MANE 3332.05

Lecture 17

Agenda

- Midterm exams are not graded; still contacting students who missed
- Linear Combination Practice Problems (assigned 10/28, due 10/30)
- Chapter Six
- Attendance
- Questions?

Question 1 (1 point)

Listen

Consider the linear combination Y = 7.925+(3.426)*X1+ (-9.744)*X2. The random variables variables X1 and X2 are independent with E(X1)=30.361, E(X2)=38.089, V(X1)=12.724, and V(X2)=16.588. What is the value of E(Y)?

- -157.4197
- -110.116
- -1425.6089
- 1724.3042
- -259.1974
- -267.1224
- The correct answer is not provided.

$$E(Y) = E\left(7.925 + 3.426X_1 + (-9.744)X_2\right)$$

$$= E(7.925) + 3.426E(X_1) - 9.744E(X_2)$$

$$= 7.925 + 3.426(30.361) - 9.744(38.089)$$

$$= -259.1974$$

Question 3 (1 point)

Listen

Consider the linear combination Y = 2.436+(2.359)*X1+(-9.803)*X2. The random variables X1 and X2 are not independent but have COV(X1,X2)=1.421 with E(X1)=20.563, E(X2)=39.131, V(X1)=18.33, and V(X2)=17.443. What is the value of E(Y)?

- -125.3173
- -1574.2473
- 1778.2558
- -332.6571
- 810.7516
- -106.8331
- The correct answer is not provided.

$$E(Y) = E(2.436 + 2.359X_1 - 9.803X_2)$$

$$= 2.436 + 2.359E(X_1) - 9.803E(X_2)$$

$$= 2.436 + 2.359(20.563) - 9.803(39.131)$$

$$= -332.65708$$

Question 5 (1 point)

Listen

Consider the linear combination Y = -1.361+(7.581)*X1+(-6.859)*X2. The random variables X1 and X2 are independent with E(X1)=20.694, E(X2)=47.648, V(X1)=16.33, and V(X2)=12.058. What is the value of V(Y)?

- 371.2314
- -171.2974
- 6.8175
- 1505.7898
- 122.8232
- The correct answer is not provided.

$V(c) = 0, \quad V(cX_1) = c^2 V(X_1) = c^2 \sigma_1^2$

$V(Y) = V(-1.361 + (7.581)X_1 + (-6.859)X_2)$

$= V(-1.361) + V(7.581 X_1) + V(-6.859 X_2)$

$V(-1.361) = 0$

$= (7.581)^2 V(X_1) + (-6.859)^2 V(X_2)$

$= (7.581)^2 (16.33) + (-6.859)^2 (12.058)$

$= 1505.7898$

Question 7 (1 point)

Listen

Consider the linear combination Y = -4.136+(1.043)*X1+(-2.103)*X2. The random variables X1 and X2 are not independent but have COV(X1,X2)=-8.368 with E(X1)=39.174, E(X2)=47.585, V(X1)=16.642, and V(X2)=19.114. What is the value of V(Y)?

- 102.6377
- 65.9285
- -63.3488
- 139.347
- -66.4298
- The correct answer is not provided.

$V(c_0 + c_1X_1 + c_2X_2) = c_1^2V(X_1) + c_2^2V(X_2) + 2c_1c_2Cov(X_1,X_2)$

$V(Y) = V(-4.136 + 1.043X_1 + (-2.103)X_2)$

$= V(1.043X_1) + V(-2.103X_2) + 2(1.043)(-2.103)Cov(X_1,X_2)$

$= 1.043^2V(X_1) + (-2.103)^2V(X_2) + 2(1.043)(-2.103)(-8.368)$

$= 1.043^2(16.642) + (-2.103)^2(19.114) + 2(1.043)(-2.103)(-8.368)$

$= 139.347$

Handouts

- Chapter 6 Slides
- Chapter 6 Slides marked

Numerical Summaries

- Called Descriptive Statistics in Chapter 6
  - Descriptive statistics help us understand the location or central tendency of data and the scatter or variability in data
  - Included in all statistical software packages, R does a good job calculating descriptive statistics

3 Characteristics
1) Location
2) Spread or Variability
3) Shape of data

Central Tendency

- Ostle, et. al. (1996) define central tendency as “the tendency of sample data to cluster about a particular numerical value”
- Population mean

