Attendance

1 - E

MANE 3332.05

Lecture 20

Agenda

- Haven’t started partial credit requests yet
- Start Chapter 8
- Chapter 8, Case 1 Practice Problems (assigned 11/6/2025, due 11/11/2025)
- Attendance
- Questions?

Handouts

- Lecture 20 Slides
- Lecture 20 Slides - marked

Week	Tuesday Lecture	Thursday Lecture
10	**11/4** - Return Midterm	**11/6** - Chapter 8 (part 1)
11	**11/11** - Chapter 8 (part 2)	**11/13** - Chapter 8 (part 3)
12	**11/18** - Chapter 8 (part 4)	**11/25** - Chapter 9 (part 1)
13	**11/25** - Chapter 9 (part 2)	**11/27** - Thanksgiving Holiday (no class)
14	**12/2** - Chapter 9 (part 3)	**12/4** - Linear Regression
15	**12/9** - Review Session	**12/11** - Study Day (no class)

The final exam for MANE 3332.01 is **Thursday December 18, 2025 at 1:15 PM - 3:00 PM**.

Chapter 8 Introduction

- Intervals are another method of performing estimation
- A confidence interval is an interval estimate on a population parameter (primary focus of this chapter)
- Three types of interval estimates
  - A confidence intervals bounds population or distribution parameters
  - A tolerance interval bounds a selected proportion of a distribution
  - A prediction interval bounds future observations from the population or distribution
- Interval estimates, especially confidence intervals are commonly used in science and engineering

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image

Confidence Interval on the Mean of a normal distribution, variance known (Case 1)

- Suppose that $X_1, X_2, \ldots, X_n$ is a random sample from a normal population with unknown mean μ and known variance σ^2
- A general expression for a confidence interval is (L, U)

$$P[L \leq \mu \leq U] = 1 - \alpha$$

- Using the sample results we calculate a $100(1-\alpha)\%$ confidence of the form

$$\rightarrow \bar{x}, n \qquad l \leq \mu \leq u$$

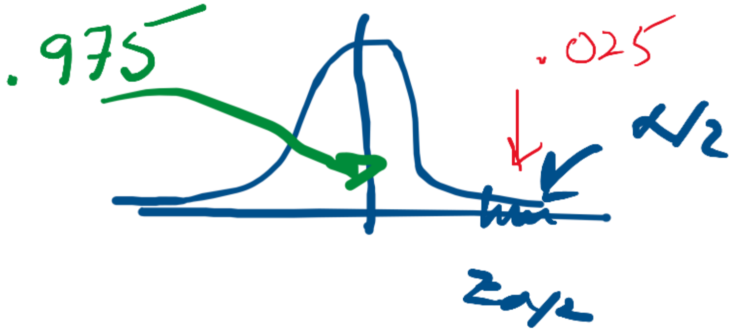
- A $100(1-\alpha)\%$ confidence interval for the mean of a normal distribution with variance known is

$$l = \bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = u$$

where $z_{\alpha/2}$ is the upper $100\alpha/2$ percentage point of the standard normal distribution.

$Z_{\alpha/2}$

95% C.I.



