2.804	3.544	4.544	5.764	7.202
15	.064	.076	.100	.128	.200	.328	.488	.684	.952	1.364	1.884	2.404	3.004	3.764	4.784	6.044	7.502
16	.068	.080	.106	.136	.212	.344	.512	.712	.992	1.444	2.004	2.544	3.164	3.944	5.004	6.284	7.762
17	.072	.084	.112	.144	.224	.360	.536	.744	1.044	1.564	2.164	2.724	3.364	4.164	5.244	6.564	8.042
18	.076	.088	.118	.152	.236	.376	.560	.772	1.084	1.644	2.284	2.864	3.524	4.344	5.444	6.784	8.322
19	.080	.092	.122	.160	.248	.392	.584	.804	1.124	1.724	2.404	3.004	3.644	4.484	5.604	6.964	8.522
20	.084	.096	.128	.168	.260	.408	.604	.832	1.164	1.804	2.524	3.144	3.804	4.644	5.784	7.164	8.722
21	.088	.100	.132	.176	.272	.424	.624	.864	1.204	1.884	2.644	3.284	3.964	4.804	5.964	7.364	8.922
22	.092	.104	.136	.184	.284	.440	.644	.892	1.244	1.964	2.764	3.424	4.124	5.004	6.164	7.584	9.122
23	.096	.108	.140	.192	.296	.456	.664	.924	1.284	2.044	2.884	3.564	4.264	5.164	6.344	7.784	9.322
24	.100	.112	.144	.200	.308	.472	.684	.944	1.324	2.124	3.004	3.704	4.404	5.324	6.524	7.944	9.522
25	.104	.116	.148	.208	.320	.488	.704	.972	1.364	2.204	3.124	3.844	4.544	5.484	6.704	8.104	9.722
26	.108	.120	.152	.216	.332	.504	.724	1.004	1.404	2.284	3.244	3.984	4.684	5.644	6.884	8.284	9.922
27	.112	.124	.156	.224	.344	.520	.744	1.032	1.444	2.364	3.364	4.124	4.824	5.804	7.044	8.444	10.122
28	.116	.128	.160	.232	.356	.536	.764	1.064	1.484	2.444	3.484	4.264	4.964	6.004	7.244	8.604	10.322
29	.120	.132	.164	.240	.368	.552	.784	1.092	1.524	2.524	3.604	4.404	5.104	6.204	7.444	8.804	10.522
30	.124	.136	.168	.248	.380	.568	.804	1.124	1.564	2.604	3.724	4.544	5.244	6.364	7.604	9.004	10.722
31	.128	.140	.172	.256	.392	.584	.824	1.152	1.604	2.684	3.844	4.684	5.384	6.524	7.764	9.204	10.922
32	.132	.144	.176	.264	.404	.600	.844	1.184	1.644	2.764	3.964	4.824	5.524	6.684	7.924	9.404	11.122
33	.136	.148	.180	.272	.416	.616	.864	1.212	1.684	2.844	4.084	4.964	5.664	6.844	8.104	9.604	11.322
34	.140	.152	.184	.280	.428	.632	.884	1.244	1.724	2.924	4.204	5.104	5.804	7.004	8.284	9.804	11.522
35	.144	.156	.188	.288	.440	.648	.904	1.272	1.764	3.004	4.324	5.244	5.944	7.164	8.484	10.004	11.722
36	.148	.160	.192	.296	.452	.664	.924	1.304	1.804	3.084	4.444	5.384	6.084	7.324	8.684	10.204	11.922
37	.152	.164	.196	.304	.464	.680	.944	1.332	1.844	3.164	4.564	5.524	6.224	7.484	8.884	10.404	12.122
38	.156	.168	.200	.312	.476	.696	.964	1.364	1.884	3.244	4.684	5.664	6.364	7.644	9.084	10.604	12.322
39	.160	.172	.204	.320	.488	.712	.984	1.392	1.924	3.324	4.804	5.804	6.504	7.804	9.284	10.804	12.522
40	.164	.176	.208	.328	.500	.728	1.004	1.424	1.964	3.404	4.924	5.944	6.644	7.964	9.484	11.004	12.722
41	.168	.180	.212	.336	.512	.744	1.024	1.452	2.004	3.484	5.044	6.084	6.784	8.124	9.684	11.204	12.922
42	.172	.184	.216	.344	.524	.760	1.044	1.484	2.044	3.564	5.164	6.224	6.924	8.324	9.884	11.404	13.122
43	.176	.188	.220	.352	.536	.776	1.064	1.512	2.084	3.644	5.284	6.364	7.064	8.524	10.084	11.604	13.322
44	.180	.192	.224	.360	.548	.792	1.084	1.544	2.124	3.724	5.404	6.504	7.204	8.724	10.284	11.804	13.522