↳ Greek Letters
N

$$\mu = \frac{1}{N}\sum_{i=1}^{N} x_i$$

- Sample mean

xbar

$$\bar{x} = \hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

- Sample median - middle value
- Sample mode - most commonly occuring number(s)

Hat Notation

$\hat{\mu}$ – sample estimator of μ

Measures of Variability

- There are several statistics that measure the variability or spread present in data
- Population variance

$$\sigma^2 = \frac{\sum_{i=1}^{N}(x_i - \mu)^2}{N}$$

σ_N

- Sample variance

$$s^2 = \hat{\sigma}^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

σ_{n-1}

- Shortcut (Computational) Formula

$$s^2 = \frac{\sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}}{n-1}$$

- Standard deviation is often used because it is measured in the original units

$$\sigma = \sqrt{\sigma^2};\ s = \sqrt{s^2}$$

R Function Summary - Data Frame

R code

`summary(midterm)`

Output is from Spring 2024 results

```
26 ```{r}
27 summary(midterm)
28 ```
```

28:4 Import Datset R Markdown

Console | Terminal | Render | Background Jobs

R 4.3.1 · /Volumes/SAMSUNG T7/wfscsBackup/Teaching2/AY_2023_2024/MANE3332_spring2024/PartTwo/l

```
> knitr::opts_chunk$set(echo = TRUE)
> library(readxl)
> midterm <- read_excel("/Volumes/NO NAME/midterm.xlsx")
> View(midterm)
> summary(midterm)
 Participation      QuizAverage      MidtermExam
 Min.   :  2.941   Min.   :  0.00   Min.   :28.00
 1st Qu.: 67.500   1st Qu.: 60.00   1st Qu.:59.75
 Median : 87.941   Median : 80.00   Median :65.00
 Mean   : 77.096   Mean   : 74.42   Mean   :66.07
 3rd Qu.: 95.147   3rd Qu.: 92.00   3rd Qu.:74.75
 Max.   :100.000   Max.   :100.00   Max.   :92.00
                                    NA's   :5
>
```

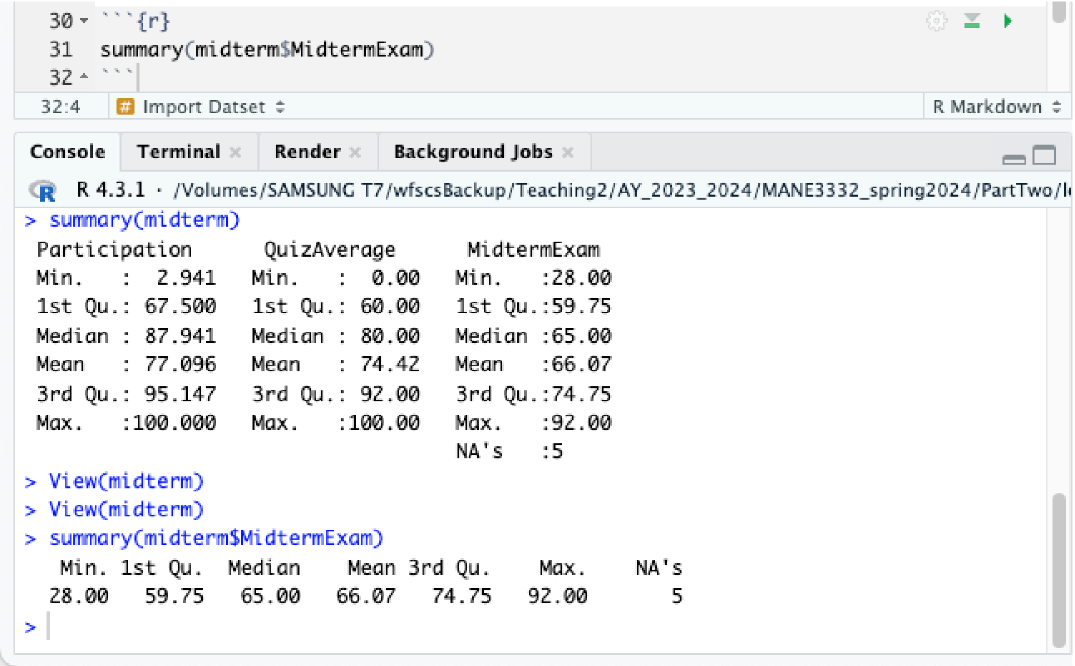
Descriptive Statistics

R Function Summary - Variable

R code

`summary(midterm$MidtermExam)`

Output is from Spring 2024 results



Descriptive Statistics

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$$

R Function Describe

Summary() does not report variability

Describe() has to be imported

Describe() is part of the package psych

R Code for descriptive statistics using psych package

```
library(psych)
describe(midterm)
```