$100(1-\alpha)\% = 95\%$

$(1-\alpha) = .95$

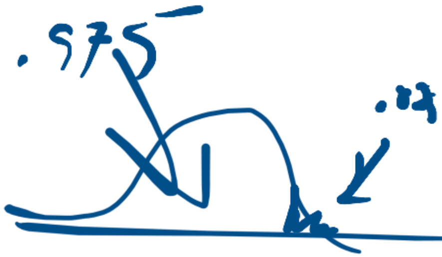
$\alpha = .05$

$Z_{\alpha/2} \to Z_{.05/2}$

$Z_{.025}$

TABLE III **Cumulative Standard Normal Distribution** *(continued)*

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738
2.2	0.986097	0.986447	0.986791	0.987126	0.987455	0.987776	0.988089	0.988396	0.988696	0.988989
2.3	0.989276	0.989556	0.989830	0.990097	0.990358	0.990613	0.990863	0.991106	0.991344	0.991576
2.4	0.991802	0.992024	0.992240	0.992451	0.992656	0.992857	0.993053	0.993244	0.993431	0.993613
2.5	0.993790	0.993963	0.994132	0.994297	0.994457	0.994614	0.994766	0.994915	0.995060	0.995201
2.6	0.995339	0.995473	0.995604	0.995731	0.995855	0.995975	0.996093	0.996207	0.996319	0.996427
2.7	0.996533	0.996636	0.996736	0.996833	0.996928	0.997020	0.997110	0.997197	0.997282	0.997365
2.8	0.997445	0.997523	0.997599	0.997673	0.997744	0.997814	0.997882	0.997948	0.998012	0.998074
2.9	0.998134	0.998193	0.998250	0.998305	0.998359	0.998411	0.998462	0.998511	0.998559	0.998605
3.0	0.998650	0.998694	0.998736	0.998777	0.998817	0.998856	0.998893	0.998930	0.998965	0.998999
3.1	0.999032	0.999065	0.999096	0.999126	0.999155	0.999184	0.999211	0.999238	0.999264	0.999289
3.2	0.999313	0.999336	0.999359	0.999381	0.999402	0.999423	0.999443	0.999462	0.999481	0.999499
3.3	0.999517	0.999533	0.999550	0.999566	0.999581	0.999596	0.999610	0.999624	0.999638	0.999650
3.4	0.999663	0.999675	0.999687	0.999698	0.999709	0.999720	0.999730	0.999740	0.999749	0.999758
3.5	0.999767	0.999776	0.999784	0.999792	0.999800	0.999807	0.999815	0.999821	0.999828	0.999835
3.6	0.999841	0.999847	0.999853	0.999858	0.999864	0.999869	0.999874	0.999879	0.999883	0.999888
3.7	0.999892	0.999896	0.999900	0.999904	0.999908	0.999912	0.999915	0.999918	0.999922	0.999925
3.8	0.999928	0.999931	0.999933	0.999936	0.999938	0.999941	0.999943	0.999946	0.999948	0.999950
3.9	0.999952	0.999954	0.999956	0.999958	0.999959	0.999961	0.999963	0.999964	0.999966	0.999967



$Z_{.025} = 1.96$

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Given/Facts

$\sigma = 25$

Problem 8-12,part a (6th edition)

$n = 20$

$\bar{x} = 1014$

95% c.i.

$Z_{\alpha/2} = Z_{.025}$

$= 1.96$

$$\bar{x} - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$100(1-\alpha)\% = 95\%$ or $(1-\alpha) = .95 \rightarrow \alpha = .05$

$$1014 - 1.96\frac{25}{\sqrt{20}} \le \mu \le 1014 + 1.96\frac{25}{\sqrt{20}}$$

$$1003.043 \le \mu \le 1024.9567$$

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Interpreting Confidence Intervals

Montgomery gives the following statement regarding the correct interpretation of confidence intervals.

> The correct interpretation lies in the realization that a CI is a random interval because in the probability statement defining the end-points of the interval, L and U are random variables. Consequently, the correct interpretation of a $100(1-\alpha)$% CI depends on the relative frequency view of probability. Specifically, if an infinite number of random samples are collected and a $100(1-\alpha)$% confidence interval for μ is computed from each sample, $100(1-\alpha)$% of these intervals will contain the true value of μ.

One-sided Confidence Bounds

It is possible to construct on-sided confidence bounds

- A $100(1-\alpha)\%$ upper-confidence bound for μ is

$$\mu \leq u = \bar{x} + z_{\alpha}\frac{\sigma}{\sqrt{n}}$$

- A $100(1-\alpha)\%$ lower-confidence bound for μ is

$$\bar{x} - z_{\alpha}\frac{\sigma}{\sqrt{n}} = l \leq \mu$$

Sample Size Considerations

If $\bar{x}$ is used as an estimate of μ, we can be $100(1-\alpha)\%$ confident that the error $|\bar{x}-\mu|$ will not exceed a specified amount E when the sample size is