45	.184	.196	.228	.368	.560	.808	1.104	1.572	2.164	3.804	5.524	6.644	7.344	8.924	10.484	12.004	13.722
46	.188	.200	.232	.376	.572	.824	1.124	1.604	2.204	3.884	5.644	6.784	7.484	9.124	10.684	12.204	13.922
47	.192	.204	.236	.384	.584	.840	1.144	1.632	2.244	3.964	5.764	6.924	7.624	9.324	10.884	12.404	14.122
48	.196	.208	.240	.392	.596	.856	1.164	1.664	2.284	4.044	5.884	7.064	7.764	9.524	11.084	12.604	14.322
49	.200	.212	.244	.400	.608	.872	1.184	1.692	2.324	4.124	5.964	7.204	7.904	9.724	11.284	12.804	14.522
50	.204	.216	.248	.408	.620	.888	1.204	1.724	2.364	4.204	6.084	7.344	8.044	9.924	11.484	13.004	14.722
51	.208	.220	.252	.416	.632	.904	1.224	1.752	2.404	4.284	6.204	7.484	8.184	10.124	11.684	13.204	14.922
52	.212	.224	.256	.424	.644	.920	1.244	1.784	2.444	4.364	6.324	7.624	8.324	10.324	11.884	13.404	15.122
53	.216	.228	.260	.432	.656	.936	1.264	1.812	2.484	4.444	6.444	7.764	8.464	10.524	12.084	13.604	15.322
54	.220	.232	.264	.440	.668	.952	1.284	1.844	2.524	4.524	6.564	7.904	8.604	10.724	12.284	13.804	15.522
55	.224	.236	.268	.448	.680	.968	1.304	1.872	2.564	4.604	6.684	8.044	8.744	10.924	12.484	14.004	15.722
56	.228	.240	.272	.456	.692	.984	1.324	1.904	2.604	4.684	6.804	8.184	8.884	11.124	12.684	14.204	15.922
57	.232	.244	.276	.464	.704	1.000	1.344	1.932	2.644	4.764	6.924	8.324	9.024	11.324	12.884	14.404	16.122
58	.236	.248	.280	.472	.716	1.016	1.364	1.964	2.684	4.844	7.044	8.464	9.164	11.524	13.084	14.604	16.322
59	.240	.252	.284	.480	.728	1.032	1.384	1.992	2.724	4.924	7.164	8.604	9.304	11.724	13.284	14.804	16.522
60	.244	.256	.288	.488	.740	1.048	1.404	2.024	2.764	5.004	7.284	8.744	9.444	11.924	13.484	15.004	16.722
61	.248	.260	.292	.496	.752	1.064	1.424	2.052	2.804	5.084	7.404	8.884	9.584	12.124	13.684	15.204	16.922
62	.252	.264	.296	.504	.764	1.080	1.444	2.084	2.844	5.164	7.524	9.024	9.724	12.324	13.884	15.404	17.122
63	.256	.268	.300	.512	.776	1.096	1.464	2.112	2.884	5.244	7.644	9.164	9.864	12.524	14.084	15.604	17.322
64	.260	.272	.304	.520	.788	1.112	1.484	2.144	2.924	5.324	7.764	9.304	10.004	12.724	14.284	15.804	17.522
65	.264	.276	.308	.528	.800	1.128	1.504	2.172	2.964	5.404	7.884	9.444	10.144	12.924	14.484	16.004	17.722
66	.268	.280	.312	.536	.812	1.144	1.524	2.204	3.004	5.484	7.964	9.584	10.284	13.124	14.684	16.204	17.922
67	.272	.284	.316	.544	.824	1.160	1.544	2.232	3.044	5.564	8.084	9.724	10.424	13.324	14.884	16.404	18.122
68	.276	.288	.320	.552	.836	1.176	1.564	2.264	3.084	5.644	8.204	9.864	10.564	13.524	15.084	16.604	18.322
69	.280	.292	.324	.560	.848	1.192	1.584	2.292	3.124	5.724	8.324	10.004	10.704	13.724	15.284	16.804	18.522
70	.284	.296	.328	.568	.860	1.208	1.604	2.324	3.164	5.804	8.444	10.144	10.844	13.924	15.484	17.004	18.722
71	.288	.300	.332	.576	.872	1.224	1.624	2.352	3.204	5.884	8.564	10.284	10.984	14.124	15.684	17.204	18.922
72	.292	.304	.336	.584	.884	1.240	1.644	2.384	3.244	5.964	8.684	10.424	11.124	14.324	15.884	17.404	19.122
73	.296	.308	.340	.592													