Psych package output from Spring 2024

```{r}
library(psych)
describe(midterm)
```

Description: df [3 × 13]

	vars	n	mean ($\bar{x}$)	sd (s)	median ($\tilde{x}$)	trimmed (trimmed mean)	mad
	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
Participation	1	33	77.10	25.65	87.94	81.35	16.13
QuizAverage	2	33	74.42	23.49	80.00	77.48	23.72
MidtermExam	3	28	66.07	13.73	65.00	66.62	13.34

3 rows | 1-8 of 13 columns

mad

mean absolute deviation

$$mad = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$$

Describe() Output

Range = max − min

Describe Output, part 2

$\Sigma (x_i - \bar{x})^3$

Skewness − 3rd Moment about mean

Kurtosis 4th moment

Description: df [3 x 13]

median	trimmed	mad	min	m...	range	skew	kurtosis	se
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
87.94	81.35	16.13	2.94	100	97.06	-1.31	0.79	4.47
80.00	77.48	23.72	0.00	100	100.00	-1.24	1.28	4.09
65.00	66.62	13.34	28.00	92	64.00	-0.46	0.39	2.59

3 rows | 6-14 of 13 columns

Se − Standard error

$$\frac{s}{\sqrt{n}}$$

Describe Output

Calculating Quantiles

Attendance 1-A

2.3.2 Sample Quantiles

In Example 2.8, we consider an ogive for the plated bracket data. The point (1.55, 0.567) is on that ogive, so we estimate that 56.7% of the sampled population of brackets weighed at most 1.55 ounces. Weights associated with other percentages can also be estimated by locating the appropriate point on the ogive. In general, if the point (x, p) is on the ogive, we can use x as an estimate of the weight with $100p$% of the population values at or below it. This estimate, called the 100*p*th *sample quantile*, is denoted x_p.

If two persons (or computer programs) use different groupings to obtain an ogive, the resulting quantiles will differ. To remedy this deficiency, an algebraic procedure is required.

THE 100pth SAMPLE QUANTILE

Several definitions of sample quantiles are used. We use the one that agrees with the default values output by the UNIVARIATE procedure in SAS®. Also, the definition used here is consistent with our definition of the sample median.

Suppose a sample of size n is obtained from some population associated with a continuous variable. For $0 < p < 1$, let $p(n + 1) = i + d$, with i the integer part of $p(n + 1)$ and $0 \le d < 1$ the decimal part. If $1 \le i < n$, and $d = 0$, the 100*p*th sample quantile is $x_{(i)}$. If $1 \le i < n$ and $0 < d < 1$, interpolate linearly between $x_{(i)}$ and $x_{(i+1)}$. In either case, the 100*p*th *sample quantile* is

$$x_p = x_{(i)} + d\,[x_{(i+1)} - x_{(i)}] \qquad (2.4)$$

when $1 \le i < n$. If $i = 0$ or n, the 100*p*th sample quantile does not exist. If $100p$ is an integer, the corresponding quantile is called a *percentile*.

EXAMPLE 2.18

Suppose we want to find the 43rd percentile of the sample of plated weights in Table 2.1. Since there are $n = 75$ observations in the sample and $p = 0.43$, we find $p(n + 1) = (0.43)(75 + 1) = 32.68$. Letting $i = 32$ and $d = 0.68$, we use Equation (2.4) to obtain $x_{0.43} = x_{(32)} + (0.68)(x_{(33)} - x_{(32)})$. The 32nd ordered value in Figure 2.1(b) is $x_{(32)} = 1.50$ and the 33rd ordered value is $x_{(33)} = 1.51$. Thus, the 43rd percentile for these data is $x_{0.43} = 1.50 + (0.68)(1.51 - 1.50) = 1.5068 \approx 1.507$. Using this as a point estimate of the population percentile, we can say that approximately 43% of the plated brackets produced on the day the data were collected had weights of 1.507 ounces or less. ■

