

## Section 1

MANE 3332.04

## Subsection 1

## Chapter Seven

# Handouts

- Chapter 7 Slides
- Chapter 7 Slides marked

## Chapter 7 Overview

- Chapter 7 contains a detailed explanation of point estimates for parameters
- Much of this chapter is of a highly statistical nature and will not be covered in this course
- Key concepts we will discuss are:
  - Statistical inference
  - Statistic
  - Sampling distribution
  - Point estimator
  - Unbiased estimate
  - MVUE estimator
  - Central limit theorem

## Statistical Inference

- Montgomery gives the following description of statistical inference. *The field of statistical inference consists of those methods used to make decisions or to draw conclusions about a population. These methods utilize the information contained in a sample from the population in drawing conclusions. This chapter begins our study of the statistical methods used for inference and decision making.*
- Statistical inference may be divided into two major areas: parameter estimation and hypothesis testing

## Point Estimate

- Montgomery states that “In practice, the engineer will use sample data to compute a number that is in some sense a reasonable value (or guess) of the true mean. This number is called a **point estimate**.”
- Discuss examples
- A formal definition of a point estimate is  
*A **point estimate** of some population parameter  $\theta$  is a single numerical value  $\hat{\theta}$  of a statistic  $\hat{\Theta}$ . The statistic  $\hat{\Theta}$  is called the point estimate.*
- Notice the use of the “hat” notation to denote a point estimate

## Statistic

- Point estimate requires a sample of random observations, say  $X_1, X_2, \dots, X_n$
- Any function of the sampled random variables is called a statistic
- The function of the random variables is itself a random variable
- Thus, the sample mean  $\bar{x}$  and the sample variance  $s^2$  are both statistics and random variables

## Properties of point estimators

- We would like point estimates to be both accurate and precise
- An unbiased estimator addresses the accuracy criteria
- A minimum variance unbiased estimator addresses the precision criteria

## Unbiased Estimator

- The point estimator  $\hat{\theta}$  is an **unbiased estimator** for the parameter  $\theta$  if

$$E(\hat{\theta}) = \theta$$

- If the point estimator is not unbiased, then the difference

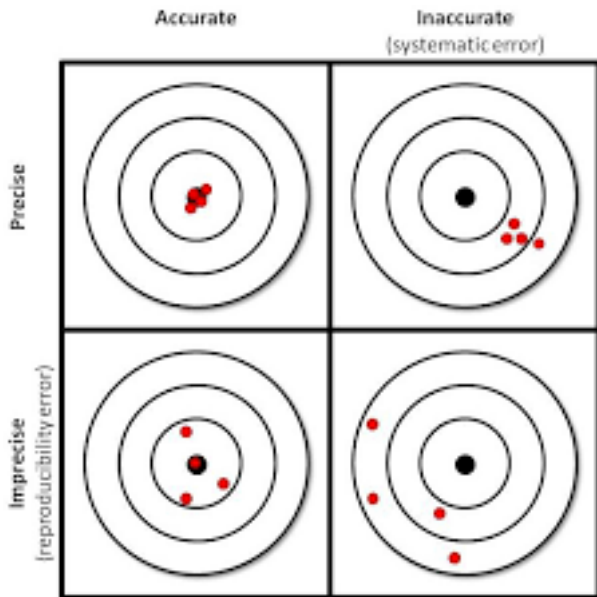
$$E(\hat{\theta}) - \theta$$

is called the **bias** of the estimator  $\hat{\theta}$

## MVUE

- Montgomery gives the following definition of a minimum variance unbiased estimator (MVUE)  
*If we consider all unbiased estimators of  $\theta$ , the one with the smallest variance is called the minimum variance unbiased estimator*
- An import fact is that the sample mean  $\bar{x}$  is the MVUE for  $\mu$  when the data comes from a normal distribution

## Accuracy vs. Precision



## Sampling Distribution

- The probability distribution of a statistic is called a **sampling distribution**

## Central Limit Theorem

- Definition of the Central Limit Theorem is  
*If  $X_1, X_2, \dots, X_n$  is a random sample of size  $n$  taken from a population (either finite or infinite) with mean  $\mu$  and finite variance  $\sigma^2$ , and if  $\bar{X}$  is the sample mean, the limiting form of the distribution of*

$$Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}}$$

*as  $n \rightarrow \infty$ , is the standard normal distribution*

- Important result because for sufficiently large  $n$ , the sampling distribution of  $\bar{X}$  is normally distribution
- This is a fundamental result that will be used extensively in the next four chapters of the textbook.