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$$

round up

$L \le \mu \le U$

$\bar{X} - Z_\alpha \sigma/\sqrt{n} \le \mu$

95% c.b. $\rightarrow \alpha = .05$

$t_{\alpha,\infty} = Z_\alpha$

$Z_{.05} = 1.645$

$1014 - 1.645 \frac{25}{\sqrt{20}} \le \mu$

$1006.481 \le \mu$

image

A Large Sample CI for μ

- When n is large (say greater than or equal to 40), the central limit theorem can be used
- It states that $\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ is approximately a standard normal random variable.
- Thus, we can replace the quantity $\sigma/\sqrt{n}$ with $S/\sqrt{n}$ and still use the quantiles of the normal distribution to construct a confidence interval.

$$\bar{x} - z_{\alpha/2}\frac{S}{\sqrt{n}} \leq \mu \leq \bar{x} + z_{\alpha/2}\frac{S}{\sqrt{n}}$$

- What assumption did we relax and why?

Chapter 8, Case 1 Practice Problems

5 Questions

3 → 2-sided c.i.

1 → upper c.b.

1 → lower c.b.

Consider a sample of size 30 from a normal distribution with mean 0.5 and sigma 0.06. What is the value of a 90.0 % lower-confidence bound for the mean?

- mu>=0.499
- mu >=0.497
- mu>=0.5
- mu>=0.5
- mu >=0.482
- mu >=0.486

$n = 30$

$\bar{x} = .5$

$\sigma = .06$

90% lower bound

$100(1-\alpha)\% = 90\%$

$(1-\alpha) = .9 \rightarrow \alpha = .1$

$L \le x$

$z_{.1} = 1.282$ (from z-table)

$\bar{x} - z_{\alpha} \sigma/\sqrt{n} \le \mu$

$.5 - 1.282 \frac{.06}{\sqrt{30}} \le \mu$

$.48596 \le \mu$

Consider a sample of size 45 from a normal distribution with mean 18.3 and sigma 4.3. What is the value of a 95.0 % upper-confidence bound for the mean?

- mu <=18.457
- mu<=19.105
- mu <=19.354
- mu<=22.834
- mu <=19.556
- mu<=18.976

$n = 45$

$\bar{x} = 18.3$

$\sigma = 4.3$

$z_{.05} = 1.645$

95% upper c.b.

$\alpha = .05$

$\mu \le u$

$\mu \le \bar{x} + z_\alpha \sigma/\sqrt{n}$

$\mu \le 18.3 + 1.645 \frac{4.3}{\sqrt{45}}$

$\mu \le 19.35446$

Consider a sample of size 23 from a normal distribution with mean 31.8 and sigma 3.62. What is the value of a two-sided 99.9 % confidence interval for the mean?

- (31.282,32.318)
- (29.925,33.675)
- (29.467,34.133)
- (29.316,34.284)
- (22.809,40.791)
- (30.039,33.561)

what is the value of α?

99.9% → .999 $\alpha = 1 - .999 = .001$

$Z_{.001/2} = Z_{.0005} = 3.291$

Confidence Interval for the mean of Normal distribution with variance unknown (Case 2)

The t distribution

- Definition.

Let $X_1, X_2, \dots, X_n$ be a random sample from a normal distribution with unknown mean μ and unknown variance σ^2. The random variable

$$T = \frac{\overline{X} - \mu}{S/\sqrt{n}}$$

has a t distribution with $n - 1$ degrees of freedom.

- A table of percentage points (quantiles) is given in Appendix A Table 5
- Figure 8-4 on page 180 shows the relationship between the t and normal distributions.
- Figure 8-5 explains the percentage points of the t distribution

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Confidence interval definition

- Using the t distribution it is possible to construct CIs

If $\bar{x}$ and s are the mean and standard deviation of a random sample from a normal distribution with unknown variance σ^2, a $100(1-\alpha)\%$ confidence interval on μ is given by

$$\bar{x} - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}} \leq \mu \leq \bar{x} + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}$$

where $t_{\alpha/2,n-1}$ is the upper $100(\alpha/2)$ percentage point of the t distribution with $n-1$ degrees of freedom.