The Sample Median Is a Percentile

Suppose we want to find the 50th percentile and the data set contains n values. When n is even, $(0.50)(n + 1) = (n/2) + (0.50)$, with $n/2$ a positive integer. Using Equation (2.4) with $i = n/2$ and $d = 0.50$, $x_{0.50} = x_{(i)} + (0.50)[x_{(i+1)} - x_{(i)}] = [x_{(i)} + x_{(i+1)}]/2$. When n is odd, $(0.50)(n + 1) = (n + 1)/2$, with $(n + 1)/2$ a positive integer. Using Equation (2.4) with $i = (n + 1)/2$ and $d = 0$, we find $x_{0.50} = x_{(i)}$. But, this is precisely how the sample median was defined. Thus, $\tilde{x} = x_{0.50}$.

SAMPLE QUARTILES

The percentiles $x_{0.25}$, $x_{0.50}$, and $x_{0.75}$ are known as the *first*, *second*, and *third sample quartiles*, respectively. These quantities are often denoted q_1, q_2, and q_3.

EXAMPLE 2.19

Consider the plated bracket weights in Table 2.1. Using the ordered stem-and-leaf display presented in Figure 2.1(b), we find the following.

(a) *First Quartile:* Since $(0.25)(75 + 1) = 19$, $q_1 = x_{0.25} = x_{(19)} = 1.46$.

(b) *Second Quartile (Median):* Since $(0.50)(75 + 1) = 38$, $q_2 = \tilde{x} = x_{0.50} = x_{(38)} = 1.53$.

reference for calculating quantiles

Quantile Example

8 Observations from binomial distribution with $n = 10$ and $p = .5$.

6, 4, 5, 7, 3, 5, 4, 6

Quantile Example

Exploratory Data (Graphical) Analysis

- Exploratory data analysis (EDA) is the use of graphical procedures to analyze data.
- John Tukey was a pioneer in this field and invented several of the procedures
- Tools include stem-and-leaf diagrams, box plots, time series plots and digidot plots

Stem and Leaf Diagram

- Excellent tool that maintains data integrity
- The stem is the leading digit or digits
- The leaf is the remaining digit
- Make sure to include units
- R Code

```
stem(midterm$MidtermExam)
```

Stem and Leaf Example

R output of a Stem and Leaf diagram

```
The decimal point is 1 digit(s) to the right of the |

2 | 8
3 |
4 | 4
5 | 11566
6 | 13334446679
7 | 2247
8 | 00147
9 | 2
```

Stem and Leaf Plot of Midterm Exam Scores

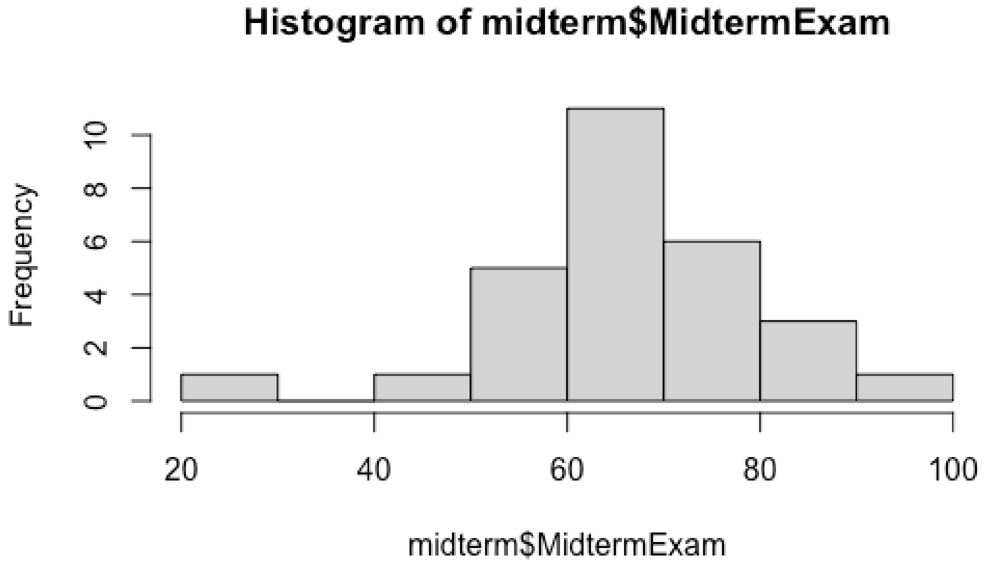
Histogram

- A histogram is a barchart displaying the frequency distribution information
- There are three types of histograms: frequency, relative frequency and cumulative relative frequency
- R code

```
hist(midterm$MidtermExam)
```