### One-sided confidence bounds

- Are easy to construct
- Use only the appropriate upper or lower bound
- Change  $\chi^2_{\alpha/2, n-1}$  to  $\chi^2_{\alpha, n-1}$  or  $\chi^2_{1-\alpha/2, n-1}$  to  $\chi^2_{1-\alpha, n-1}$
- See eqn (8-20) on page 184

## Chapter 8, Case 3 Practice Problems

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## Large-Sample CI for a Population Proportion (Case 4)

- Recall from chapter 4, that the sampling distribution of  $\hat{P}$  is approximately normal with mean  $p$  and variance  $p(1-p)/n$ , if  $n$  is not too close to either 0 or 1 and if  $n$  is relatively large.
- Typically, we require both  $np \geq 5$  and  $n(1-p) \geq 5$

If  $n$  is large, the distribution of

$$Z = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}} = \frac{\hat{P} - p}{\sqrt{np(1-p)}/n}$$

is approximately standard normal.

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If  $\hat{p}$  is the proportion of observations in a random sample of size  $n$  that belongs to a class of interest, an approximate  $100(1 - \alpha)\%$  confidence interval on the proportion  $p$  of the population that belongs to this class is

$$\hat{p} - z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2} \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

where  $z_{\alpha/2}$  is the upper  $\alpha/2$  percentage point of the standard normal distribution

### Other Considerations

- We can select a sample so that we are  $100(1 - \alpha)\%$  confident that error  $E = |p - \hat{P}|$  using

$$n = \left( \frac{z_{\alpha/2}}{E} \right)^2 p(1 - p)$$

- An upper bound on  $n$  is given by

$$n = \left( \frac{z_{\alpha/2}}{E} \right)^2 (0.25)$$

- One-sided confidence bounds are given in eqn (8-26) on page 187

### Guidelines for Constructing Confidence Intervals

- Review excellent guide given in Table 8-1

### Other Interval Estimates

- When we want to predict the value of a single value in the future, a **prediction interval** is used
- A **tolerance interval** captures  $100(1 - \alpha)\%$  of observations from a distribution

### Prediction Interval for a Normal Distribution

- Excellent discussion on pages 189 - 190
- A  $100(1 - \alpha)\%$  PI on a single future observation from a normal distribution is given by

$$\bar{x} - t_{\alpha/2, n-1} s \sqrt{1 + \frac{1}{n}} \leq X_{n+1} \leq \bar{x} + t_{\alpha/2, n-1} s \sqrt{1 + \frac{1}{n}}$$

### Tolerance Intervals for a Normal Distribution

- A **tolerance interval** to contain at least  $\gamma\%$  of the values in a normal population with confidence level  $100(1 - \alpha)\%$  is

$$\bar{x} - ks, \bar{x} + ks$$

where  $k$  is a tolerance interval factor for the normal distribution found in appendix A Table XII. Values are given for  $1 - \alpha = 0.9, 0.95$  and  $0.99$  confidence levels and for  $\gamma = .90, .95$ , and  $.99\%$  probability of coverage