Problem 8-30 (6th edition)

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One-sided confidence bounds

- Are easy to construct
- Use only the appropriate upper or lower bound
- Change $t_{\alpha/2,n-1}$ to $t_{\alpha,n-1}$

Chapter 8, Case 2 Practice Problems

Confidence Interval for σ^2 and σ (Case 3)

- Section 8-3 presents a CI for σ^2 or σ
- Requires the χ^2 (chi-squared) distribution

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with mean μ and variance σ^2 and let S^2 be the sample variance. Then the random variable

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-square (χ^2) distribution with $n-1$ degrees of freedom

- A table of the upper percentage points of the χ^2 distribution are given in Table 4 in the appendix
- Figure 8-9 on page 183 explains the percentage points of the χ^2 distribution

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Confidence Intervals for σ^2 and σ

If s^2 is the sample variance from a random sample of n observations from a normal distribution with unknown variance σ^2, then a $100(1-\alpha)\%$ confidence interval on σ^2 is

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} \leq \sigma^2 \leq \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

where $\chi^2_{\alpha/2,n-1}$ and $\chi^2_{1-\alpha/2,n-1}$ are the upper and lower $100\alpha/2$ percentage points of the χ^2-distribution with $n-1$ degrees of freedom

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Problem 8-36 (6th edition)

image

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One-sided confidence bounds

- Are easy to construct
- Use only the appropriate upper or lower bound
- Change $\chi^2_{\alpha/2,n-1}$ to $\chi^2_{\alpha,n-1}$ or $\chi^2_{1-\alpha/2,n-1}$ to $\chi^2_{1-\alpha,n-1}$
- See eqn (8-20) on page 184

Chapter 8, Case 3 Practice Problems

Large-Sample CI for a Population Proportion (Case 4)

- Recall from chapter 4, that the sampling distribution of $\hat{P}$ is approximately normal with mean p and variance $p(1-p)/p$, if n is not too close to either 0 or 1 and if n is relatively large.
- Typically, we require both $np \geq 5$ and $n(1-p) \geq 5$

f n is large, the distribution of

$$Z = \frac{X - np}{\sqrt{np(1-p)}} = \frac{\hat{P} - p}{\sqrt{\frac{p(1-p)}{n}}}$$

is approximately standard normal.

If $\hat{p}$ is the proportion of observations in a random sample of size n that belongs to a class of interest, an approximate $100(1-\alpha)\%$ confidence interval on the proportion p of the population that belongs to this class is

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

where $z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution

Other Considerations

- We can select a sample so that we are $100(1-\alpha)\%$ confident that error $E = |p - \hat{P}|$ using

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p)$$

- An upper bound on is given by

$$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 (0.25)$$

- One-sided confidence bounds are given in eqn (8-26) on page 187

Guidelines for Constructing Confidence Intervals

- Review excellent guide given in Table 8-1

Other Interval Estimates

- When we want to predict the value of a single value in the future, a **prediction interval** is used
- A **tolerance interval** captures $100(1-\alpha)\%$ of observations from a distribution

Prediction Interval for a Normal Distribution

- Excellent discussion on pages 189 - 190
- A $100(1-\alpha)\%$ PI on a single future observation from a normal distribution is given by

$$\bar{x} - t_{\alpha/2,n-1}s\sqrt{1+\frac{1}{n}} \le X_{n+1} \le \bar{x} + t_{\alpha/2,n-1}s\sqrt{1+\frac{1}{n}}$$

Tolerance Intervals for a Normal Distribution

- A **tolerance interval** to contain at least $\gamma\%$ of the values in a normal population with confidence level $100(1-\alpha)\%$ is

$$\bar{x}-ks, \bar{x}+ks$$

where k is a tolerance interval factor for the normal distribution found in appendix A Table XII. Values are given for $1-\alpha$=0.9, 0.95 and 0.99 confidence levels and for $\gamma = .90, .95$, and $.99\%$ probability of coverage

- One-sided tolerance bounds can also be computed. The factors are also in Table XII