Histogram Example

R output of histogram



Histogram of Midterm Exam Scores

Boxplot

Graphical display that simultaneously describes several important features of a data set such as center, spread, departure from symmetry and outliers

Requires the calculation of quantiles (quartiles)

Box Plot 1

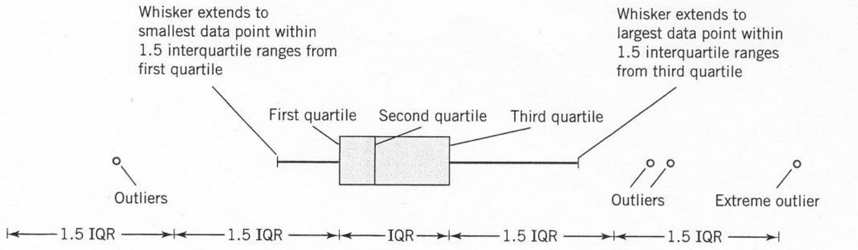


Figure 2-11 Description of a box plot.

Box plot with explanation

Box Plot 2

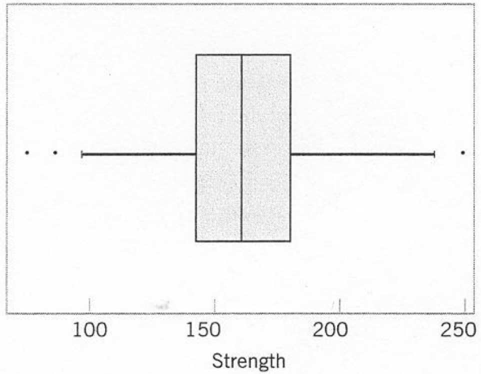


Figure 2-12 Box plot for compressive strength data in Table 2-2.

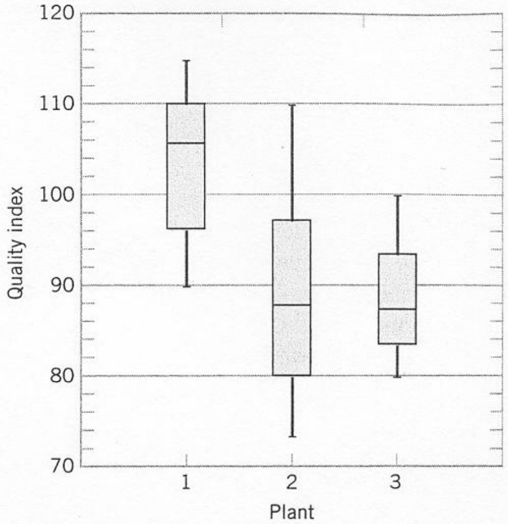


Figure 2-13 Comparative box plots of a quality index at three plants.