- One-sided tolerance bounds can also be computed. The factors are also in Table XII

find 95% C.I. Chapter 8, Case 1

Given  $n = 20, \sigma = 25, \bar{x} = 1014$

$$\bar{x} - Z_{\alpha/2} \sigma / \sqrt{n} \leq \mu \leq \bar{x} + Z_{\alpha/2} \sigma / \sqrt{n}$$

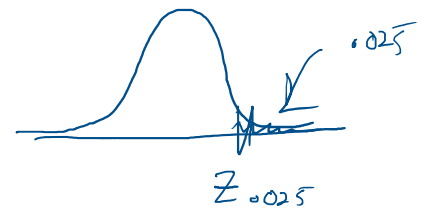
$$100(1-\alpha)\% = 95\%$$

$$(1-\alpha) = .95$$

$$\alpha = .05$$

Normal Distribution (Z)

$$Z_{.025} = 1.96$$



Method 1  
std. Normal table (not recommended)

Method 2  
use t-tables  $t_{\infty, .025} = Z_{.025}$

Method 3 - Memorize

$$\bar{X} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{X} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$1014 - 1.96 \frac{25}{\sqrt{20}} \leq \mu \leq 1014 + 1.96 \frac{25}{\sqrt{20}}$$

$$1003.043 \leq \mu \leq 1024.957$$

$$100(1-\alpha)\% = 90\%$$

$$1-\alpha = .9$$

$$\alpha = .1 \rightarrow \alpha/2 = .05$$

**QUESTION 3**

$n$

$\bar{x}$   $\sigma$

Consider a sample of size 27 from a normal distribution with mean 0.5 and sigma 0.07. What is the value of a two-sided 90.0 % confidence interval for the mean?

- ☐ (0.5,0.5)
- ☐ (0.5,0.5)
- ☐ (0.496,0.504)
- ☐ (0.478,0.522)
- ☐ (0.483,0.517)
- ☐ (0.498,0.502)

$$Z_{0.025} = 1.645$$

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$.5 - 1.645 \frac{.07}{\sqrt{27}} \leq \mu < .5 + 1.645 \frac{.07}{\sqrt{27}}$$

$$.48188 \leq \mu \leq .52216$$

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QUESTION 4

Consider a sample of size 42 from a normal distribution with mean 0.5 and sigma 0.05. What is the value of a 99.9 % upper-confidence bound for the mean?

- ☒ mu <= 0.524
- ☐ mu <= 0.504
- ☐ mu <= 0.5
- ☐ mu <= 0.525
- ☐ mu <= 0.5
- ☐ mu <= 0.501

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$\bar{x}$



$$\alpha = .001$$

$$100(1-\alpha)\% = 99.9\%$$

$$(1-\alpha) = .999$$

$$\alpha = .001$$

~~$\mu \leq u$~~

$$\mu \leq \bar{x} + z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

$$\mu \leq .5 + z_{.001} \frac{.05}{\sqrt{42}}$$

$$\mu \leq .5 + 3.09 \frac{.05}{\sqrt{42}}$$

$$\mu \leq .52384$$

$$z_{.001} = 3.09$$

$$\frac{100(1-\alpha)\%}{100} = 99.5\%$$

$$(1-\alpha) = .995$$

$$\alpha = .005$$

QUESTION 5

Consider a sample of size 36 from a normal distribution with mean 11.8 and sigma 1.73. What is the value of a 99.5% lower-confidence bound for the mean?

- ☐ mu >= 11.651
- ☐ mu >= 10.851
- ☐ mu >= 11.526
- ☐ mu >= 10.909
- ☐ mu >= 10.259
- ☐ mu >= 11.543

$$\mu \leq 11.543$$

$$\bar{X} - Z_{\alpha} \frac{\sigma}{\sqrt{n}}$$

$$99.5\%$$

$$Z_{.005} = 2.576$$

$$11.8 - Z_{.005} \frac{1.73}{\sqrt{36}} \leq \mu$$

$$11.8 - 2.576 \frac{1.73}{\sqrt{36}} \leq \mu$$

$$11.057 \leq \mu$$

attending p  
1-C

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