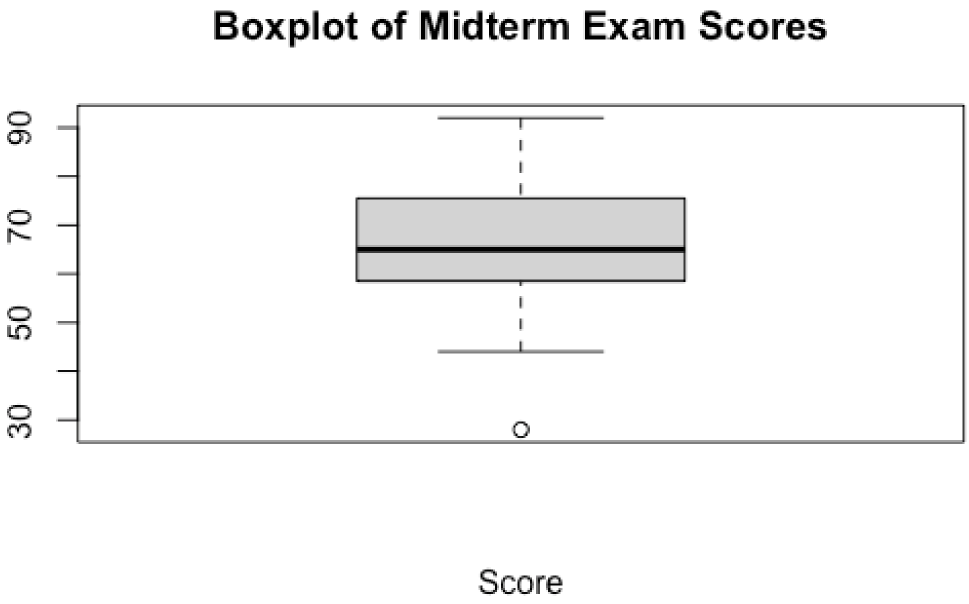
examples of boxplots

Box Plot 3

R code for Box Plot

```
boxplot(midterm$MidtermExam,xlab='Score',main='Boxplot of Midterm Exam Scores')
```

R Box Plot output



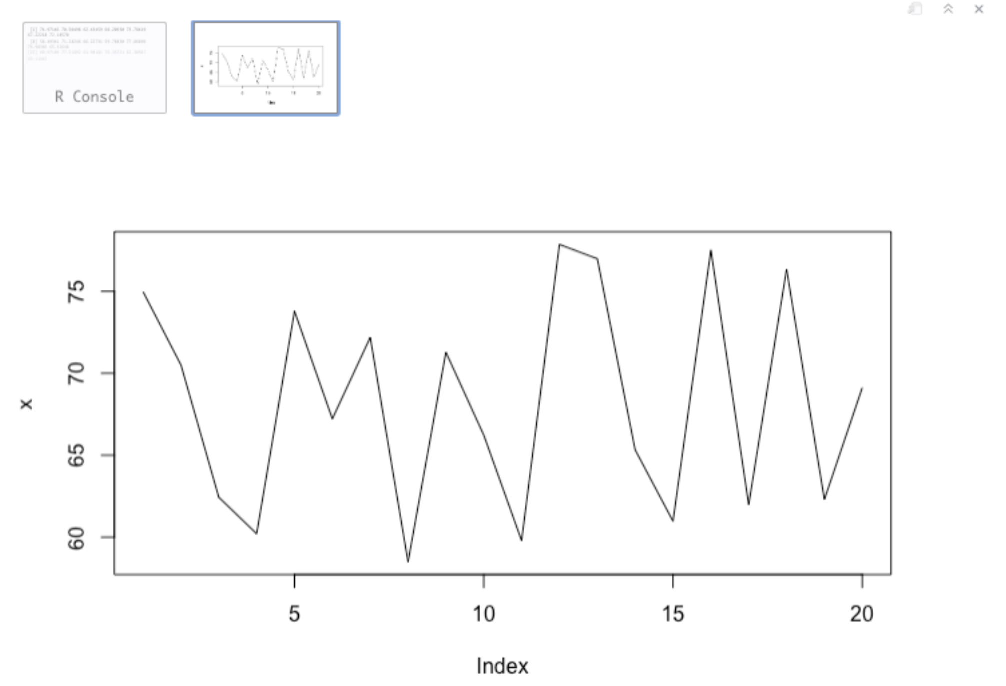
Boxplot of Midterm Exam Scores

Time Series Plot

- A **time series plot** is a graph in which the vertical axis denotes the observed value of the variable (say x) and the horizontal axis denotes time
- Excellent tool for detecting:
  - trends,
  - cycles,
  - other non-random patterns

Time Series Plot in R

```{r}
x<-rnorm(20,70,10)
print(x)
plot(x,type='l')
```



Time Series Plot

Probability Plotting

- **Probability plotting** is a graphical method of determining whether sample data conform to a hypothesized distribution
- Used for validating assumptions
- Alternative to hypothesis testing

Construction

1. Sort the data from smallest to largest, .

2. $x_{(1)}, x_{(2)}, \ldots, x_{(n)}$

3. Calculate the observed cumulative frequency $(j - 0.5)/n$

For the normal distribution find z_j that satisfies

$$\frac{j - 0.5}{n} = P(Z \leq z_j) = \Phi(z_j)$$

3. Plot z_j versus $x_{(j)}$ on special graph paper

Usage

If the data plots as a straight line, the assumed distribution is correct

6-7 PROBABILITY PLOTS 215

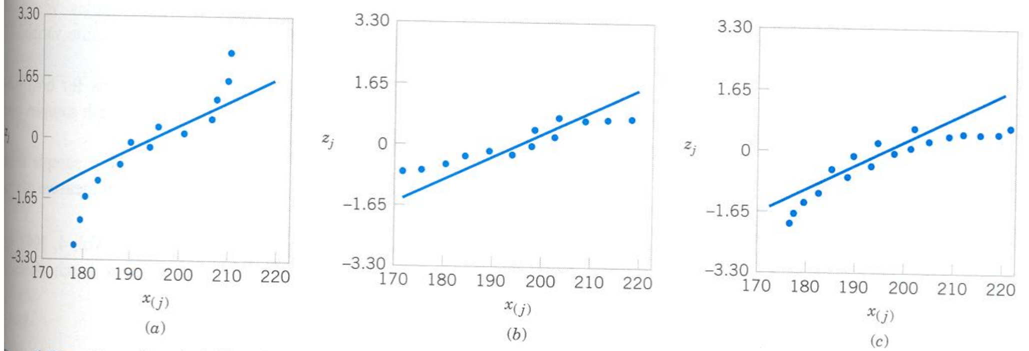
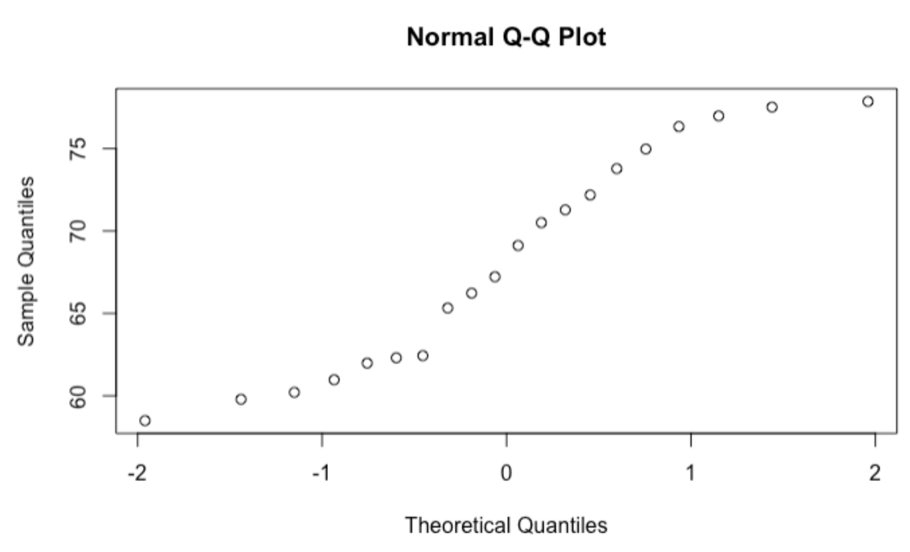


Figure 6-21 Normal probability plots indicating a nonnormal distrubution. (a) Light-tailed distribution. (b) Heavy-tailed distribution. (c) A distribution with positive (or right) skew.

normal probability plots from textbook, figure 6.21 on page 215

Probability Plot Example 1 in R

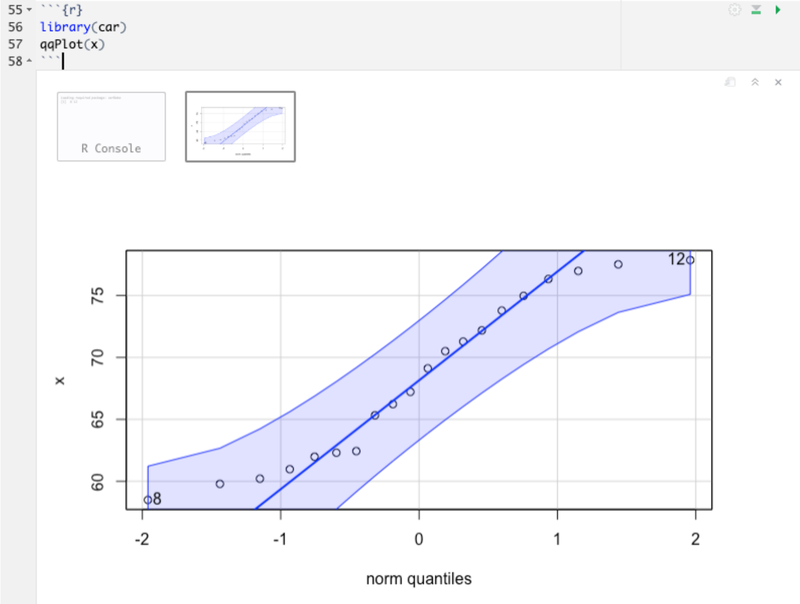
````
```{r}
qqnorm(x)
```
````



Normal Probability Plot

Probability Plot Example 2

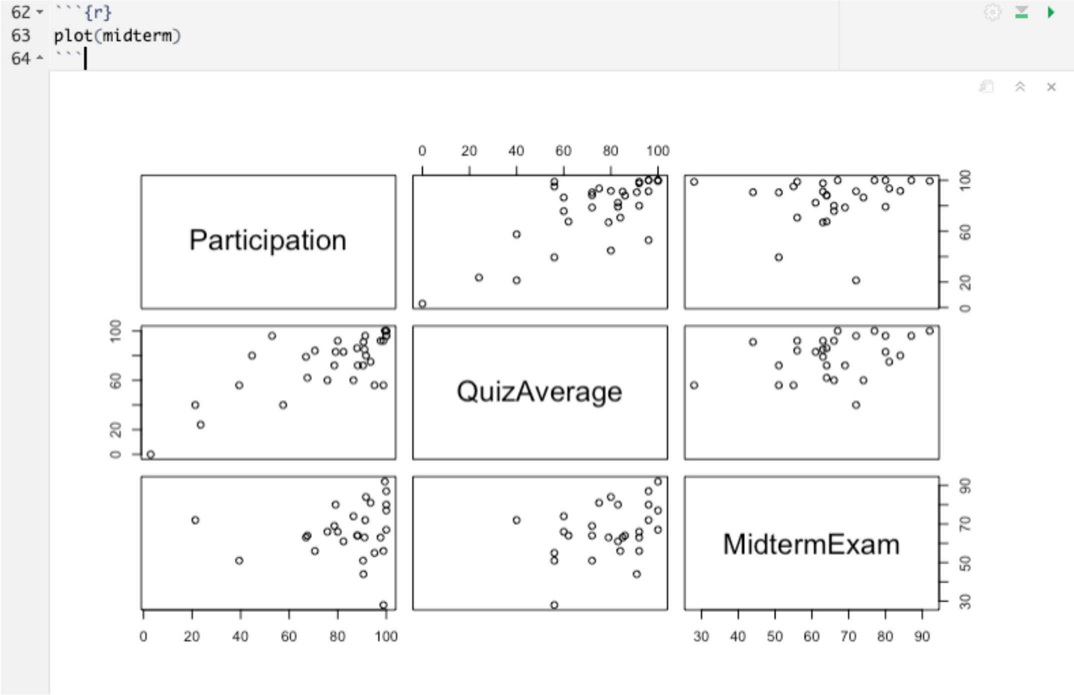
- Difficulty from example one is how close to straight is "good enough"
- Add confidence bands to normal probability plot
  - Requires package car to be added to R
  - If all points are within the band, we are 95% confident that the sample is from a normal distribution. However if one or more points are not within band, the data is not from a normal distribution



QQ Plot with band

Multivariate Data

Matrix of Scatter Plot in R



Scatter Plots

Covariance in R

```{r}
midterm_NA <- na.omit(midterm)
print(cov(midterm_NA))
```

```
             Participation QuizAverage MidtermExam
Participation    340.16778    193.7847    28.75699
QuizAverage      193.78474    269.0899    81.17460
MidtermExam       28.75699     81.1746   188.43915
```

Covariance Matrix

Correlation

```{r}
print(cor(midterm_NA))
```

```
              Participation QuizAverage MidtermExam
Participation     1.0000000   0.6405076   0.1135825
QuizAverage       0.6405076   1.0000000   0.3604839
MidtermExam       0.1135825   0.3604839   1.0000000
```

Correlation Matrix